# 小型空調系統設計

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# Psychrometrics
# - Properties of Moisture Air

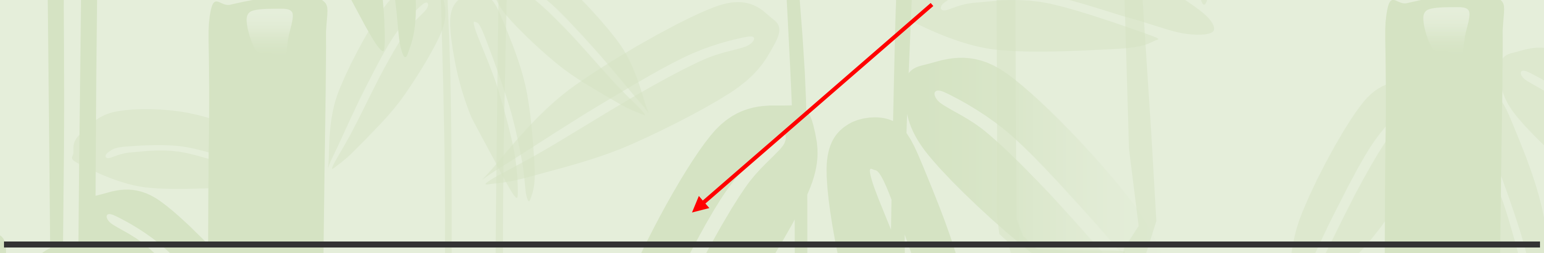
Psychrometrics is the relationship of the physical and thermal properties of an air-vapor mixture.

Water vapor is moisture in the air.

Dry air has no moisture.

# 1. What is a dry bulb temperature?

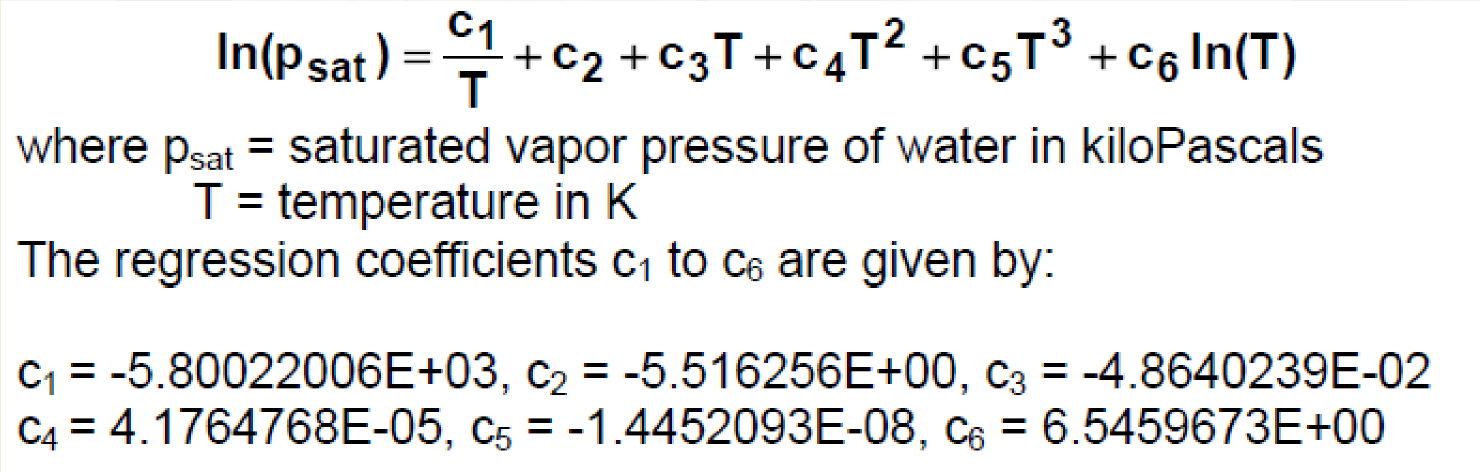
- A. It is measured with an ordinary thermometer.
- B. It is independent of moisture.
- C. It is located on the "X" axis of a psychometric chart.



0 $^0$C DRY BULB 35 $^0$C

# Important psychrometric properties

- Dry bulb temperature (DBT) is the temperature of the moist air as measured by a standard thermometer or other temperature measuring instruments.
- Saturated vapor pressure ($p_{sat}$) is the saturated partial pressure of water vapor at the dry bulb temperature. This is readily available in thermodynamic tables and charts. ASHRAE suggests the following regression equation for saturated vapor pressure of water, which is valid for 0 to 100$^{o}$C.



$$\ln(p_{sat}) = \frac{c_1}{T} + c_2 + c_3 T + c_4 T^2 + c_5 T^3 + c_6 \ln(T)$$

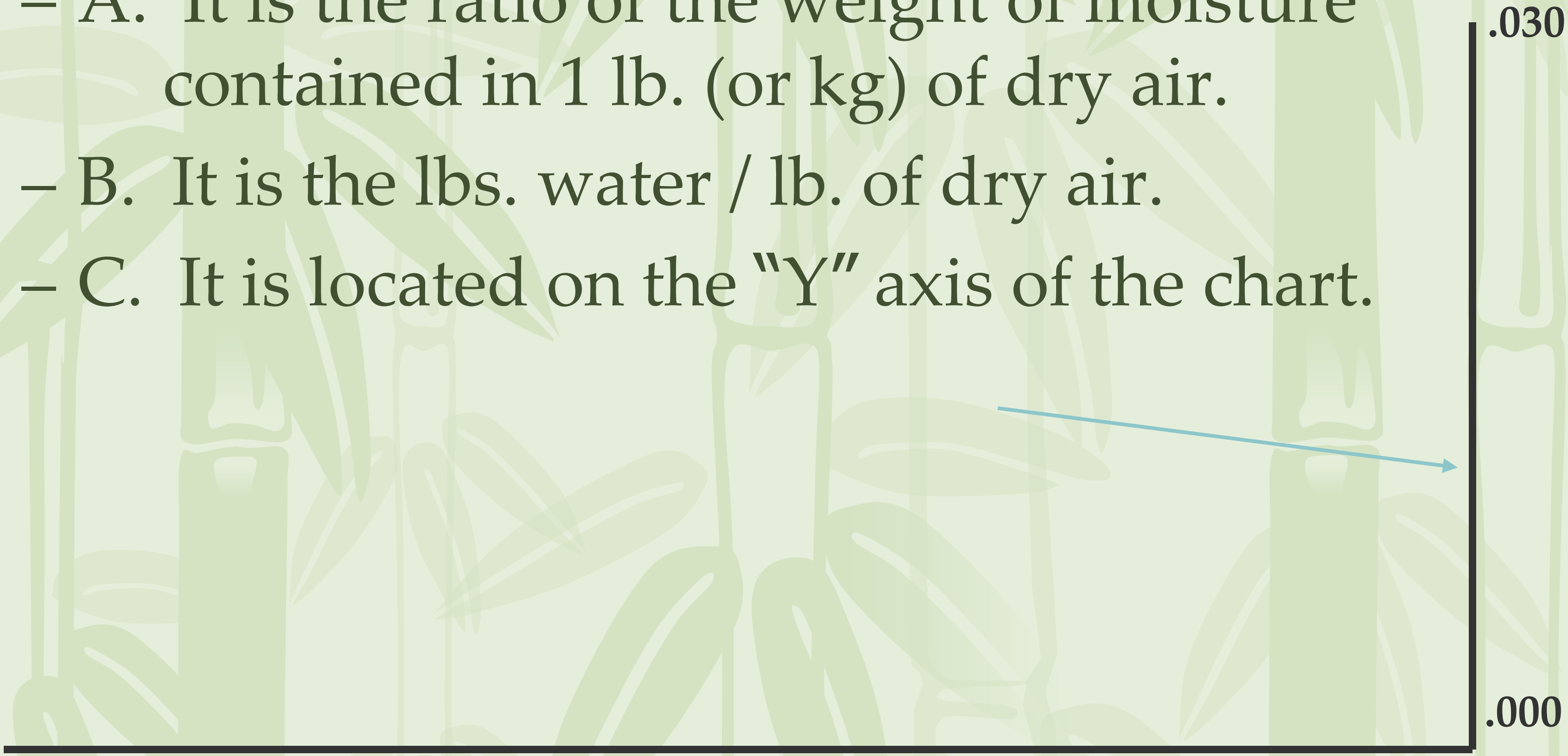
where $p_{sat}$ = saturated vapor pressure of water in kiloPascals
$T$ = temperature in K

The regression coefficients $c_1$ to $c_6$ are given by:

$c_1$ = -5.80022006E+03, $c_2$ = -5.516256E+00, $c_3$ = -4.8640239E-02
$c_4$ = 4.1764768E-05, $c_5$ = -1.4452093E-08, $c_6$ = 6.5459673E+00

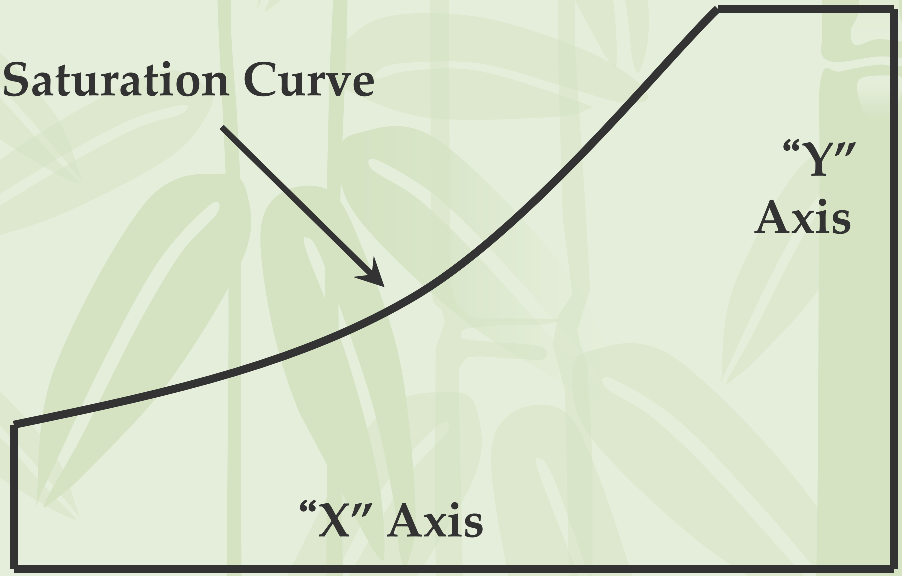
# 2. What is humidity ratio (absolute humidity)?

- A. It is the ratio of the weight of moisture contained in 1 lb. (or kg) of dry air.
- B. It is the lbs. water / lb. of dry air.
- C. It is located on the "Y" axis of the chart.



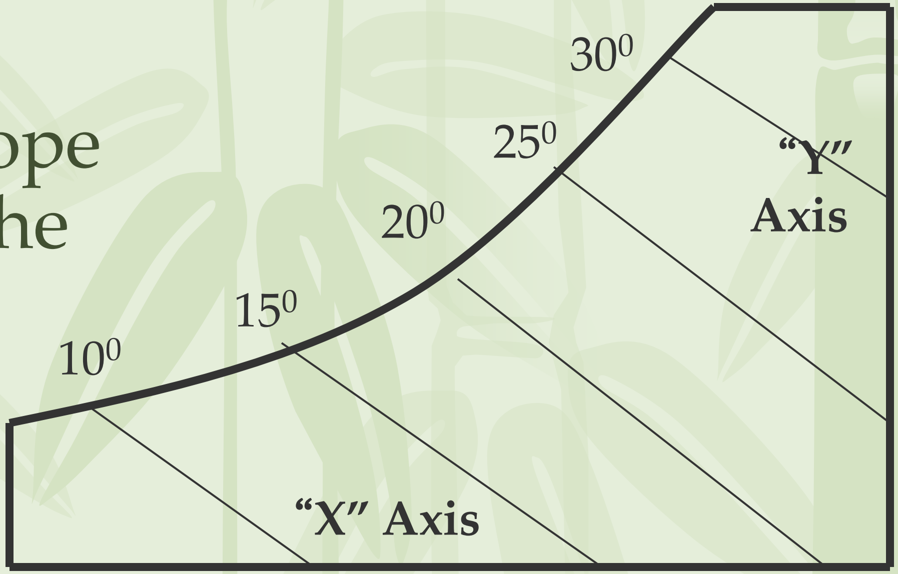
# 3. What is the saturation curve?

- A. It includes the wet bulb and dew point temperatures.
- B. It completes the psychrometric chart outline.

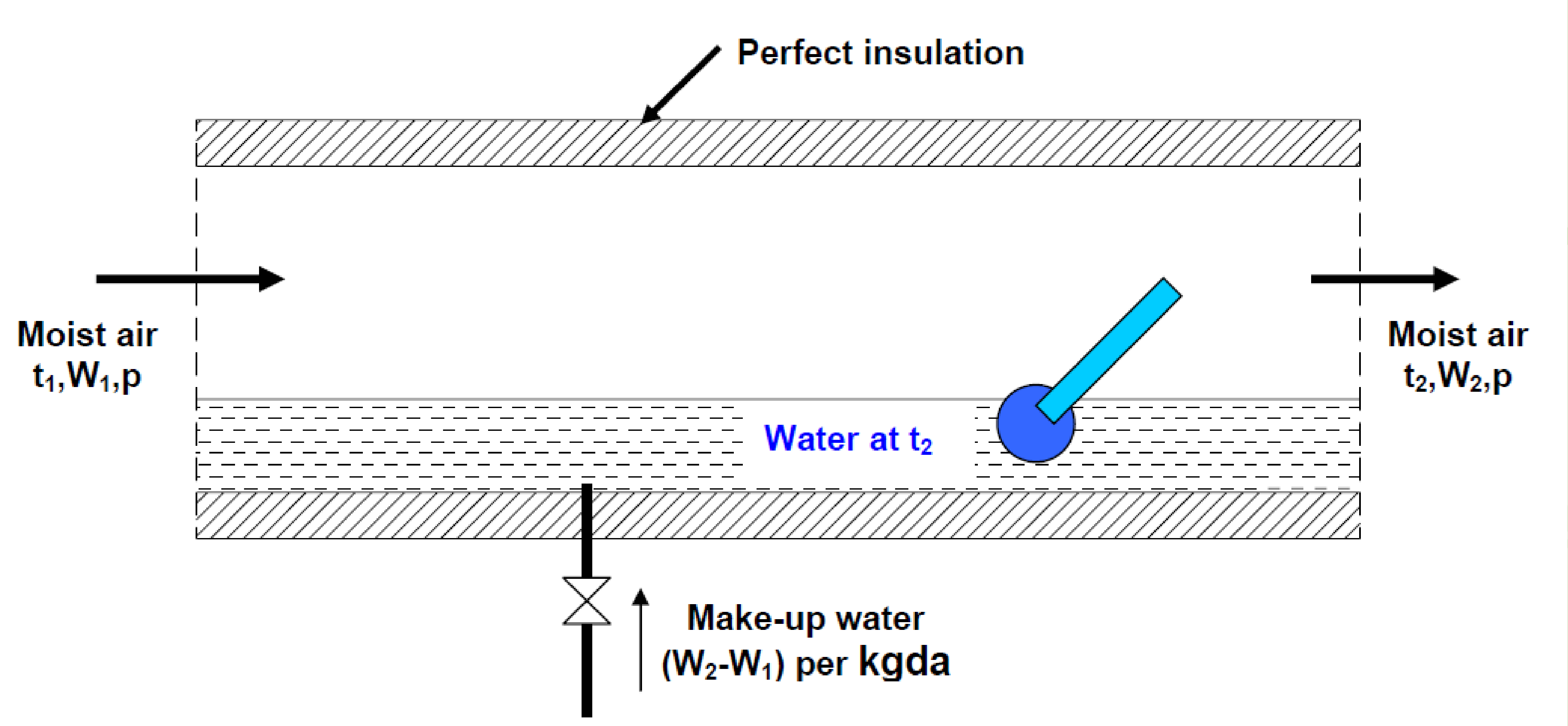


# 4. What is the wet bulb temperature?

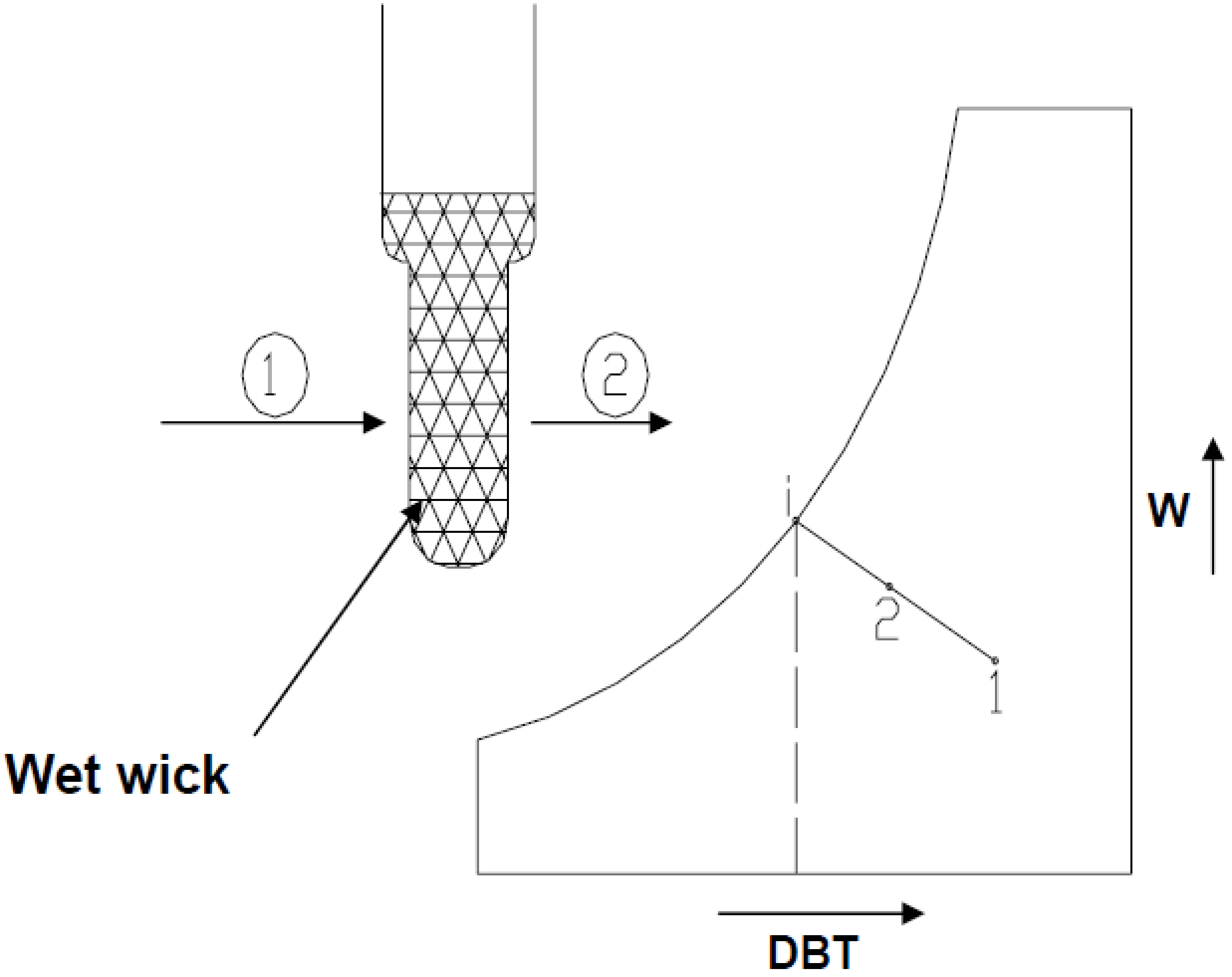
- A. The temperature at which an equilibrium exists between an air-vapor mixture and water.
- B. It is dependent on moisture in the air.
- C. Values are on the saturation curve.
- D. The lines slope downward to the "X" axis.



# Adiabatic saturation and thermodynamic wet bulb temperature



# Wet-Bulb Thermometer



# Basic gas laws for moist air

$$p_1 = \frac{n_1 R_u T}{V};\ p_2 = \frac{n_2 R_u T}{V};\ p_3 = \frac{n_3 R_u T}{V}\ \ldots\ldots$$

$$p_t = p_1 + p_2 + p_3 + \ldots\ldots$$

where $n_1, n_2, n_3, \ldots$ are the number of moles of gases 1,2,3,…

Applying this equation to moist air.

$$p = p_t = p_a + p_v$$

where $p = p_t$ = total barometric pressure

$p_a$ = partial pressure of dry air

$p_v$ = partial pressure of water vapour

# Calculation of psychrometric properties from p, DBT and WBT

**i) Modified Apjohn equation:**

$$p_v = p_v' - \frac{1.8p(t - t')}{2700}$$

**ii) Modified Ferrel equation:**

$$p_v = p_v' - 0.00066p(t - t')\left[1 + \frac{1.8t}{1571}\right]$$

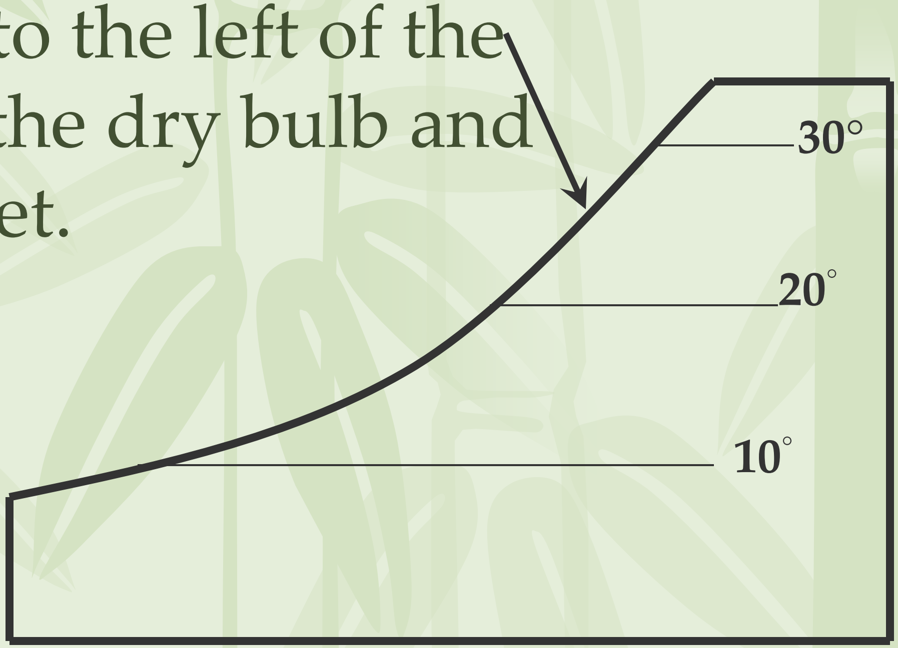
**iii) Carrier equation:**

$$p_v = p_v' - \frac{1.8(p - p_v')(t - t')}{2800 - 1.3(1.8t + 32)}$$

where $t$ = dry bulb temperature, °C
$t'$ = wet bulb temperature, °C
$p$ = barometric pressure
$p_v$ = vapor pressure
$p_v'$ = saturation vapor pressure at wet-bulb temperature

# 5. What is dew point?

- A. It is the temperature at which condensation occurs as heat is removed from an air-vapor mixture.
- B. The answer is read on the saturation curve horizontally to the left of the point where the dry bulb and wet bulb meet.



- Dew-point temperature: If unsaturated moist air is cooled at constant pressure, then the temperature at which the moisture in the air begins to condense is known as *dew-point temperature (DPT)* of air. An approximate equation for dew-point temperature is given by:

$$\mathrm{DPT} = \frac{4030(\mathrm{DBT} + 235)}{4030 - (\mathrm{DBT} + 235)\ln\phi} - 235$$

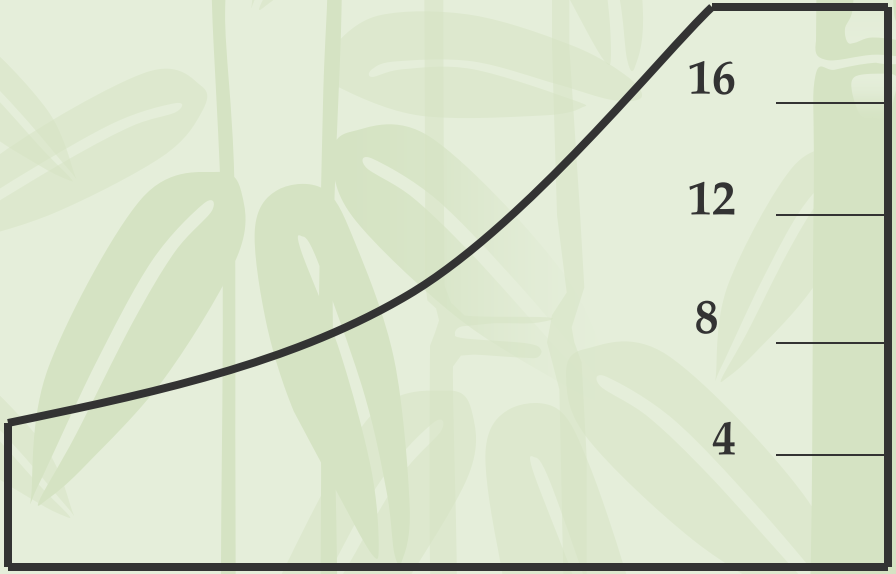
where Φ is the relative humidity (in fraction). DBT & DPT are in $^{o}$C.

- Degree of saturation μ: The degree of saturation is the ratio of the humidity ratio W to the humidity ratio of a saturated mixture $W_s$ at the same temperature and pressure

$$\mu = \left|\frac{W}{W_s}\right|_{t,P}$$

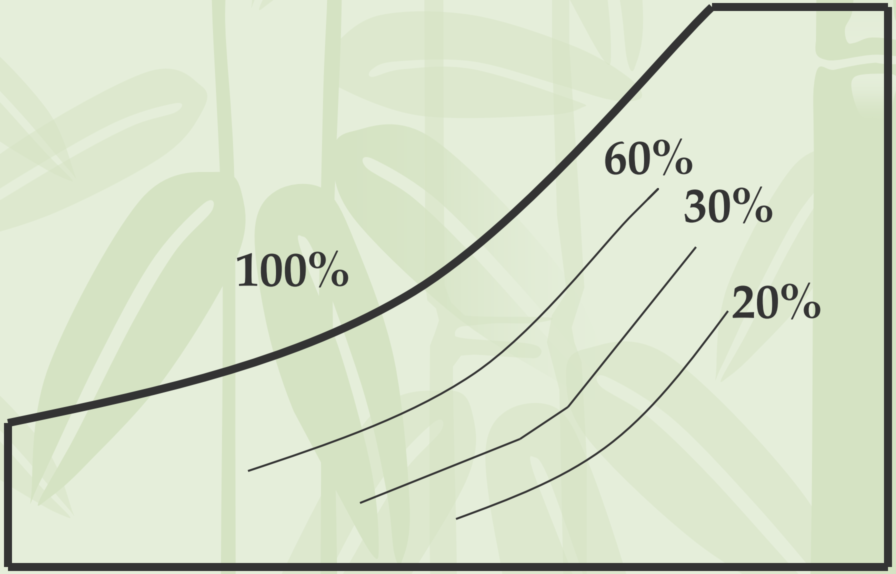
# 6. What is absolute humidity? (humidity ratio)

- A. It is the weight of water vapor in 1 kg of dry air (kg). (normally with g/kg dry air)
- B. It is also called Humidity Ratio or Specific Humidity.



# 7. What is relative humidity?

- A. It is the ratio of actual pressure of water vapor in the air to the pressure if the air were saturated and with a constant temperature.



- *Relative humidity (Φ)* is defined as the ratio of the mole fraction of water vapor in moist air to mole fraction of water vapor in saturated air at the same temperature and pressure. Using perfect gas equation we can show that

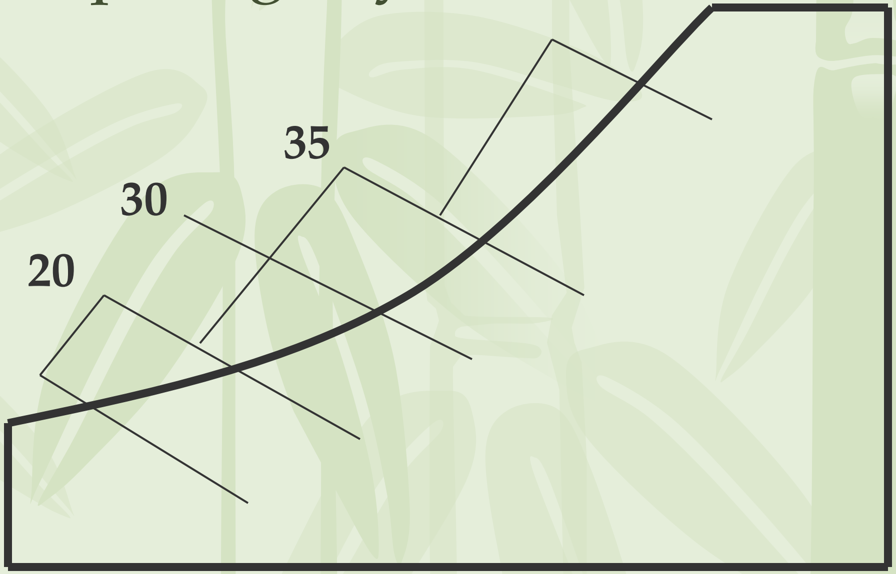
$$\phi = \frac{\text{partial pressure of water vapour}}{\text{saturation pressure of pure water vapour at same temperature}} = \frac{p_v}{p_{sat}}$$

- Humidity ratio (W): The humidity ratio (or specific humidity) W is the mass of water associated with each kilogram of dry air. Assuming both water vapor and dry air to be perfect gases, the humidity ratio is given by:

$$W = \frac{\text{kg of water vapour}}{\text{kg of dry air}} = \frac{p_v V / R_v T}{p_a V / R_a T} = \frac{p_v / R_v}{(p_t - p_v) / R_a} = 0.622 \frac{p_v}{p_t - p_v}$$

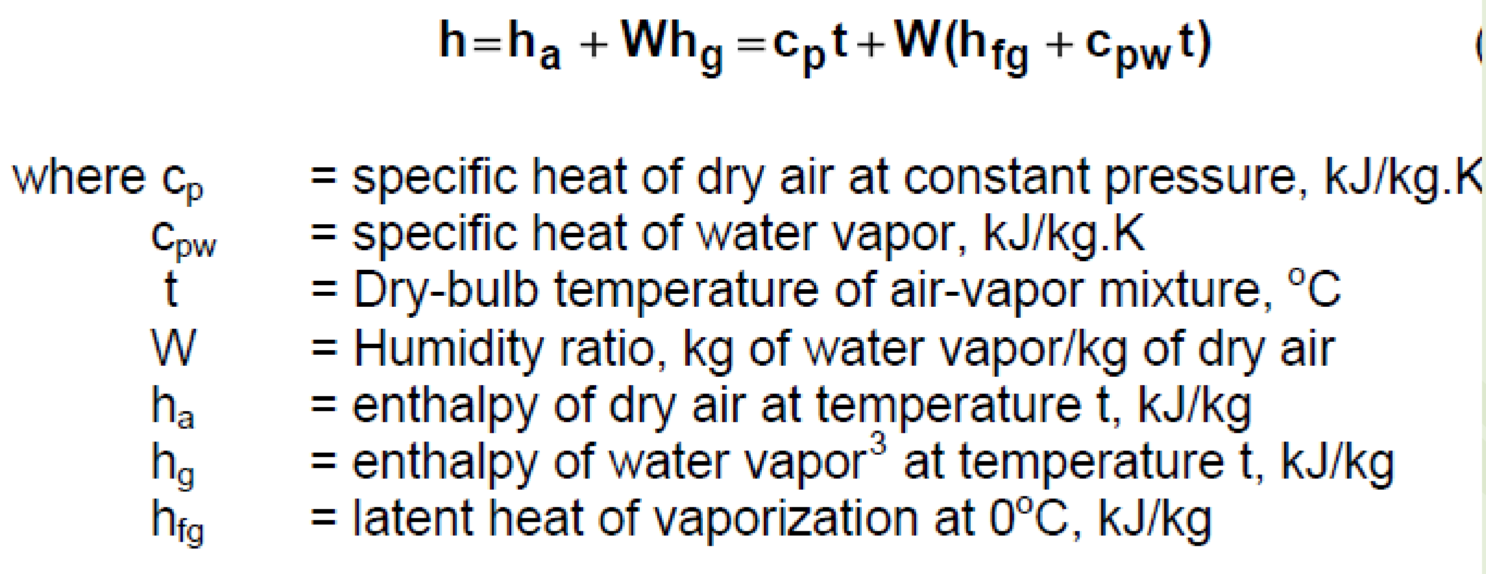
# 8. What are enthalpy lines?

- A. Enthalpy is a thermal (heat) property.
- B. It is the heat in an air vapor mixture.
- C. Lines are parallel to the wet bulb temp. lines.
- D. Values are in kJ's per kg dry air.



**kJ s per kg of dry air**

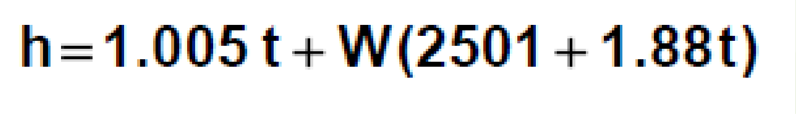
- Enthalpy: The enthalpy of moist air is the sum of the enthalpy of the dry air and the enthalpy of the water vapor. Enthalpy values are always based on some reference value. For moist air, the enthalpy of dry air is given a zero value at 0°C, and for water vapor the enthalpy of saturated water is taken as zero at 0°C. The enthalpy of moist air is given by:



$$h = h_a + Wh_g = c_p t + W(h_{fg} + c_{pw} t)$$

where $c_p$ = specific heat of dry air at constant pressure, kJ/kg.K
$c_{pw}$ = specific heat of water vapor, kJ/kg.K
$t$ = Dry-bulb temperature of air-vapor mixture, °C
$W$ = Humidity ratio, kg of water vapor/kg of dry air
$h_a$ = enthalpy of dry air at temperature t, kJ/kg
$h_g$ = enthalpy of water vapor$^3$ at temperature t, kJ/kg
$h_{fg}$ = latent heat of vaporization at 0°C, kJ/kg

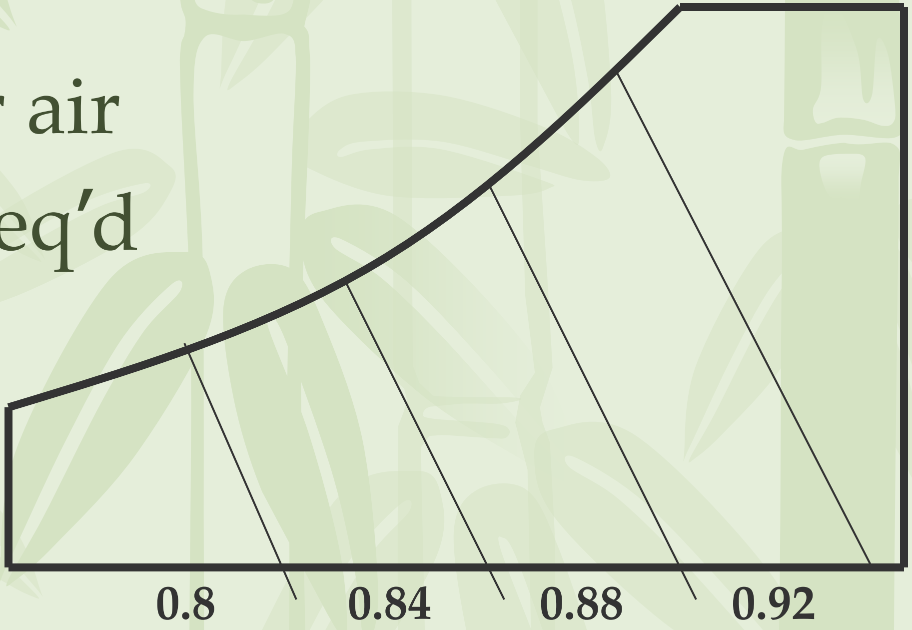
- The unit of h is kJ/kg of dry air. Substituting the approximate values of $c_p$ and $h_g$, we obtain:



$$h = 1.005\,t + W(2501 + 1.88t)$$

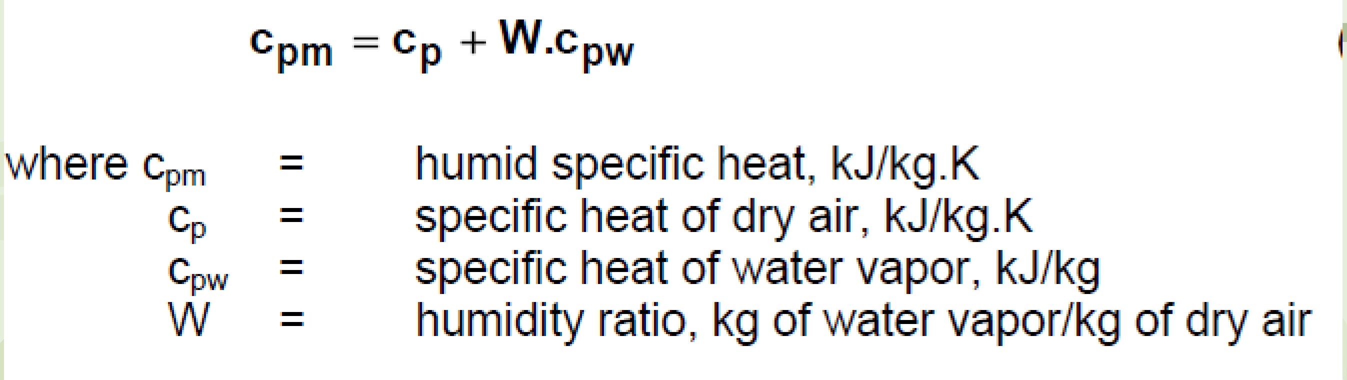
# 9. What is specific volume?

- A. The volume occupied by 1 kg of dry air.
- B. It represents the m$^3$/kg of dry air.
- C. The values are read below the dry bulb readings.
- D. Cooler, dryer air
  = less volume req'd



**Specific Volume** = m$^3$/kg **dry air**

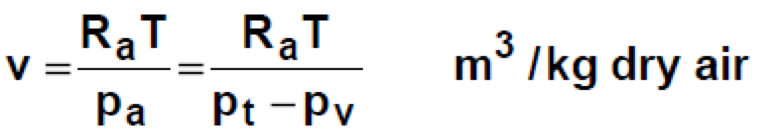
- Humid specific heat: From the equation for enthalpy of moist air, the humid specific heat of moist air can be written as:



$$c_{pm} = c_p + W.c_{pw}$$

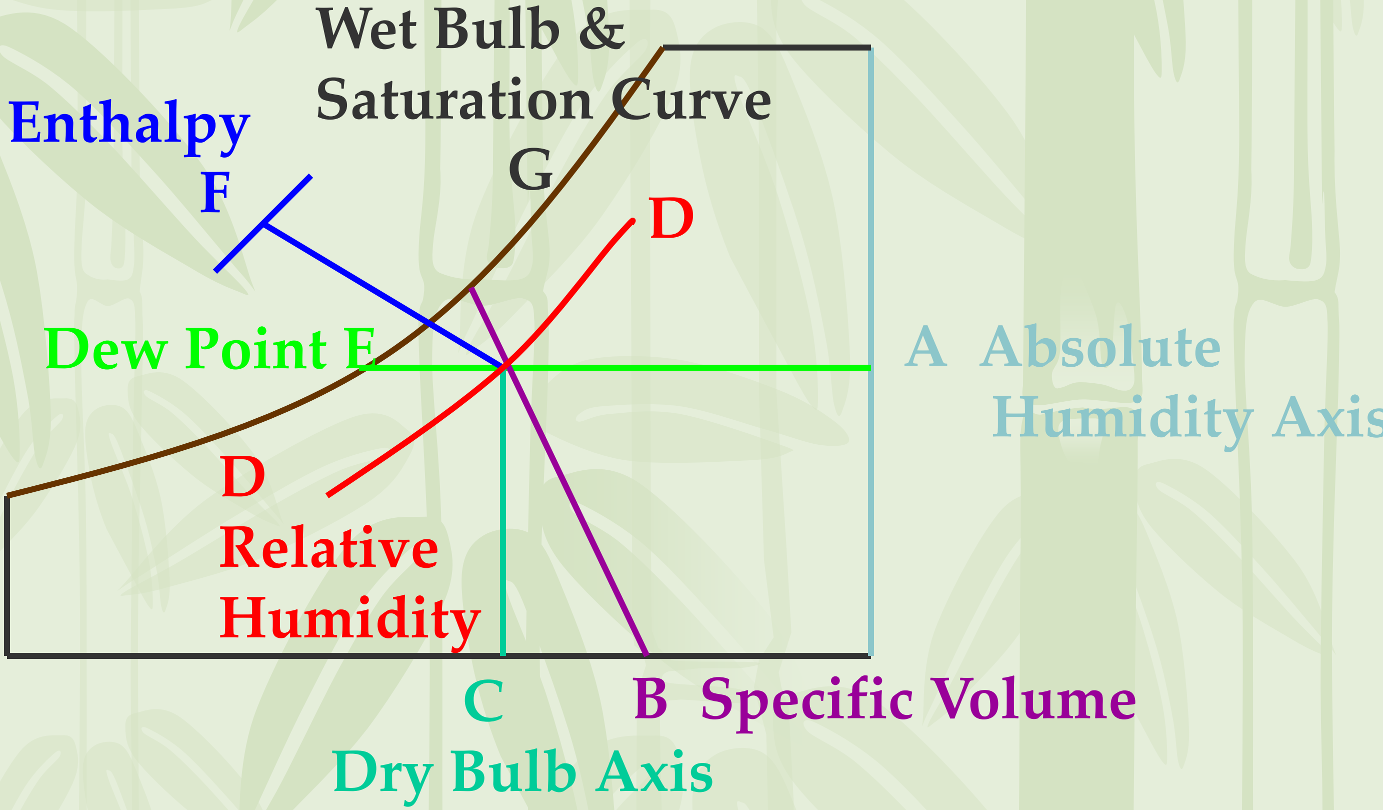
where $c_{pm}$ = humid specific heat, kJ/kg.K
$c_p$ = specific heat of dry air, kJ/kg.K
$c_{pw}$ = specific heat of water vapor, kJ/kg
$W$ = humidity ratio, kg of water vapor/kg of dry air

- Since the second term in the above equation ($w.c_{pm}$) is very small compared to the first term, for all practical purposes, the humid specific heat of moist air, $c_{pm}$ can is as 1.0216 kJ/kg dry air.K
- Specific volume: The specific volume is defined as the number of cubic meters of moist air per kilogram of dry air. From perfect gas equation since the volumes occupied by the individual substances are the same, the specific volume is also equal to the number of cubic meters of dry air per kilogram of dry air

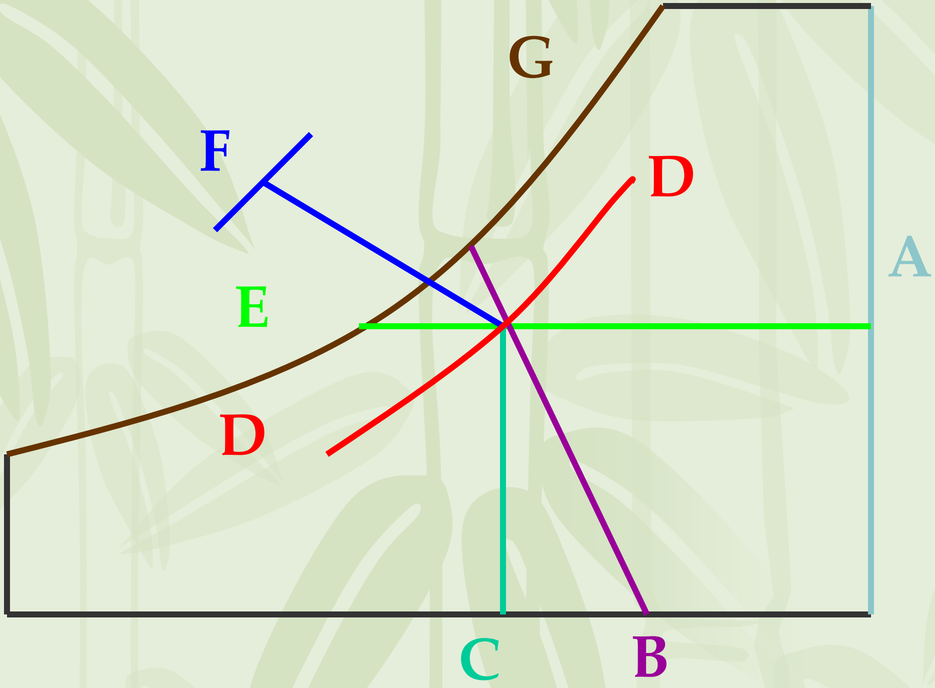


$$v = \frac{R_a T}{p_a} = \frac{R_a T}{p_t - p_v} \qquad m^3 /kg \text{ dry air}$$

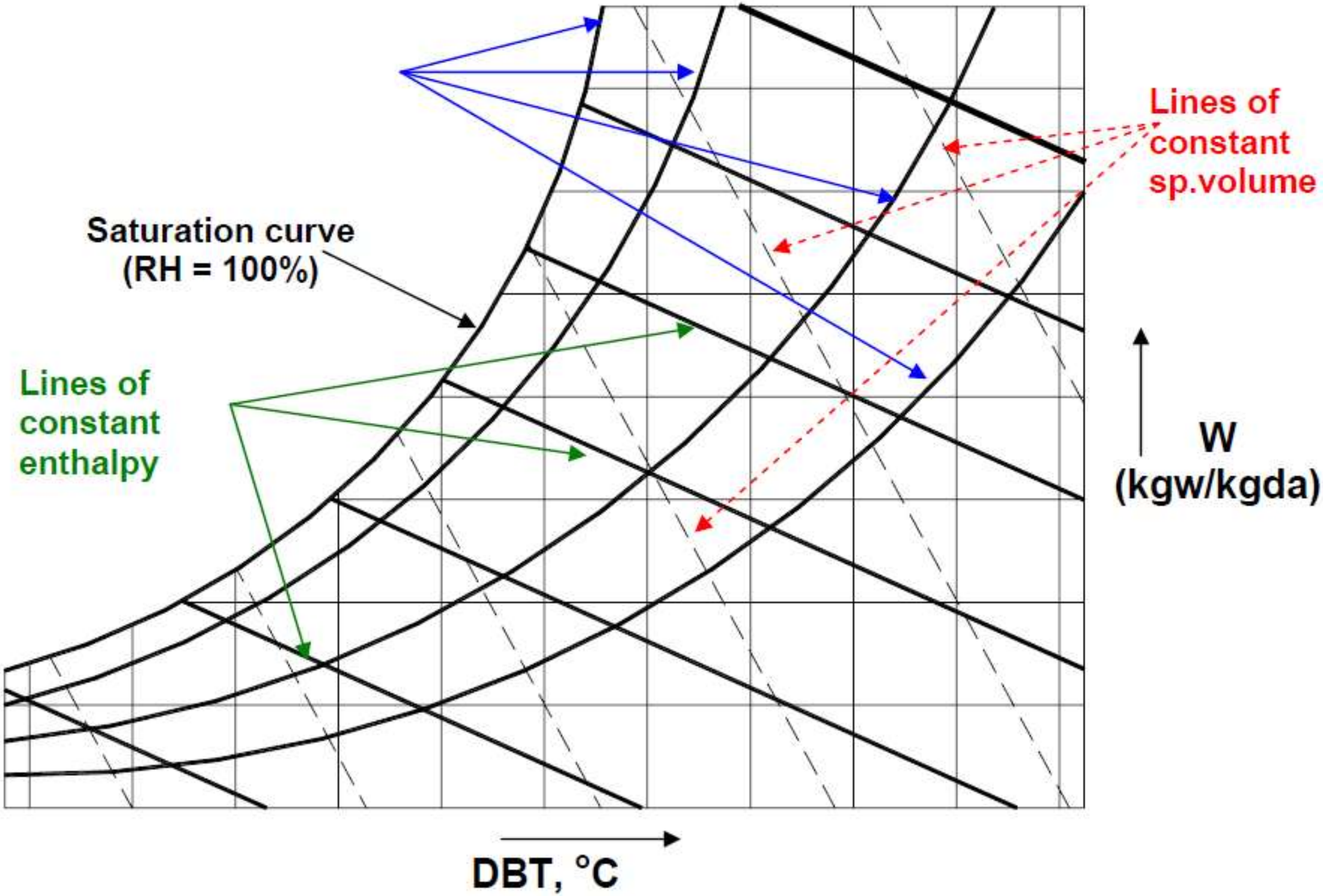
# The psychrometric chart has seven lines.



# Using the psychrometric chart

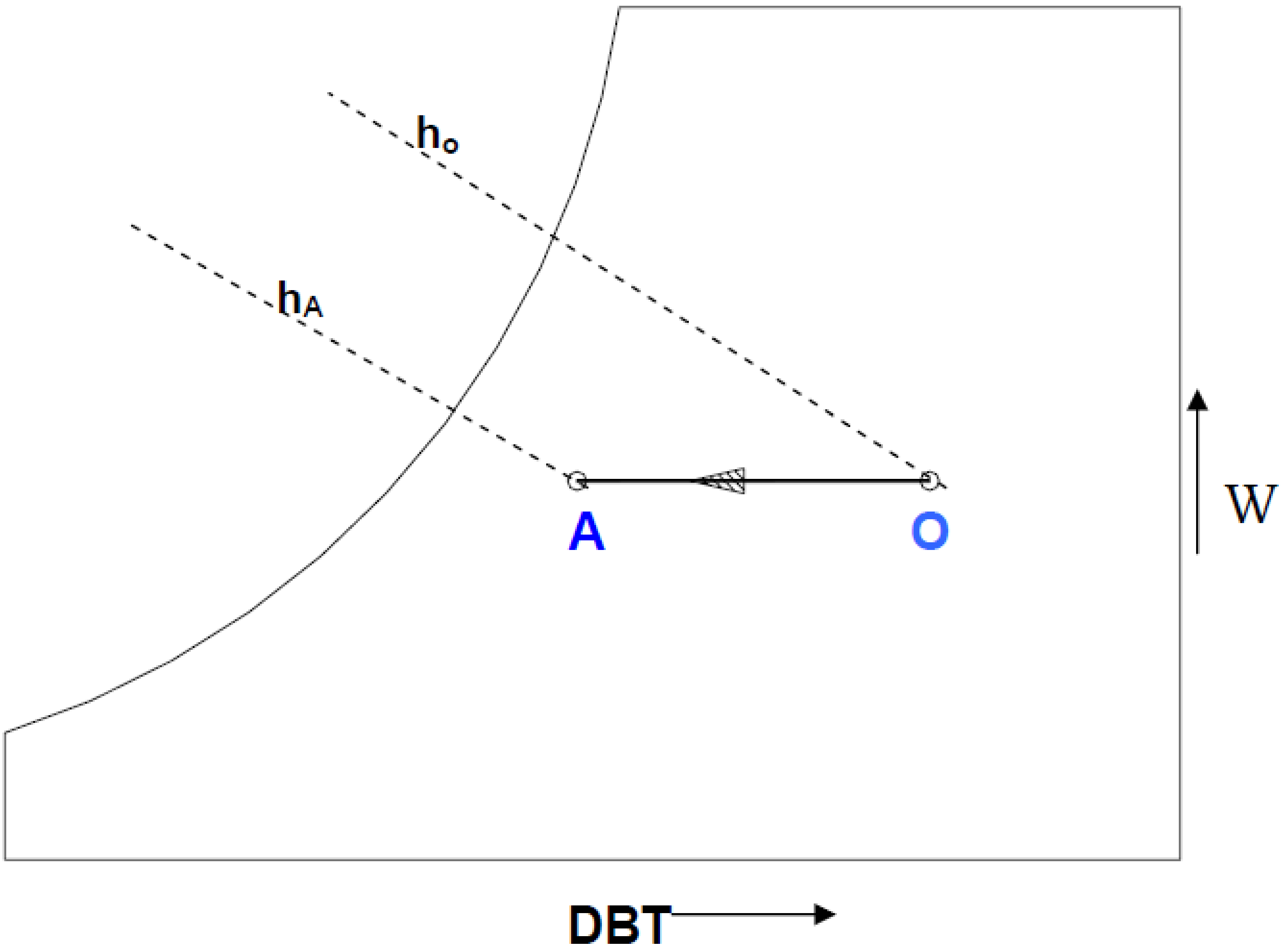


You have been learning by looking at one line at a time.
Take the psychrometric chart given you. You will notice that it has many lines. Don't panic. Find the values.

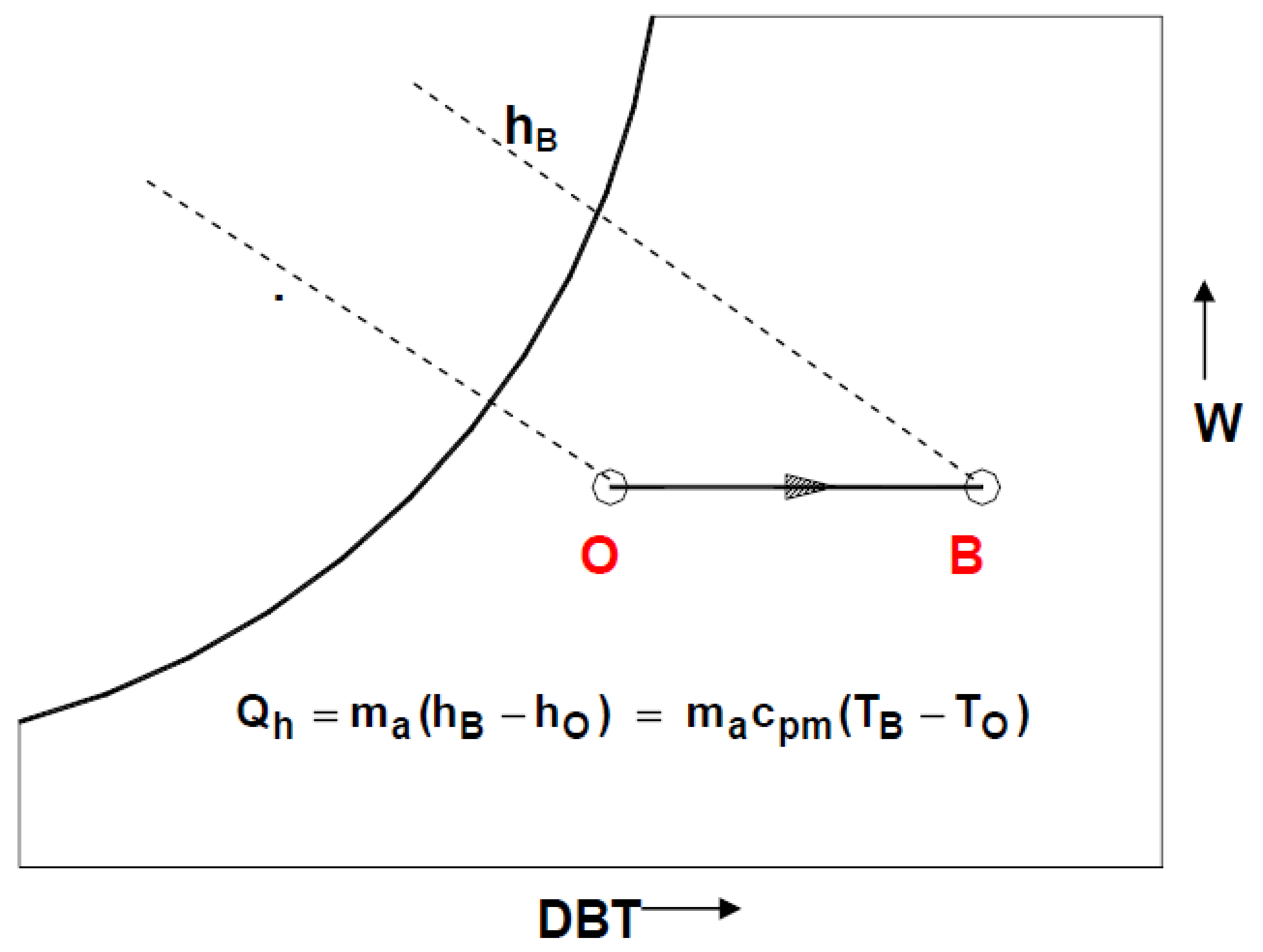

Saturation curve
(RH = 100%)
Lines of constant enthalpy
Lines of constant sp.volume
W
(kgw/kgda)
DBT, °C

# Sensible cooling

$$Q_c = m_a(h_O - h_A) = m_a c_{pm}(T_O - T_A)$$



# Sensible heating



$$Q_h = m_a(h_B - h_O) = m_a c_{pm}(T_B - T_O)$$

# Cooling and dehumidification

By applying mass balance for the water:

$$m_a.w_O = m_a.w_C + m_w$$

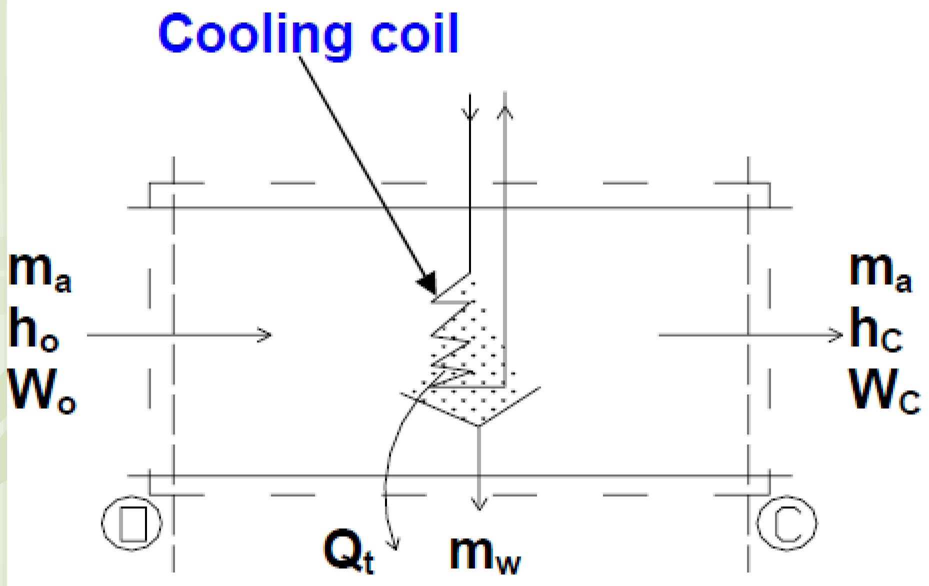
By applying energy balance:

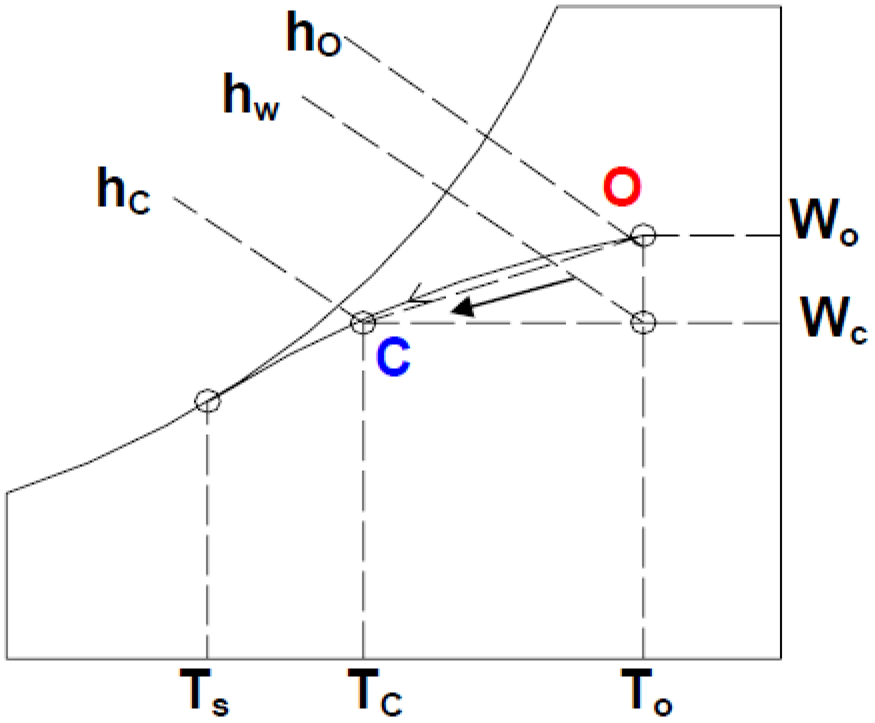
$$m_a.h_O = Q_t + m_w.h_w + m_a.h_C$$

$$Q_t = m_a(h_O - h_C) - m_a(w_O - w_C)h_w$$

- the 2 $^{nd}$ term on the RHS of the above equation is normally smal compared to the other terms, so it can be neglected. Hence,

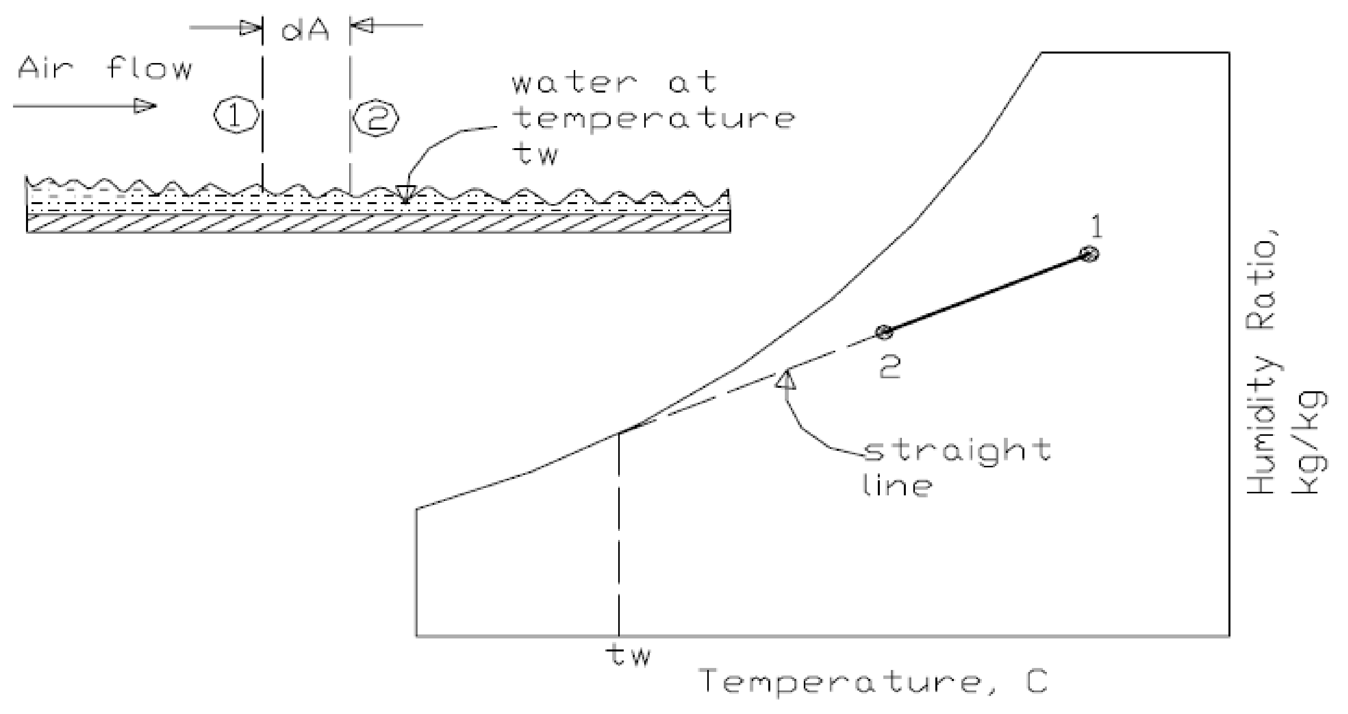
$$Q_t = m_a(h_O - h_C)$$





# Combined heat and mass transfer; the straight line law

- The straight line law states that *“when air is transferring heat and mass (water) to or from a wetted surface, the condition of air shown on a psychrometric chart drives towards the saturation line at the temperature of the wetted surface”*.



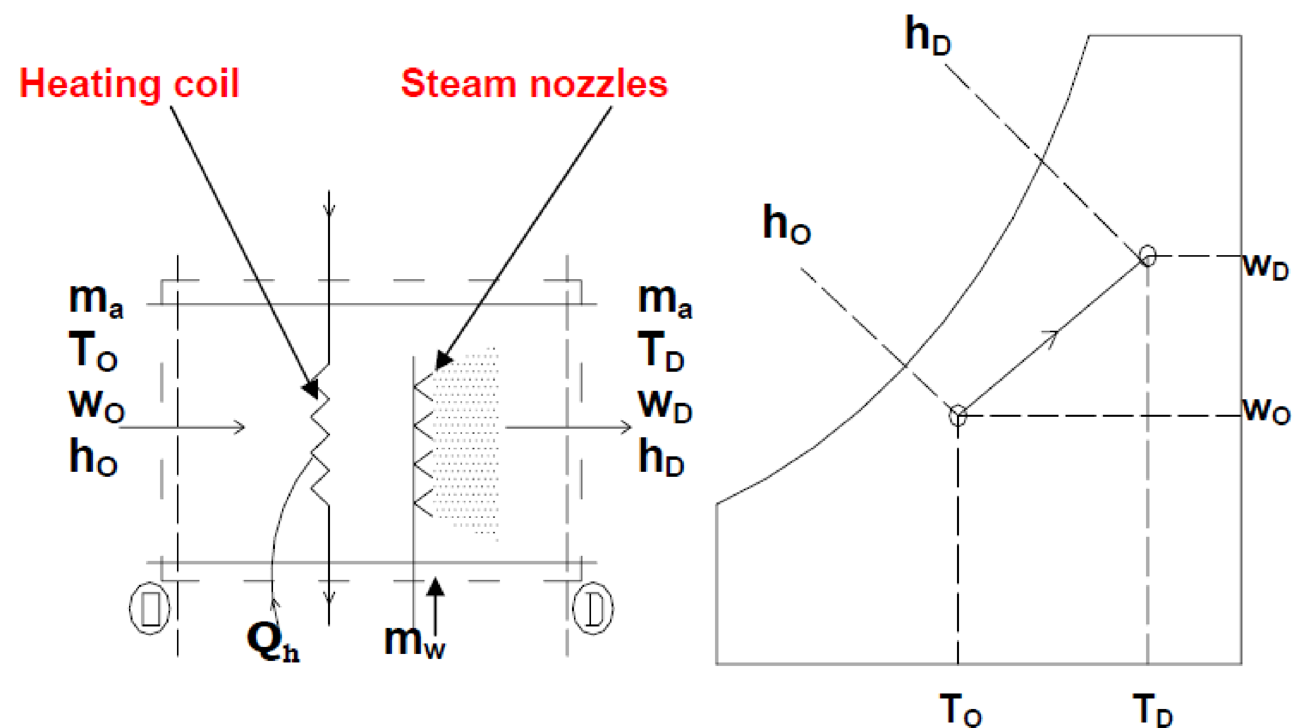
# Heating and Humidification

- During winter it is essential to heat and humidify the room air for comfort.

$$m_w = m_a(w_D - w_O)$$

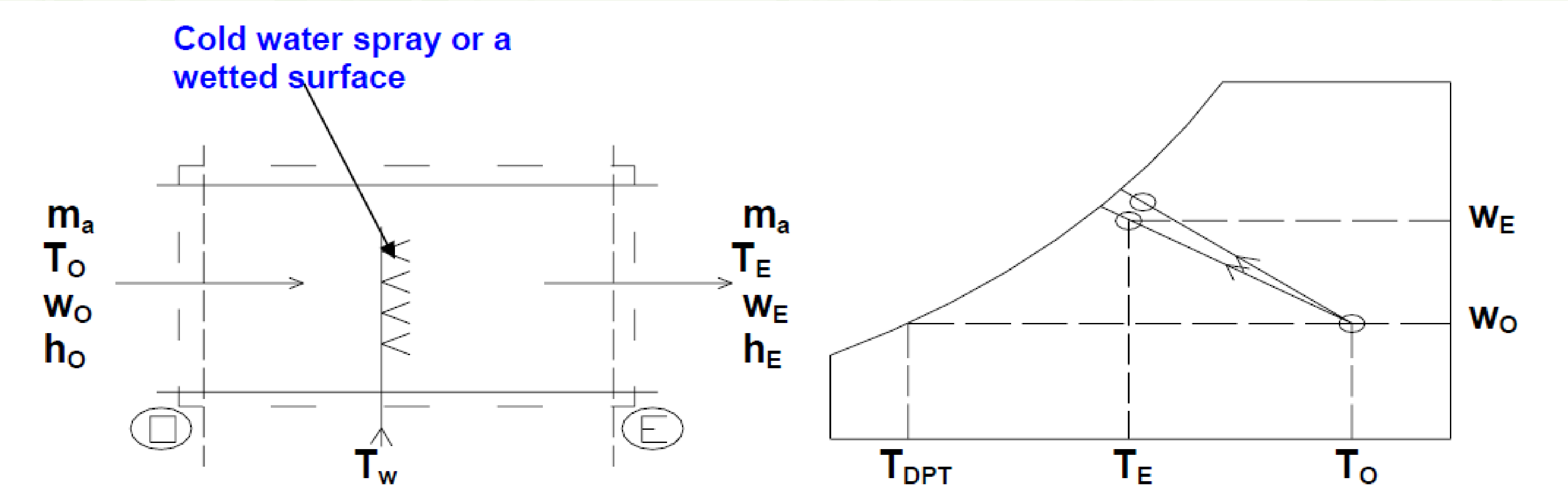
$$Q_h = m_a(h_D - h_O) - m_w h_w$$

- where $Q_h$ is the heat supplied through the heating coil and $h_w$ is the enthalpy of steam

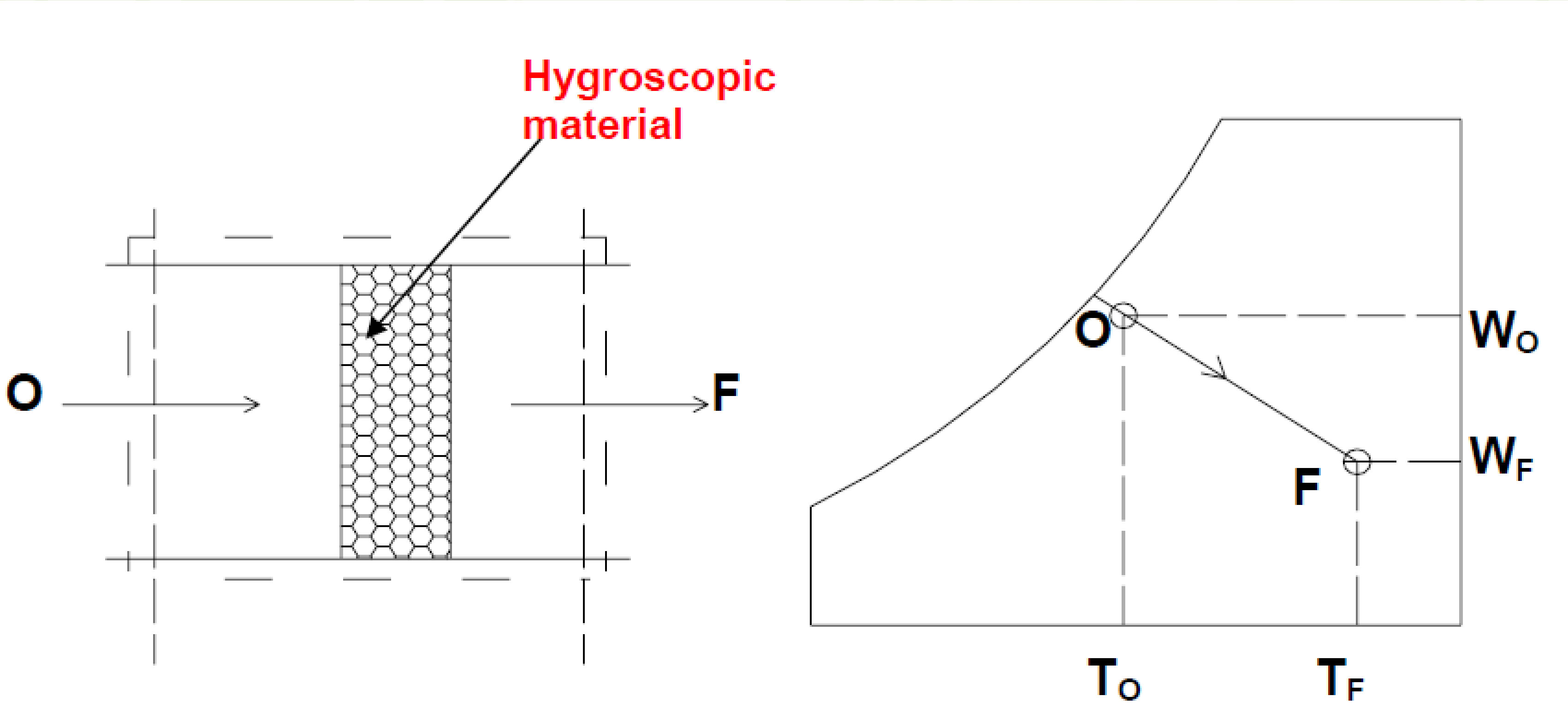


# Cooling & humidification

- If the water temperature is greater than WBT, then there will be a net heat transfer from water to air. If the water temperature is less than WBT, then the net heat transfer will be from air to water.



# Heating and de-humidification



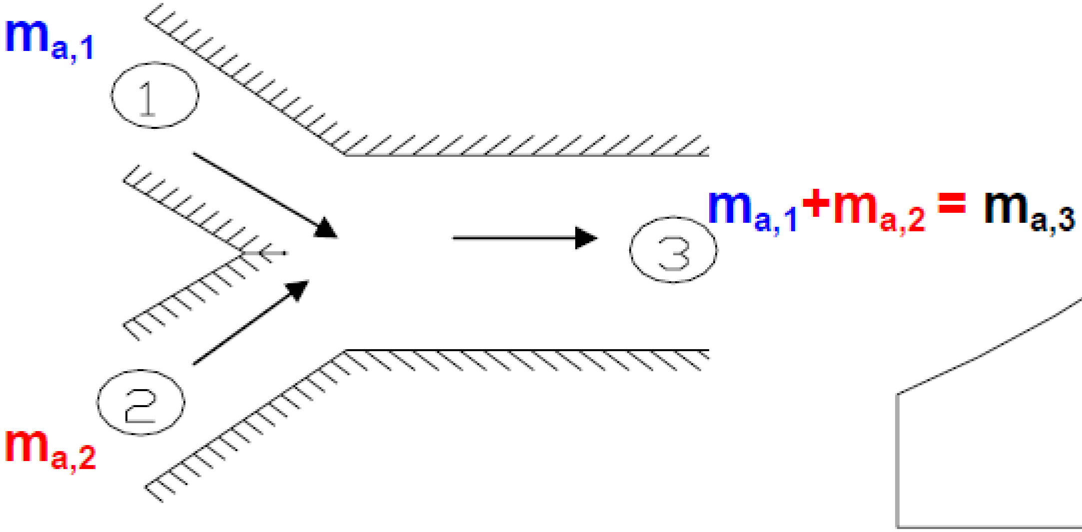
# Mixing of air streams:

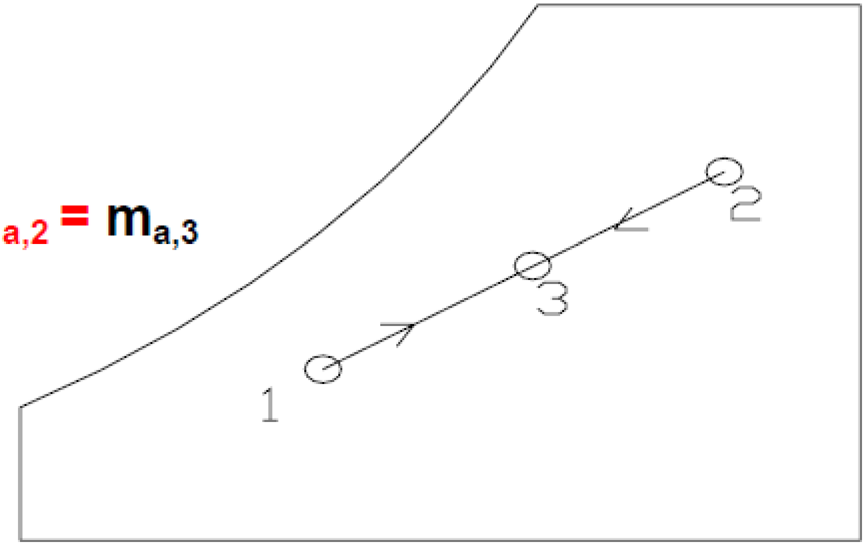
From the mass balance of dry air and water vapor:

$$m_{a,1}w_1 + m_{a,2}w_2 = m_{a,3}w_3 = (m_{a,1} + m_{a,2})w_3$$

From energy balance:

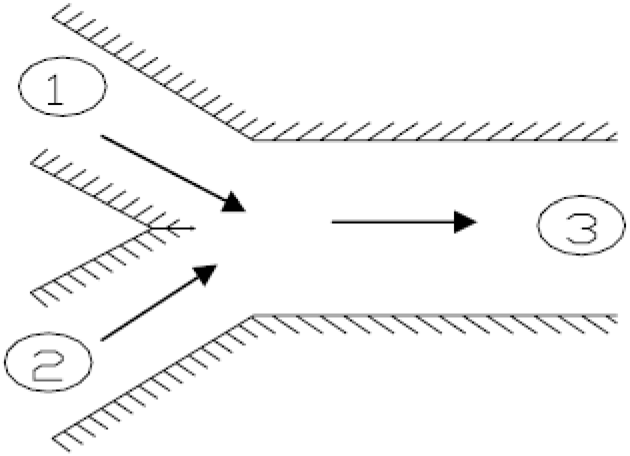
$$m_{a,1}h_1 + m_{a,2}h_2 = m_{a,3}h_3 = (m_{a,1} + m_{a,2})h_3$$

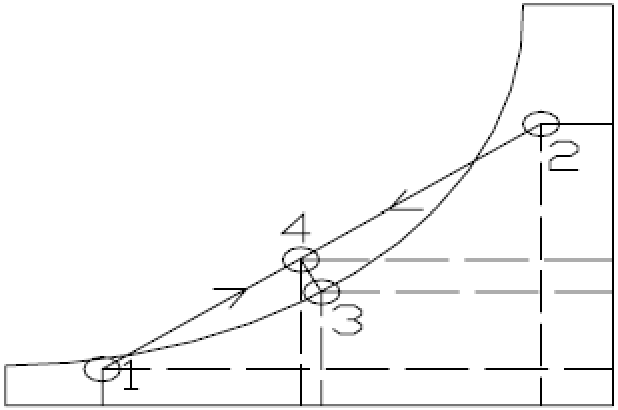




# Mixing with condensation

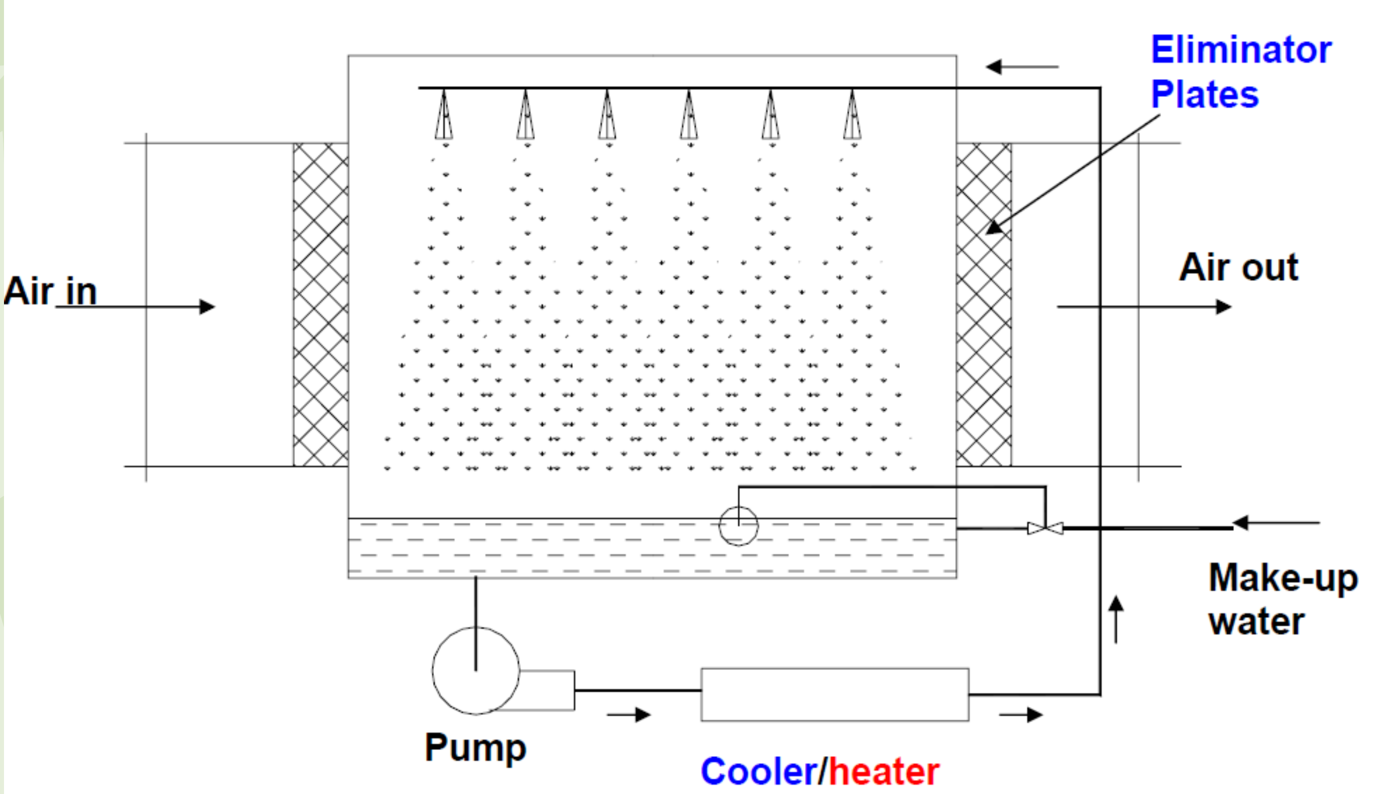
- Dry air mixes with warm air at high relative humidity, the resulting mixture condition may lie in the two-phase region, as a result there will be condensation of water vapor and some amount of water will leave the system as liquid water. Due to this, the humidity ratio of the resulting mixture (point 3) will be less than that at point 4. Corresponding to this will be an increase in temperature of air due to the release of latent heat of condensation. This process rarely occurs in an air conditioning system, but this is the phenomenon which results in the formation of fog or frost (if the mixture temperature is below $0^{o}$C).





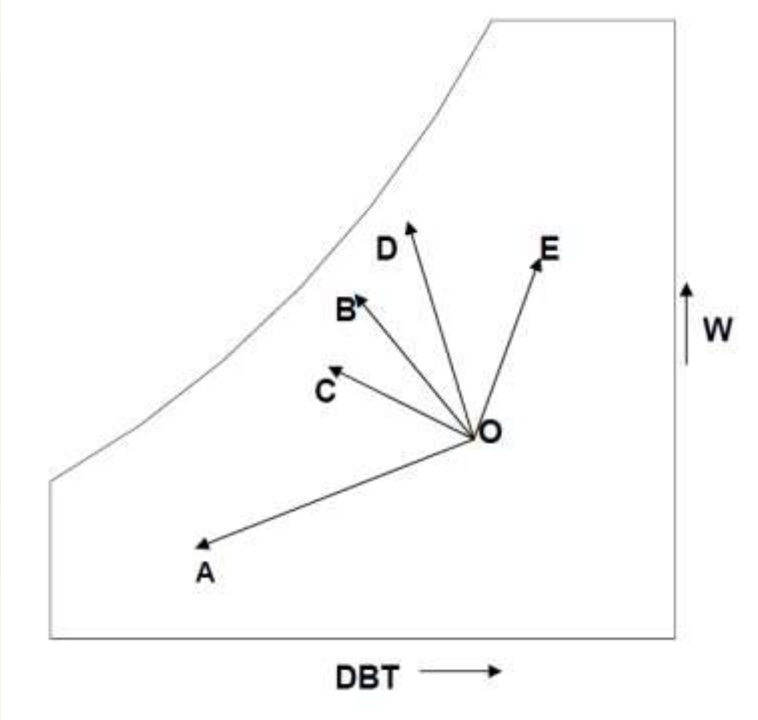
# Air Washers

- An air washer is a device for conditioning air. As shown in the figure, in an air washer air comes in direct contact with a spray of water and there will be an exchange of heat and mass (water vapor) between air and water. The outlet condition of air depends upon the temperature of water sprayed in the air washer .



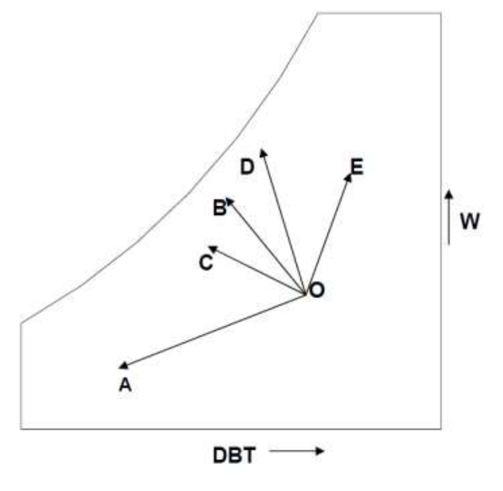
# Various psychrometric processes that can take place in an air washer

- A) Cooling and dehumidification: $t_{water} < T_{dew}$.
- B) Adiabatic saturation: $t_{water} = WBT$.
- C) Cooling and humidification: $T_{dew} < t_{water} < WBT$
- D) Cooling and humidification: $WBT < t_{water} < DBT$
- E) Heating and humidification: $t_{water} > DBT$



- **A) Cooling and dehumidification: $t_{water} < T_{dew}$** Since the exit enthalpy of air is less than its inlet value, from energy balance it can be shown that there is a transfer of total energy from air to water. Hence to continue the process, water has to be externally cooled. Here both latent and sensible heat transfers are from air to water.
- **B) Adiabatic saturation: $t_{water} = WBT$**. Here the sensible heat transfer from air to water is exactly equal to latent heat transfer from water to air. Hence, no external cooling or heating of water is required. That is this is a case of pure water recirculation. This the process that takes place in a perfectly insulated evaporative cooler.
- **C) Cooling and humidification: $T_{dew} < t_{water} < WBT$**. Here the sensible heat transfer is from air to water and latent heat transfer is from water to air, but the total heat transfer is from air to water, hence, water has to be cooled externally.

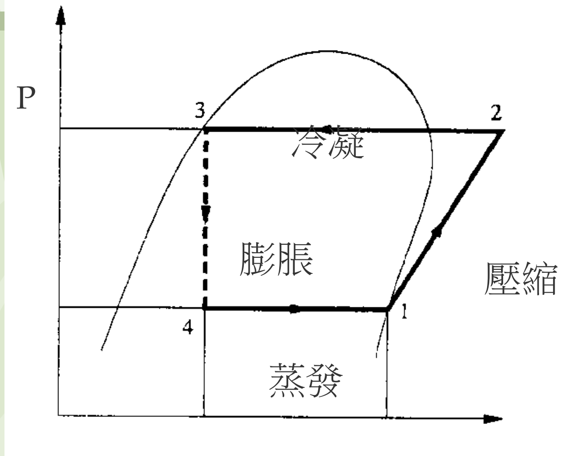
- **D) Cooling and humidification: WBT < $t_{water}$ < DBT.** Here the sensible heat transfer is from air to water and latent heat transfer is from water to air, but the total heat transfer is from water to air, hence, water has to be heated externally. This is the process that takes place in a cooling tower. The air stream extracts heat from the hot water coming from the condenser, and the cooled water is sent back to the condenser.
- **E) Heating and humidification: $t_{water}$ > DBT.** Here both sensible and latent heat transfers are from water to air, hence, water has to be heated externally.

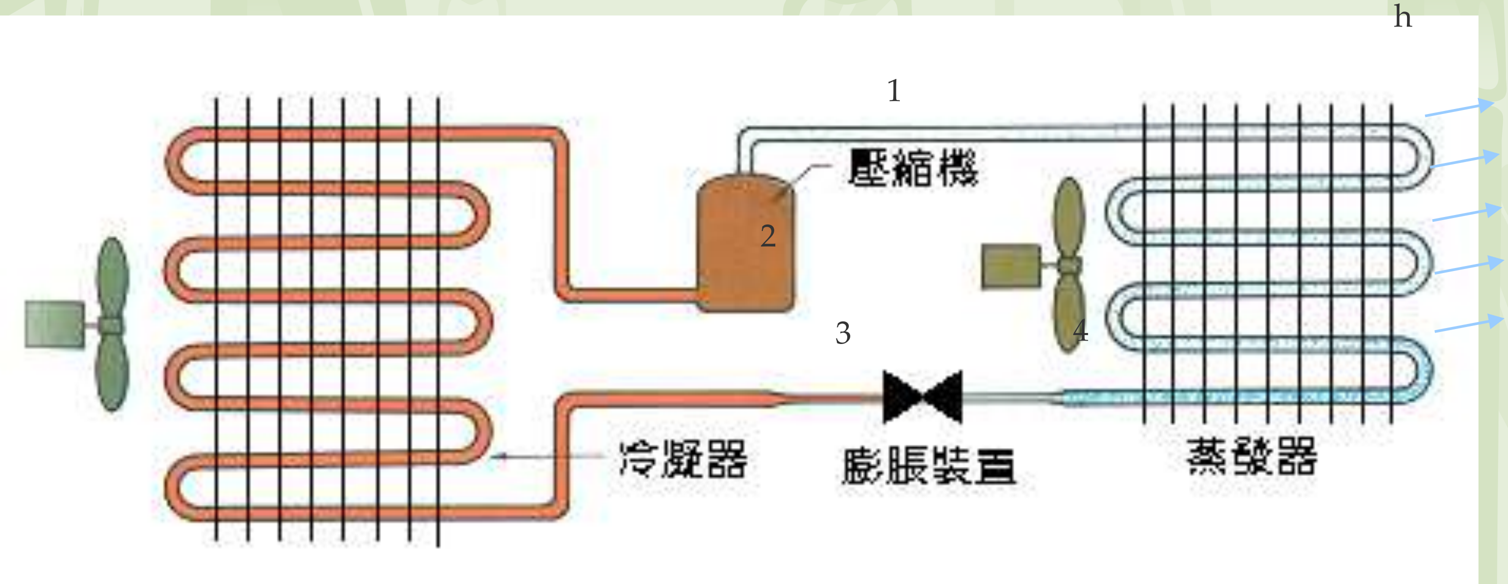


# 『冷凍空調基本架構』

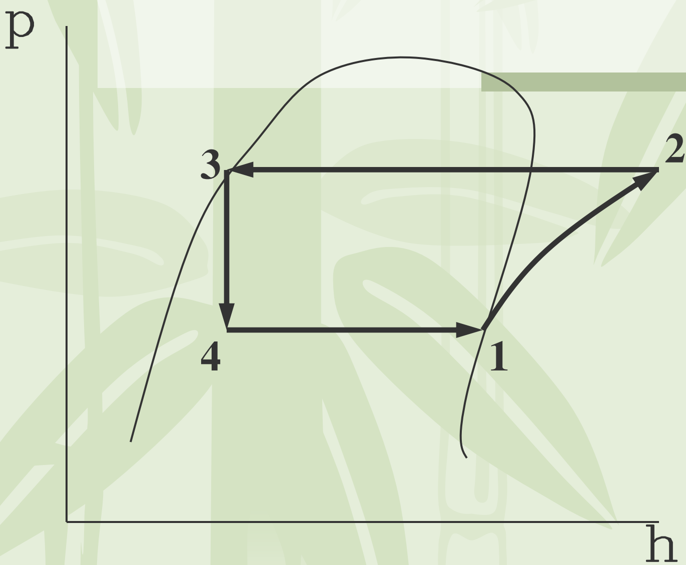
冷凍空調五大元件：

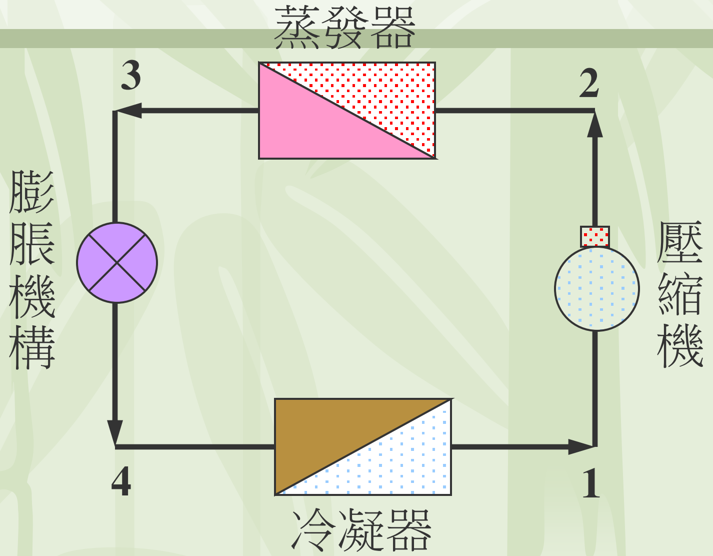
- 壓縮機 (Compressor)
- 冷凝器 (Condenser)
- 蒸發器 (Evaporator)
- 膨脹裝置 (Expansion Device)
- 送風裝置 (Fan Device)

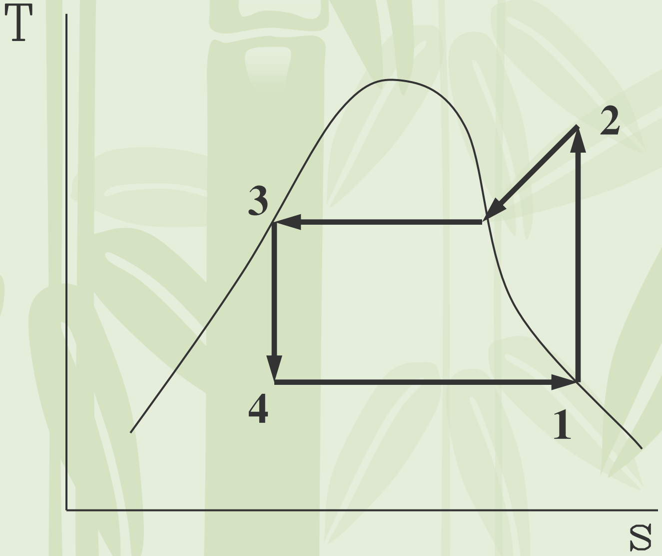


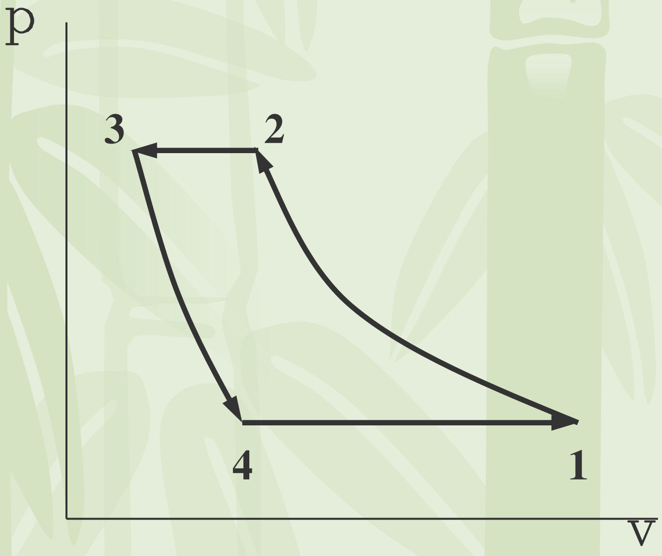


# 冷媒特性與系統運轉匹配圖

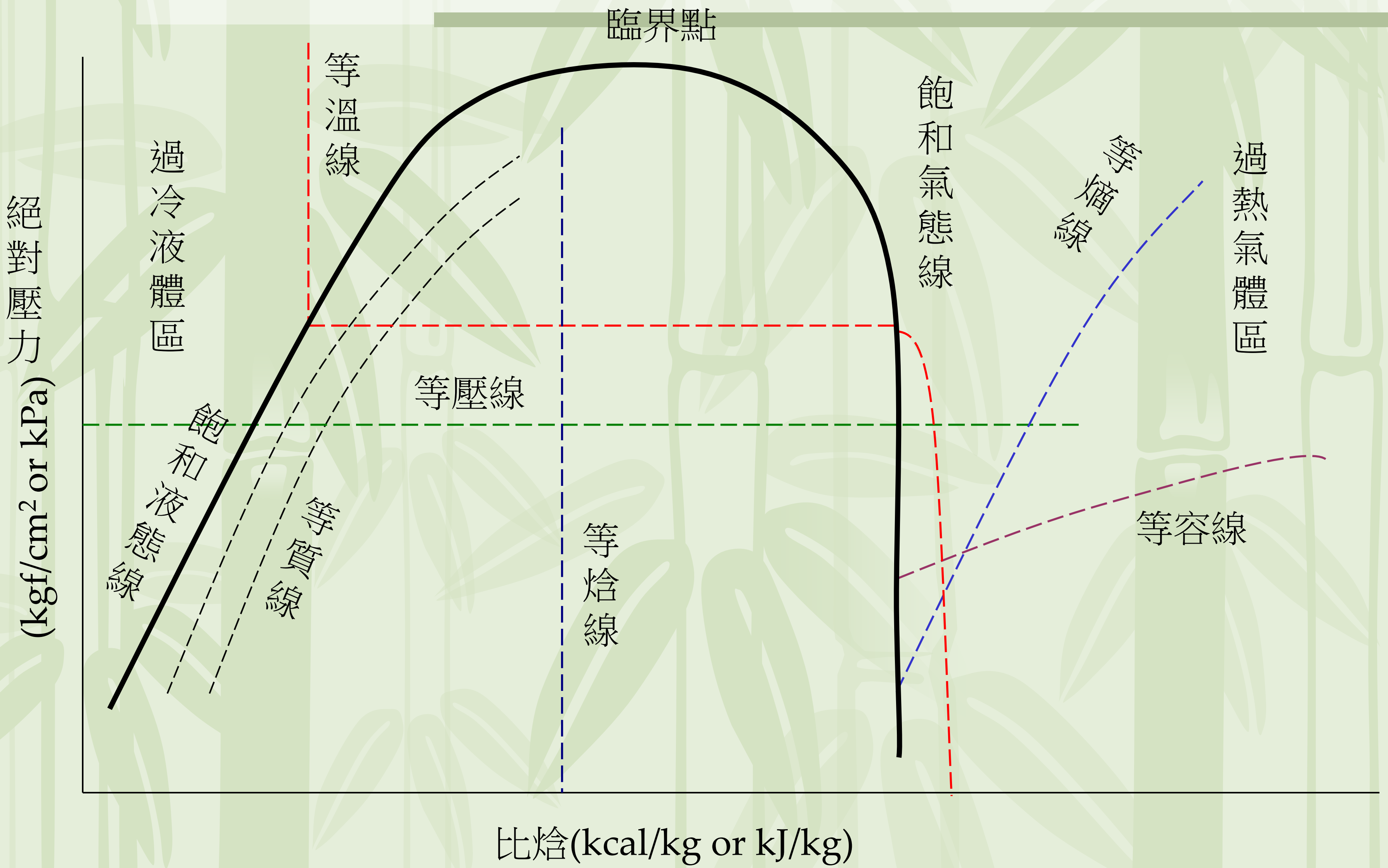




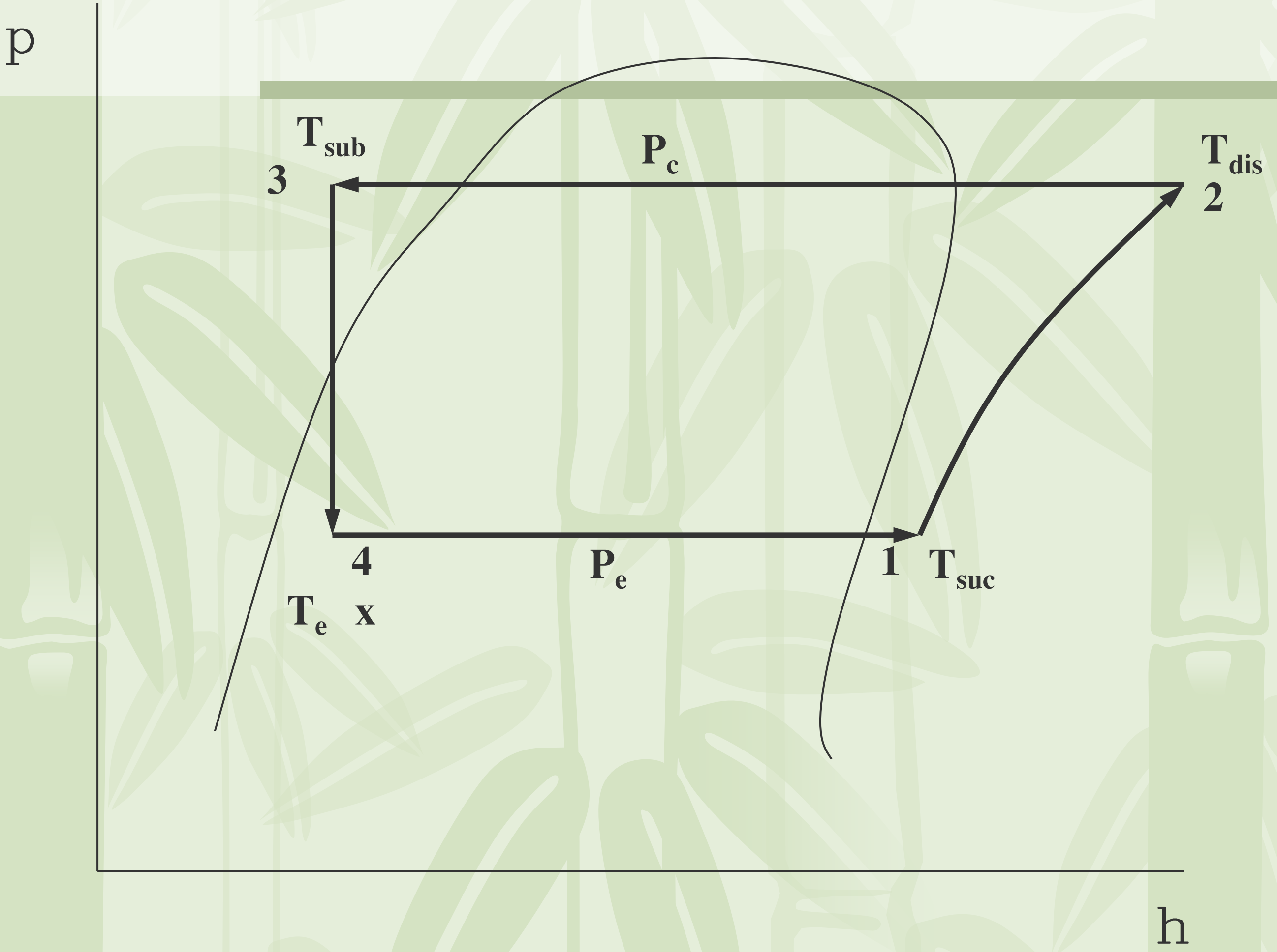




# 冷凍空調系統心電圖
## （莫氏-Mollier曲線圖）



# 空調系統平衡設計



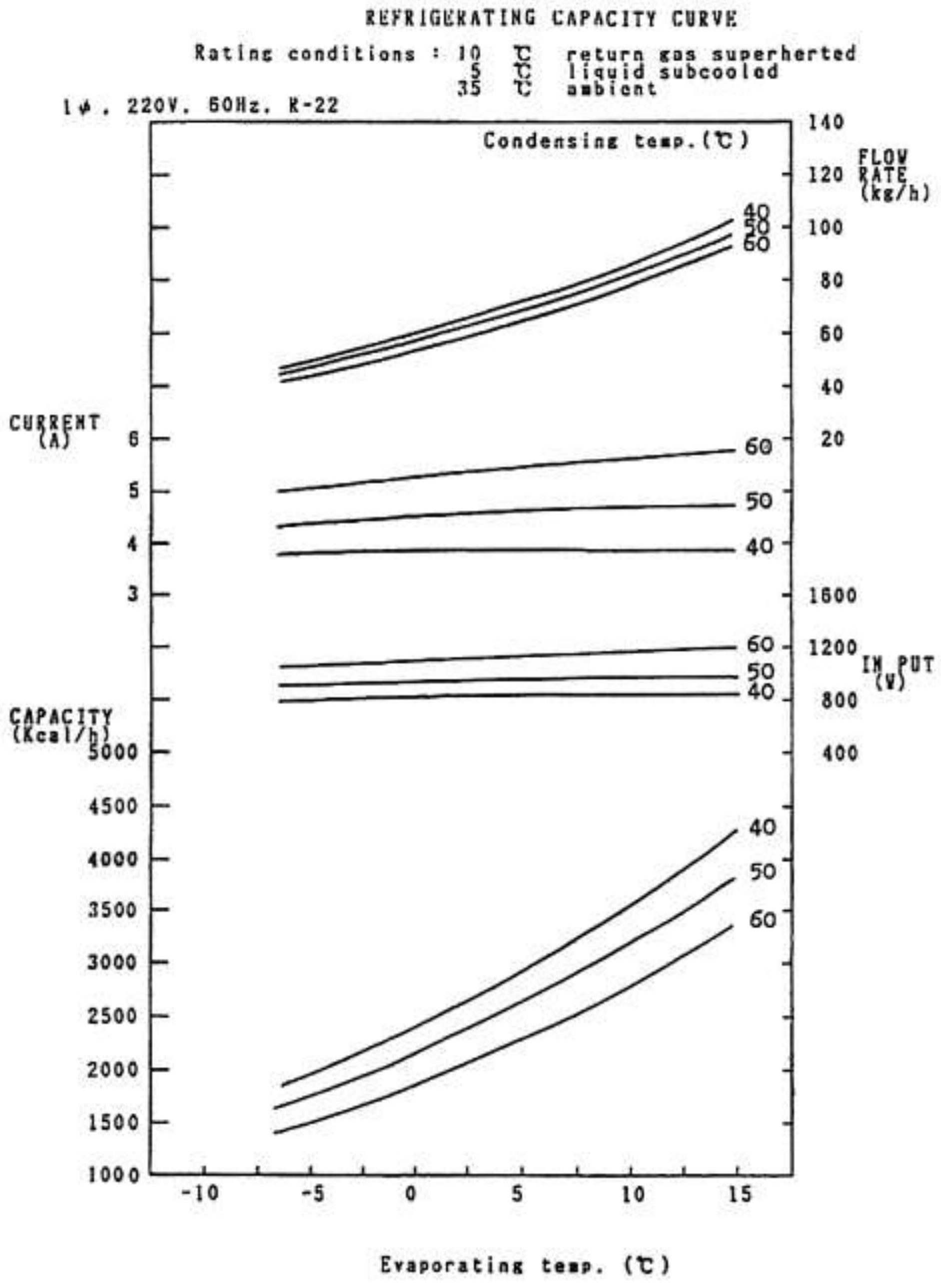
主要參數

- 蒸發壓力,低壓 (Evaporation pressure, $P_c$)
- 冷凝壓力, 高壓(Condensing Pressure, $P_e$)
- 過熱溫度、過冷度

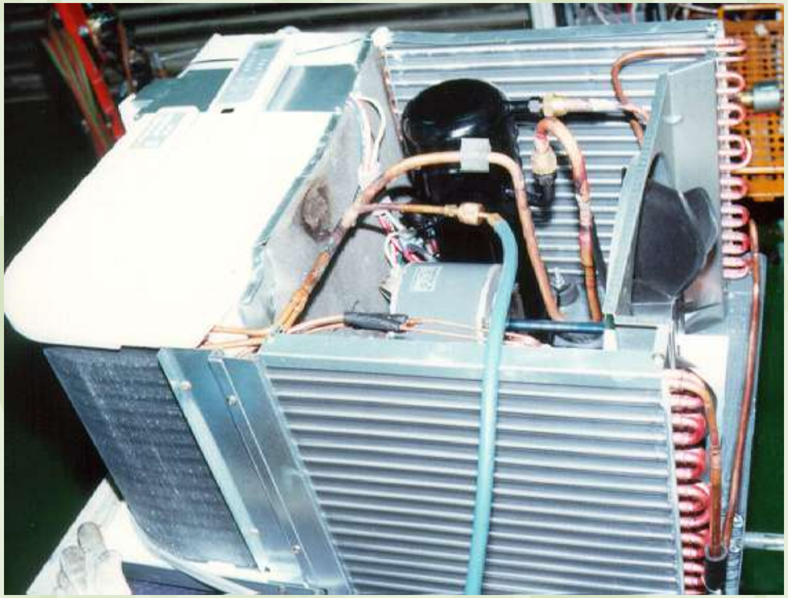
# 傳統系統設計（案例）

| | |
|---|---|
| 已知條件 | 1.所需冷房能力($Q_c$)<br>2.使用冷媒:<br>3.設計條件: Tech.條件-<br>Te:7.2℃ 、Tc:54.4℃ 、Ts:35℃ 、$T_l$:46.1℃ 、Ta:35℃<br>以R-22 而言，對應Pse:6.38kgf/$cm^2$、Psc:21.89kgf/$cm^2$、壓縮比 =21.89/6.38=3.431 |
| 查圖表計算 | 相對於R22之p-h圖，匹配系統循環可知：<br>點1:由蒸發器吸熱等壓昇溫為氣態冷媒而進入壓縮機時<br>其溫度為Ts:35℃ 、壓力為6.38kgf/$cm^2$<br>查圖表可得比容($v_1$)=0.0434$m^3$/kg、比焓($h_1$)=154kcal/kg<br>點2:假設等熵壓縮之壓縮機的排氣溫度(Td)為102℃<br>排氣壓力為21.89kgf/$cm^2$<br>點3:經過冷凝器等壓放熱成為液態時，其液體溫度$T_l$為46.1℃<br>此時壓力仍為21.89kgf/$cm^2$<br>點4:經過膨脹裝置降溫與降壓而欲進入蒸發器時<br>其溫度為Ts:7.2℃ 、壓力為6.38kgf/$cm^2$<br>查圖表可得比焓($h_4$)=113kcal/kg |

| | |
|---|---|
| 後續計算 | 1.經過蒸發器之焓差(Δh)= $h_1$- $h_4$=154-113=41kcal/kg<br>2.系統質量流量$m=Q_c/\Delta h$<br>3.進入壓縮機之氣態冷媒的體積流量<br>$v_h= v_1$・m=0.0434・365.85=4.假設壓縮機的轉速約為3450rpm或207000rev/hr<br>則壓縮機的吸入容積為$v_{rev}=15.88x10^6/207000=76.7$cc/rev<br>5.假設壓縮機的容積效率$\eta_v$為0.9，則實際應吸入冷媒的體積為<br>$v=v_{rev}/\eta_v=76.7/0.9=85.2$cc/rev<br>6.可經由上述之計算式，來計算此壓縮機之每cc的冷房能力<br>$q_c=\Delta h/ v_1=41/0.0434=944.7$kcal/m$^3$=9.447x10$^{-4}$kcal/cc<br>7.則每轉一圈之每cc的冷房能力為<br>$q_{cv}$=9.447x10$^{-4}$x207000x0.9=176(kcal/hr／cc/rev)<br>8.匹配各種壓縮機的機構，由v與進排氣之溫度與壓力開始，計算此壓縮機的泵之各元件尺寸與所需馬達的尺寸，再考慮此壓縮機的機械效率$\eta_{me}$、馬達效率$\eta_{mo}$、壓縮效率$\eta_c$，可得馬達所需的實際之輸入功$P_{motor}$<br>9.最後可計算出壓縮機的E.E.R.=$Q_c/P_{motor}$ |


REFRIGERATING CAPACITY CURVE
Rating conditions : 10 ℃ return gas superherted
5 ℃ liquid subcooled
35 ℃ ambient
1φ, 220V, 60Hz, R-22
Condensing temp.(℃)
FLOW RATE (kg/h)
140
120
100
80
60
40
20
40
50
60
CURRENT (A)
6
5
4
3
60
50
40
1600
1200
800
400
IN PUT (W)
60
50
40
CAPACITY (Kcal/h)
5000
4500
4000
3500
3000
2500
2000
1500
1000
40
50
60
-10
-5
0
5
10
15
Evaporating temp. (℃)


Discharge Line
(0.506)
12.91/12.81 I.D.
x 12.0 Deep (0.472)
ø163.3
(6.43)
Suction Line
(0.756)
19.26/19.16 I.D.
x 12.0 Deep
(0.472)
71.7
(2.82)
"C"
"B"
"A"
"D"
MILLENNIUM
12.3
(0.484)



# 實際計算平衡過程

給訂條件如下：

- 壓縮機 (Compressor)　轉速，容積效率
- 冷凝器 (Condenser) 詳細尺寸大小，入口風量與溫度
- 蒸發器 (Evaporator) 詳細尺寸大小，入口風量與乾溼球溫度
- 膨脹裝置 (Expansion Device) 毛細管尺寸長度

計算：

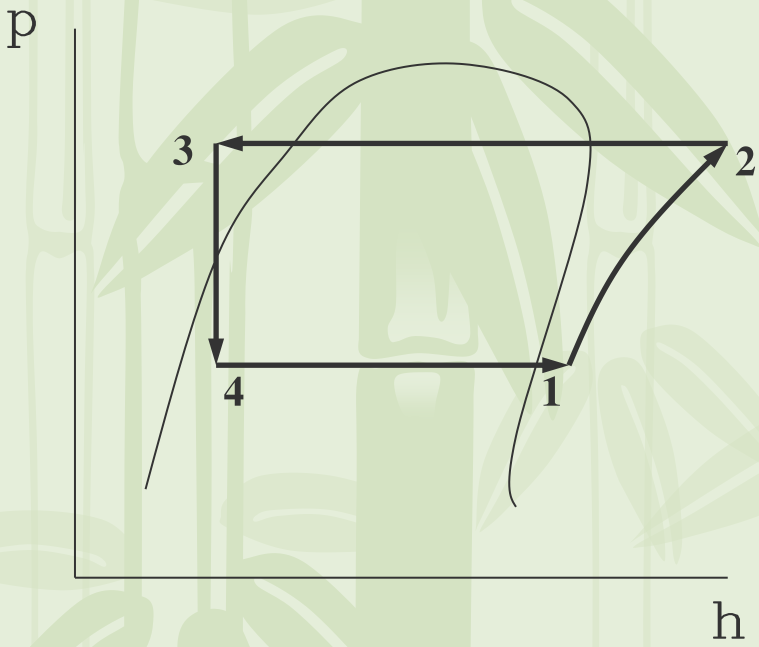
- 能力 (冷凝器,蒸發器) 高低壓 ($P_{con}$, $P_{eva}$)
- 壓縮機吸入與吐出溫度 ($T_{suc}$, $T_{dis}$)
- 冷凝器出口過冷度 ($T_{sub}$)

主要未知參數

高低壓 ($P_{con}$, $P_{eva}$)　與壓縮機吸入與吐出溫度 $T_{suc}$

# 計算流程

假設三個主要為之參數($P_{con}$, $P_{eva}$)與壓縮機吸入與吐出溫度 $T_{suc}$



(1) 由 $P_{eva}$ & $T_{suc}$ 算出壓縮機入口冷媒密度

(2) 由壓縮機轉速與壓縮機壓縮機的吸入容積與壓縮機容積效率算出吸入冷媒流量 m

(3) 由此一蒸發器出口條件，根據給定的蒸發器尺寸，反算出蒸發能力$Q_{eva}$ 與蒸發器入口乾度 x

(4) 假設壓縮機為等熵過程，由猜測的$P_{con}$ & S 得知冷凝器的入口溫度與壓縮功 $W = m(h_2 - h_1)$

(5) 由冷凝器的入口條件，算出出口冷媒溫度與冷凝器能力 $Q_{con}$

(6) 由冷凝器出口條件與給定毛細管算出出口毛細管壓力 ($P_e$)與出口乾度 $x_e$

(7) 檢查

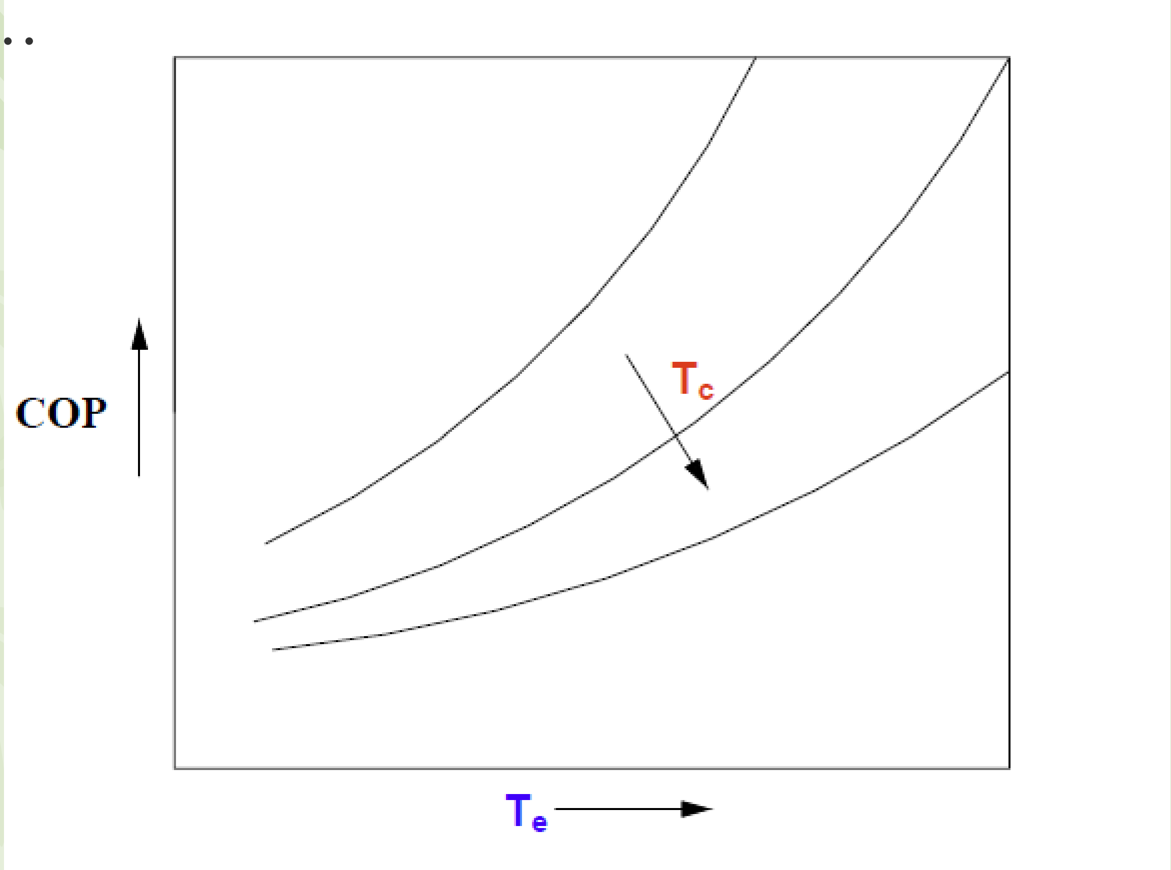
(1) $P_e = P_{eva}$ ?

(2) $Q_{con} = Q_{eva} + W$?

(3) $x_e = x$ ?

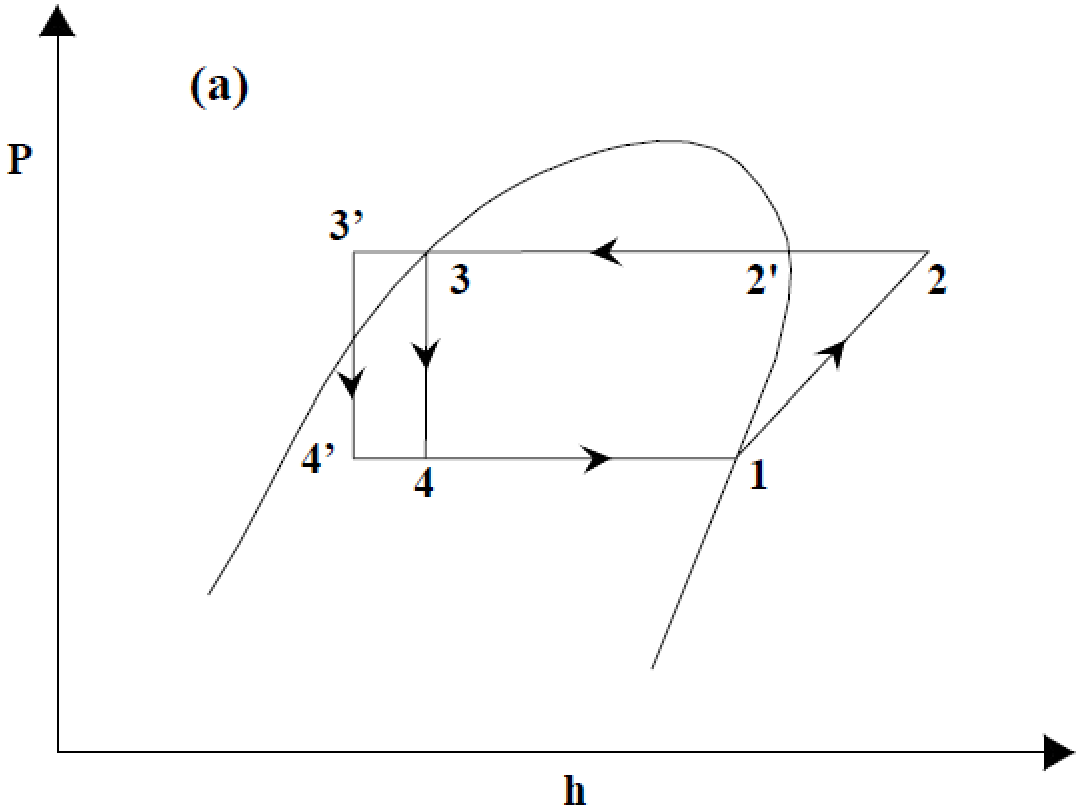
# Some Typical Methods for Enhancing System COP

Improve components efficiency: increase Heat Exchanger Size, Improve HX performance, improve fan performance, use high efficiency compressor…



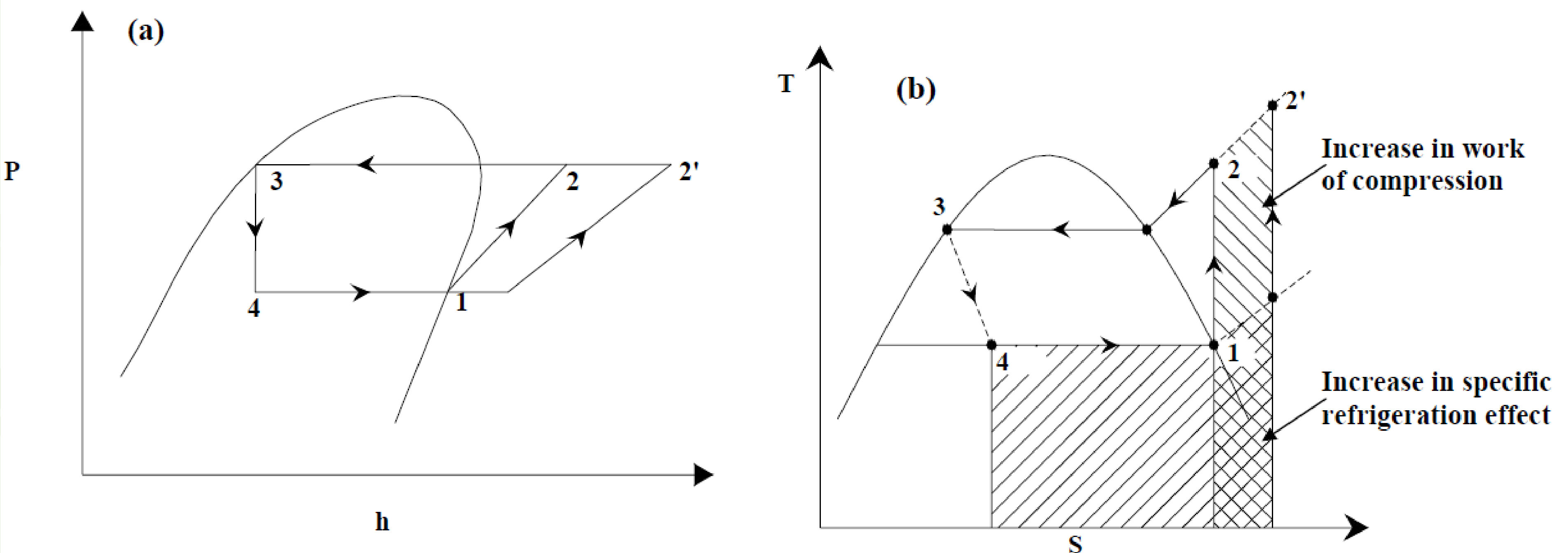
# Increase subcooling at the condenser

(without additional compression work)

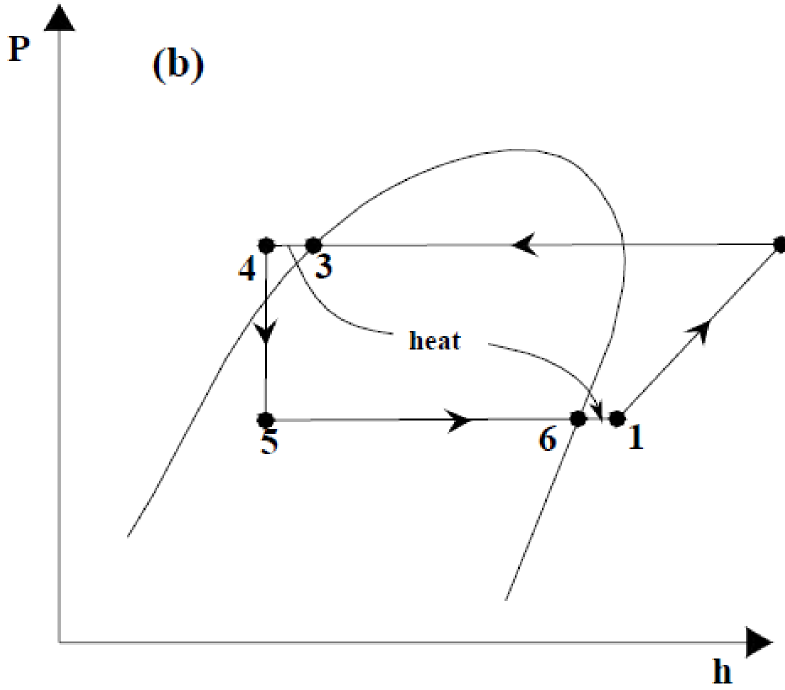


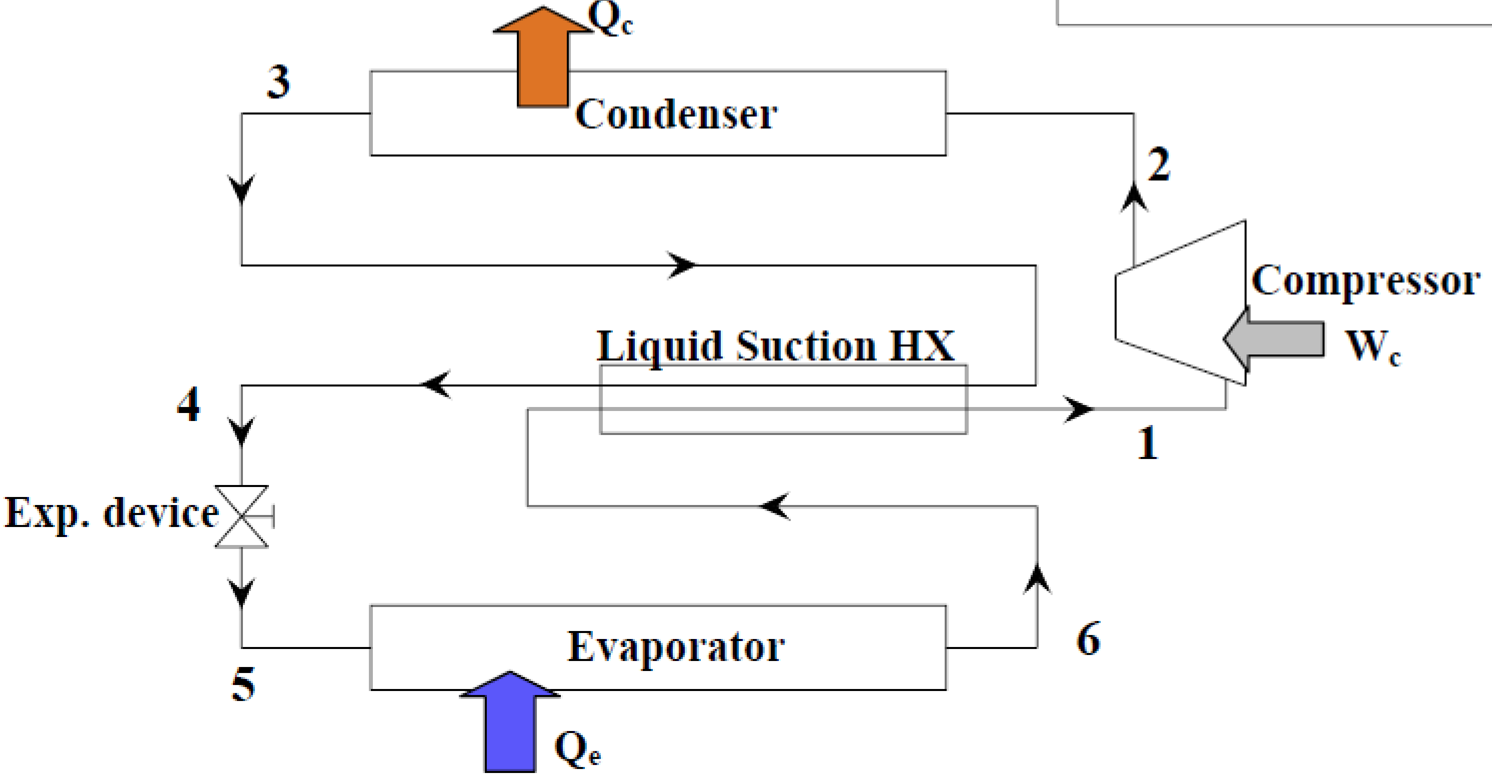
# Useful superheat

Useful superheating increases both the refrigeration effect as well as the work of compression. Hence the COP (ratio of refrigeration effect and work of compression) may or may not increase with superheat, depending mainly upon the nature of the working fluid. Even though useful superheating may or may not increase the COP of the system, a minimum amount of superheat is desirable as it prevents the entry of liquid droplets into the compressor.

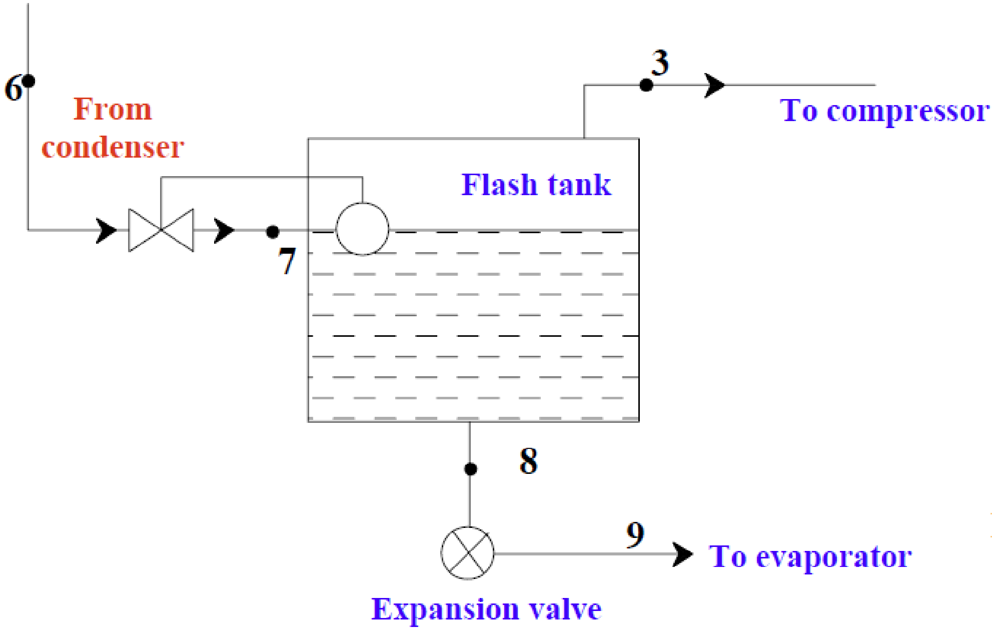


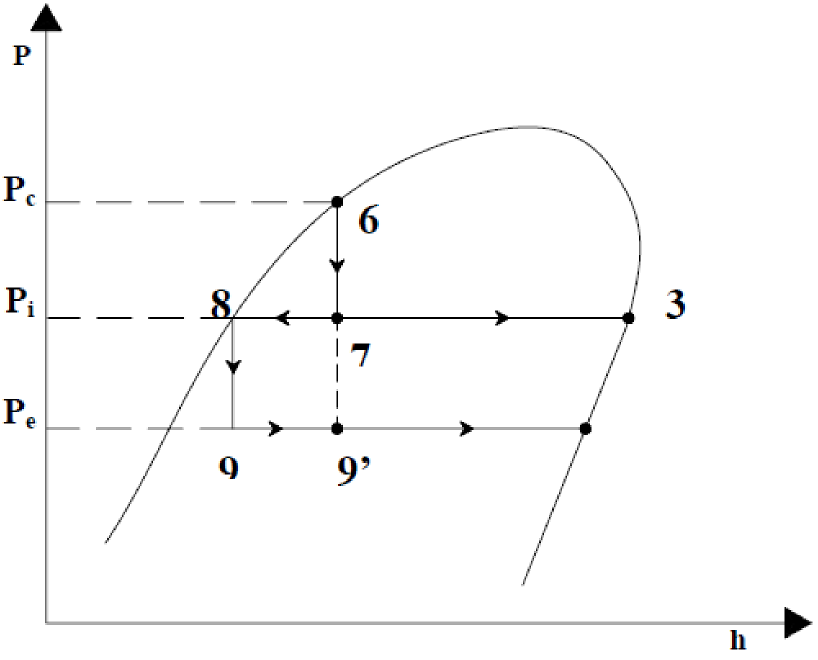
# Use of liquid-suction heat exchanger



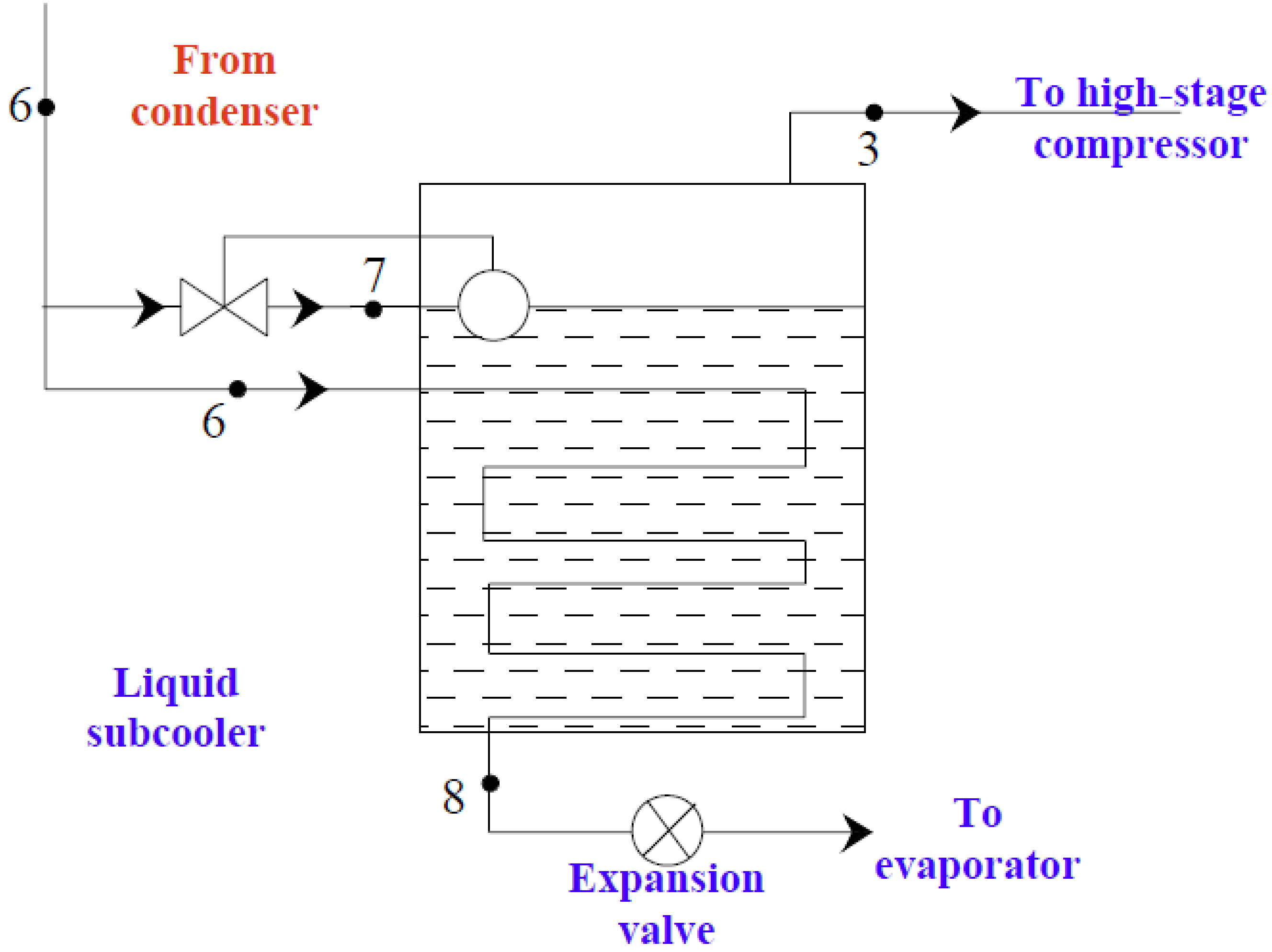


# Flash gas removal using flash tank (receiver)

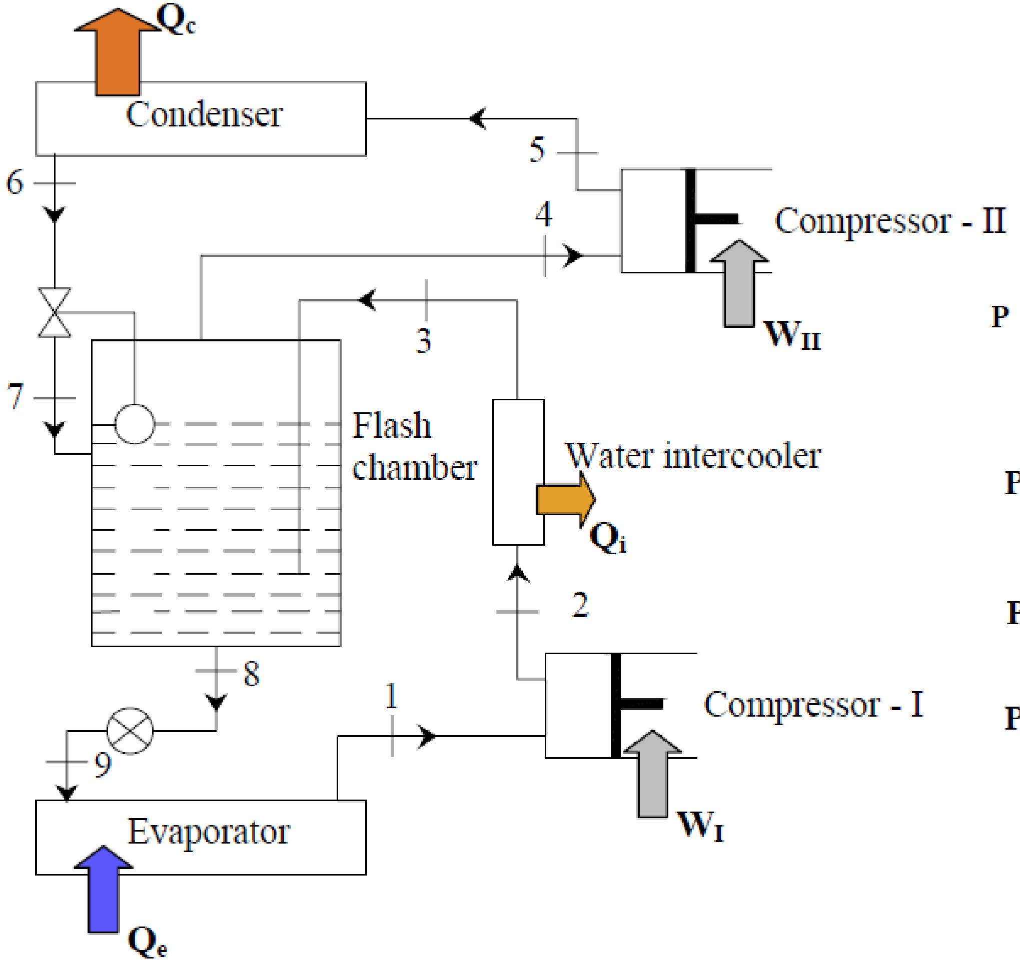


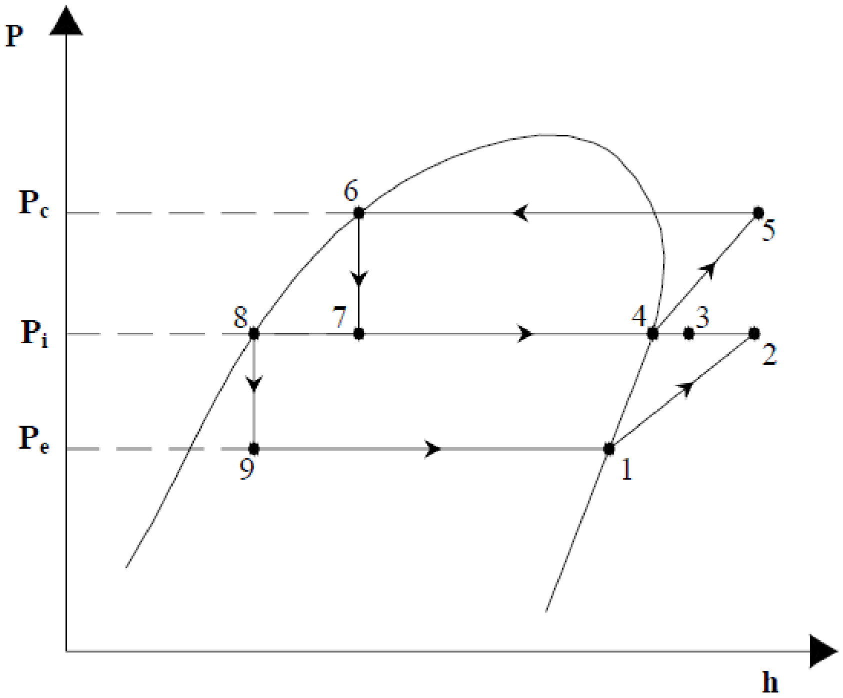


# Refrigeration system with liquid subcooler

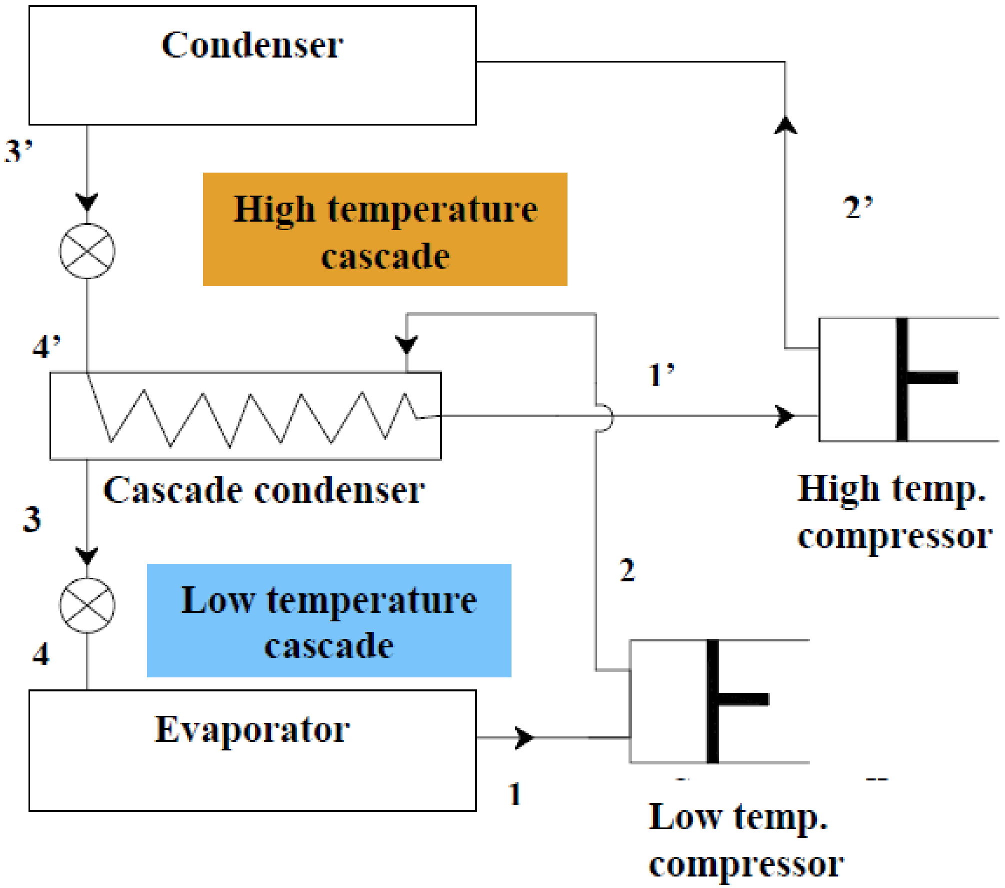


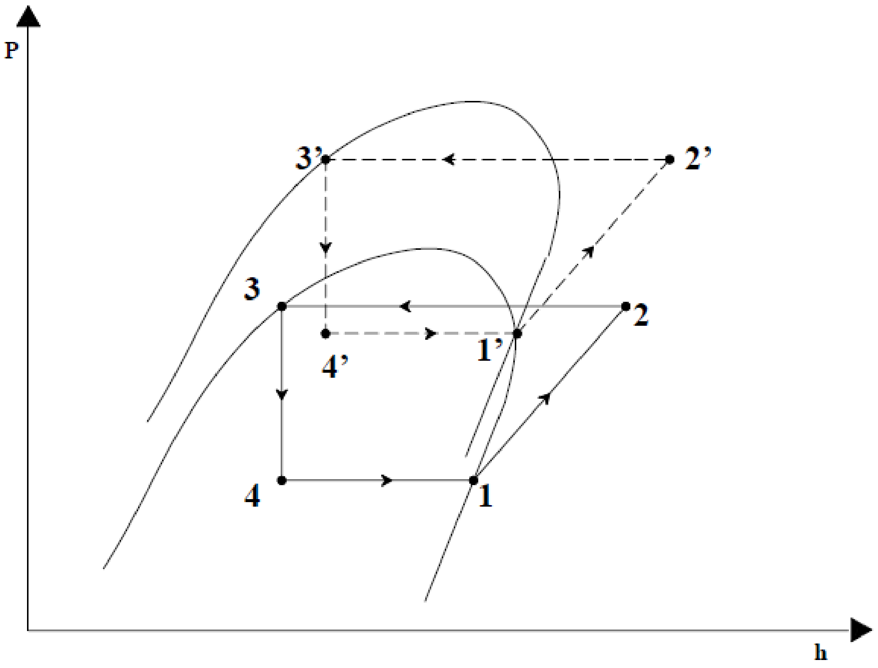
# Multi-stage system with flash gas removal and intercooling

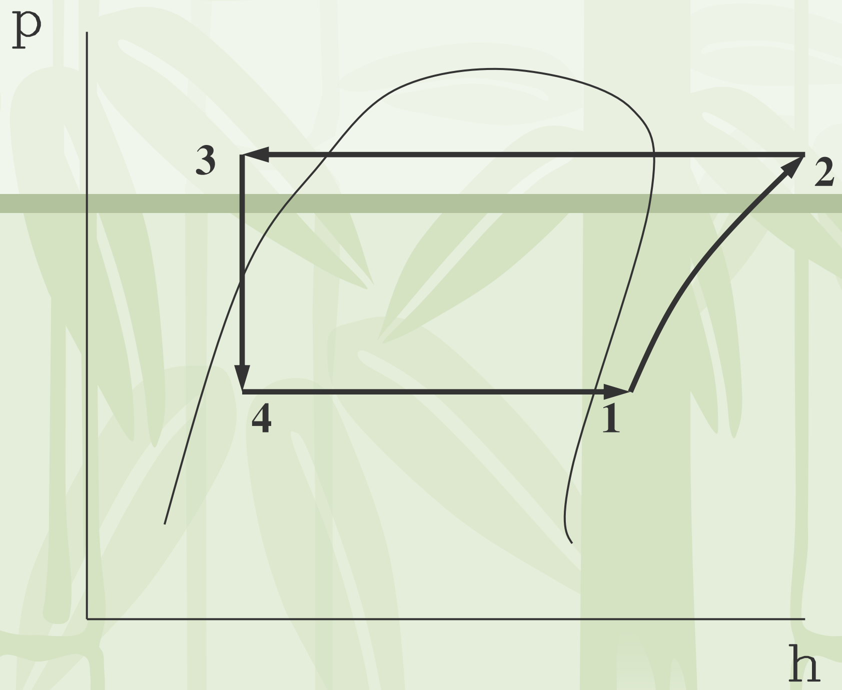




# Cascade refrigeration system



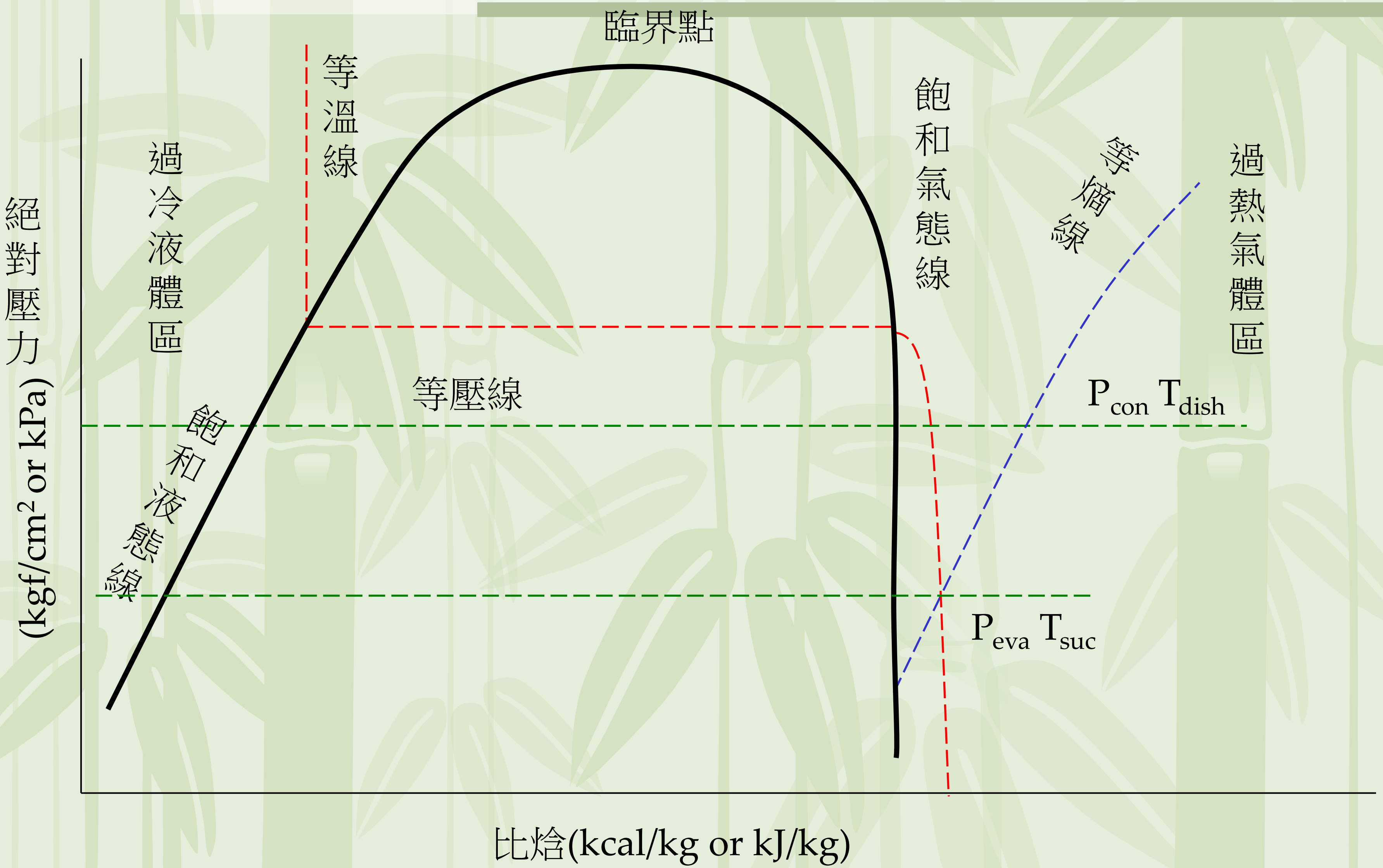




# 壓縮機流程 1→2

# Process 1-2

## 由 S = CONST 與 $P=P_{con}$ 得到 $T_{dish}$



# Types of compressors

- Positive displacement type
- Roto-dynamic type

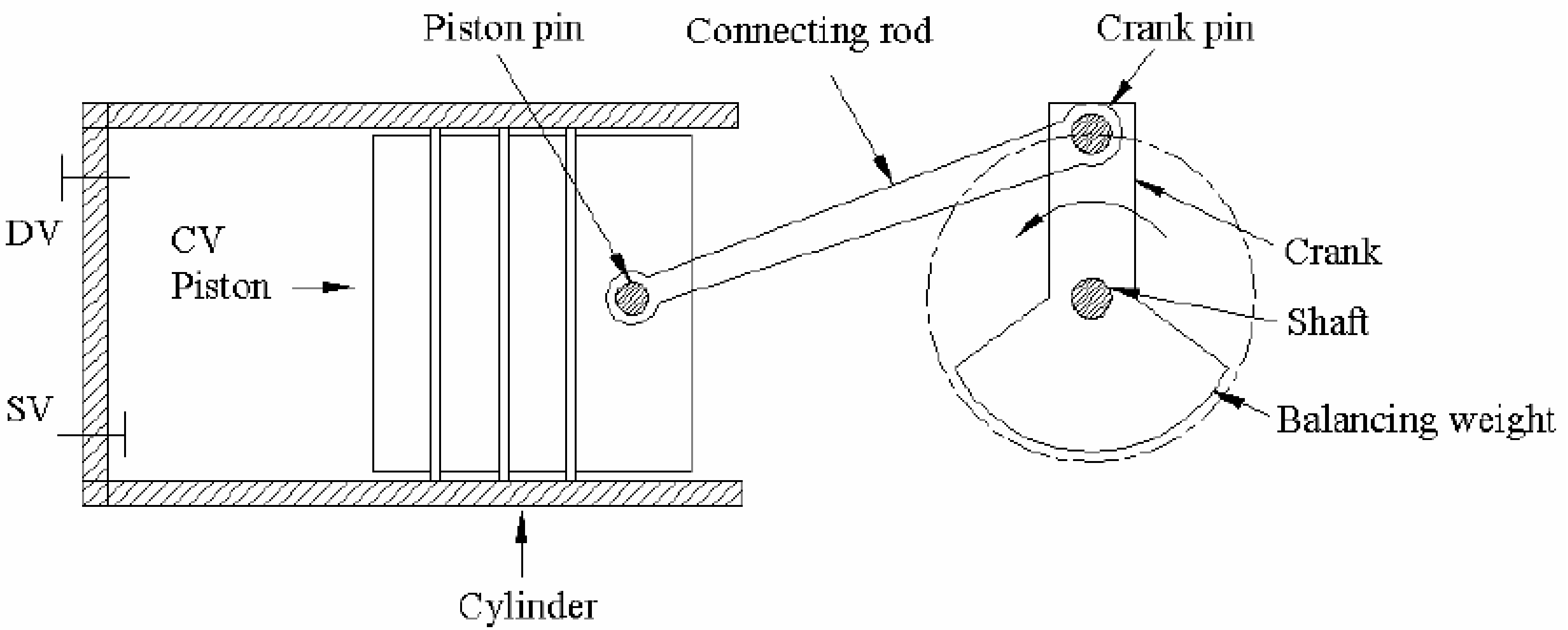
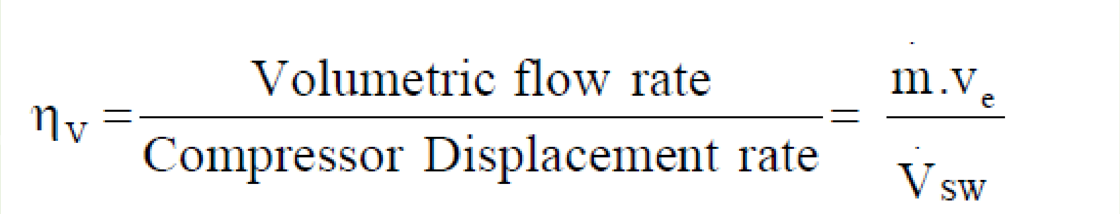


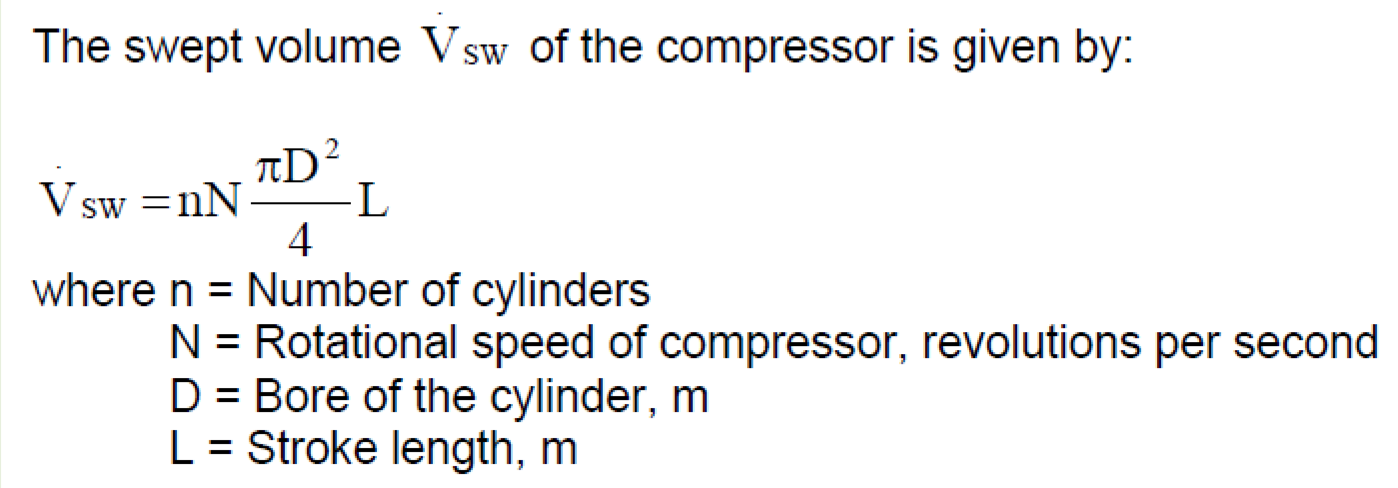
**Fig 18.1**: *Schematic of a reciprocating compressor*

# Volumetric efficiency

- The mass flow rate decides the refrigeration capacity of the system and for a given compressor inlet condition, it depends on the volumetric efficiency of the compressor. The volumetric efficiency, $\eta_v$ is defined as the ratio of volumetric flow rate of refrigerant to the maximum possible volumetric flow rate, which is equal to the compressor displacement rate, i.e.,



$$\eta_V = \frac{\text{Volumetric flow rate}}{\text{Compressor Displacement rate}} = \frac{\dot{m}.v_e}{\dot{V}_{SW}}$$

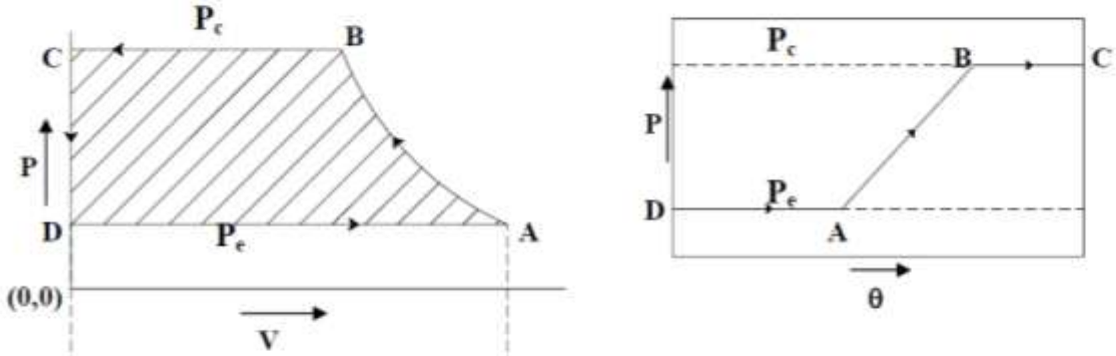


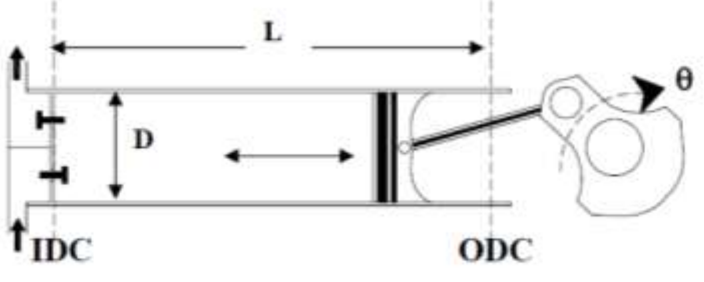
The swept volume $\dot{V}_{SW}$ of the compressor is given by:

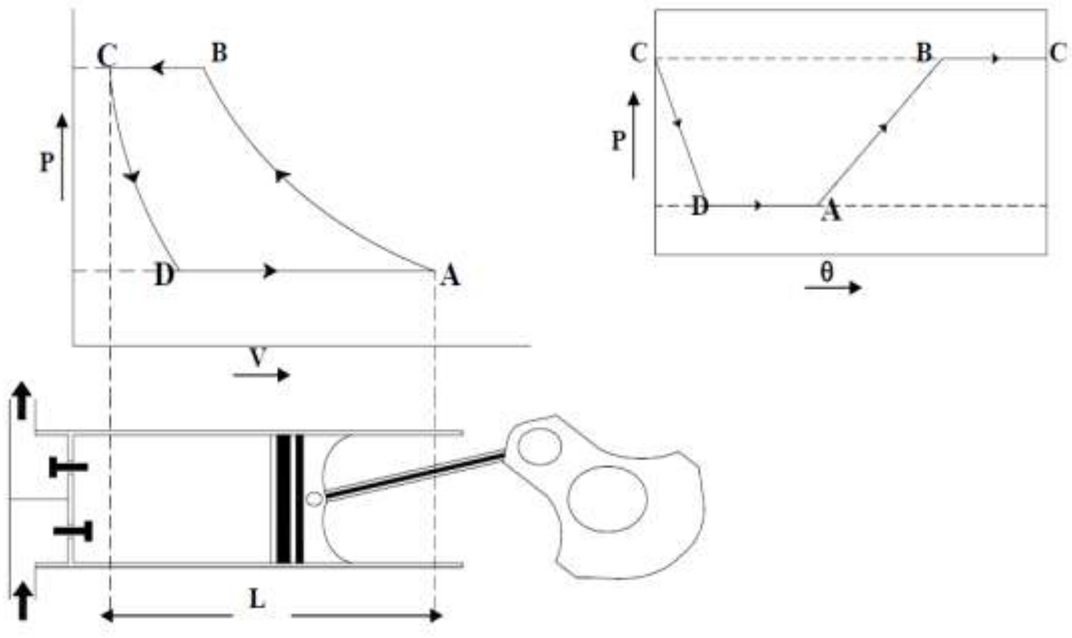
$$\dot{V}_{SW} = nN\frac{\pi D^2}{4}L$$

where n = Number of cylinders
N = Rotational speed of compressor, revolutions per second
D = Bore of the cylinder, m
L = Stroke length, m

# Ideal reciprocating compressor vs. real compression with clearance







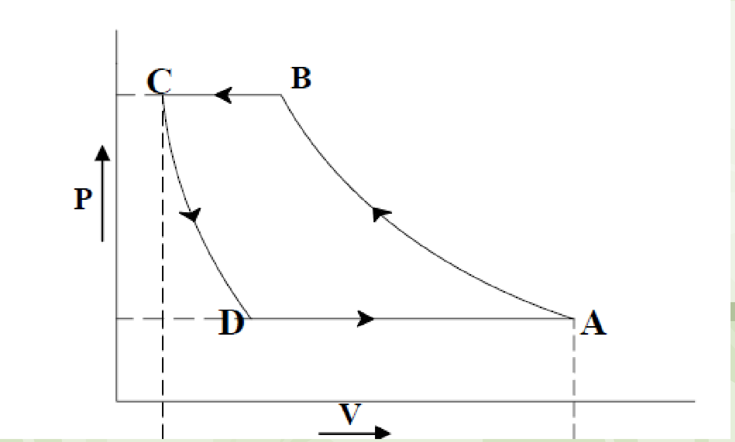
# Ideal compressor with clearance - Reciprocating Compressor

- In actual compressors, a small clearance is left between the cylinder head and piston to accommodate the valves and to take care of thermal expansion and machining tolerances. As a thumb rule, the clearance C in millimetres is given by:

C = (0.005L + 0.5) mm, where L is stroke length in mm

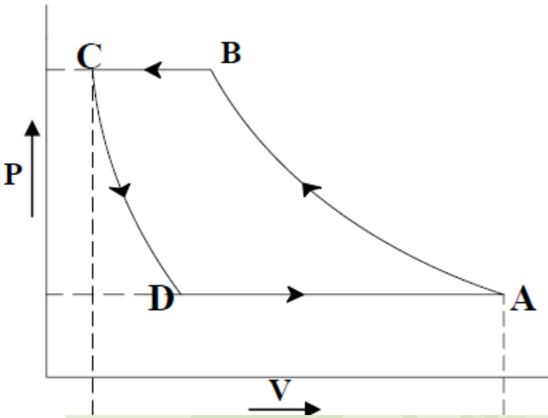
- This space along with all other spaces between the closed valves and the piston at the inner dead center (IDC) is called as Clearance volume, Vc. The ratio of the clearance volume to the swept volume is called as Clearance ratio, ε, i.e.,

$$\varepsilon = \frac{V_c}{V_{sw}}$$



- Due to the presence of the clearance volume, at the end of the discharge stroke, some amount of refrigerant at the discharge pressure $P_c$ will be left in the clearance volume. As a result, suction does not begin as soon as the piston starts moving away from the IDC, since the pressure inside the cylinder is higher than the suction pressure ($P_c > P_e$)
- As a result, the volumetric efficiency of the compressor with clearance, $\eta_{V,cl}$ is less than 100 percent

$$\eta_{V,cl} = \frac{\text{Actual volume of refrigerant compressed}}{\text{Swept volume of the compressor}} = \left(\frac{V_A - V_D}{V_A - V_C}\right)$$



- the clearance volumetric efficiency can be written as:

$$\eta_{V,cl}=\left(\frac{V_A-V_D}{V_A-V_C}\right)=\frac{(V_A-V_C)+(V_C-V_D)}{(V_A-V_C)}=1+\left(\frac{(V_C-V_D)}{(V_A-V_C)}\right)$$

$$\varepsilon=\frac{Vc}{V_{SW}}=\frac{V_C}{V_A-V_C}\Rightarrow(V_A-V_C)=\frac{V_C}{\varepsilon}$$

- Substituting the above equation in the expression for clearance volumetric efficiency; we can show that:

$$\eta_{V,cl}=1+\left(\frac{(V_C-V_D)}{(V_A-V_C)}\right)=1+\frac{\varepsilon(V_C-V_D)}{V_C}=1+\varepsilon-\varepsilon\left(\frac{V_D}{V_C}\right)$$

- Since the mass of refrigerant in the cylinder at points C and D are same, we can express the ratio of cylinder volumes at points D and C in terms of ratio of specific volumes of refrigerant at D and C, i.e.,

$$\left(\frac{V_D}{V_C}\right)=\left(\frac{v_D}{v_C}\right)$$

- the clearance volumetric efficiency is given by

$$\eta_{V,cl} = 1+\varepsilon-\varepsilon\left(\frac{V_D}{V_C}\right)=1+\varepsilon-\varepsilon\left(\frac{v_D}{v_C}\right)$$

- If we assume the re-expansion process also to follow the equation $Pv^k$ = constant, then:

$$\left(\frac{v_D}{v_C}\right)=\left(\frac{P_C}{P_D}\right)^{1/k}=\left(\frac{P_c}{P_e}\right)^{1/k}$$

- the clearance volumetric efficiency is given by:

$$\eta_{V,cl} = 1+\varepsilon-\varepsilon\left(\frac{P_c}{P_e}\right)^{1/k}=1-\varepsilon\left[r_p{}^{1/k}-1\right]$$

- where r is the pressure ratio, $P_c/P_e$.

- This limiting pressure ratio is obtained from the equation:

$$\eta_{V,cl} = 1-\varepsilon\left[r_p^{\,1/n}-1\right]=0$$

$$\Rightarrow r_{p,max} = \left[\frac{1+\varepsilon}{\varepsilon}\right]^{n}$$

- The mass flow rate of refrigerant compressed with clearance, $\dot{m}_{cl}$, is given by:

$$\dot{m}_{cl} = \eta_{V,cl}\frac{\dot{V}_{SW}}{v_e}$$

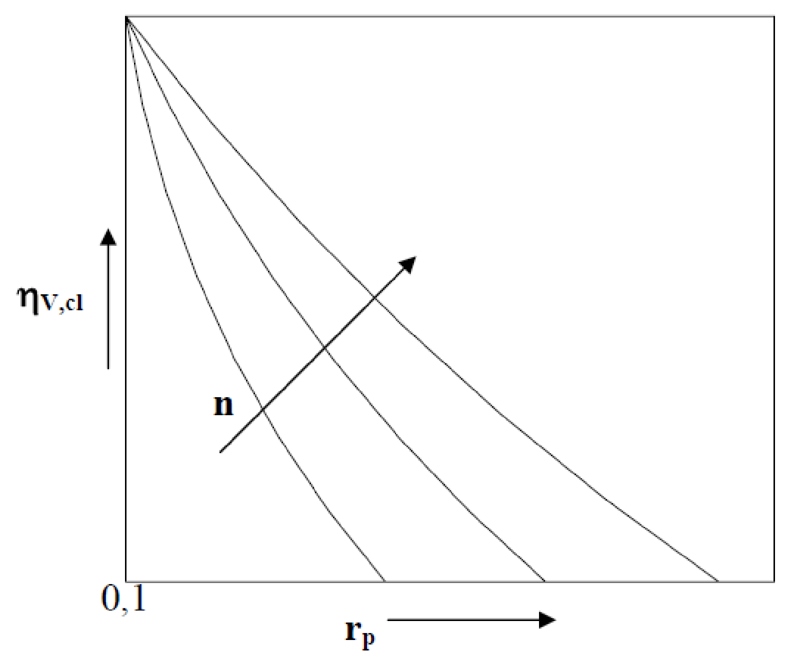
- Thus the mass flow rate and hence the refrigeration capacity of the system decreases as the volumetric efficiency reduces, in other words, the required size of the compressor increases as the volumetric efficiency decreases.

$$\eta_{V,cl} = 1 + \varepsilon - \varepsilon\left(\frac{P_c}{P_e}\right)^{1/n} = 1 - \varepsilon\left[r_p^{\ 1/n} - 1\right]$$

- The above expression shows that $\eta_{V,cl}$ ↓ as $r_p$ ↑ and ε ↑
- It can also be seen that for a given compressor with fixed clearance ratio ε, there is a limiting pressure ratio at which the clearance volumetric efficiency becomes zero. This limiting pressure ratio is obtained from the equation:

$$\eta_{V,cl} = 1 - \varepsilon\left[r_p^{\ 1/n} - 1\right] = 0$$

$$\Rightarrow r_{p,max} = \left[\frac{1+\varepsilon}{\varepsilon}\right]^n$$



# Work input to the compressor with clearance

- If we assume that both compression and expansion follow the same equation $Pv^n$ = constant (i.e., the index of compression is equal to the index of expansion), then the extra work required to compress the vapor that is left in the clearance volume will be exactly equal to the work output obtained during the re-expansion process. Hence, the clearance for this special case does not impose any penalty on work input to the compressor. The total work input to the compressor during one cycle will then be equal to the area A-B-C-D-A on P-V diagram.
- The specific work with and without clearance will be given by the same expression:

$$W_{id} = \int_{Pe}^{Pc} v.dP = P_e v_e \left(\frac{n}{n-1}\right)\left[\left(\frac{Pc}{Pe}\right)^{\frac{n-1}{n}} - 1\right]$$

- Since the mass of refrigerant compressed during one cycle is different with and without clearance, the power input to the compressor will be different with and without clearance. The power input to the compressor and mean effective pressure (mep) with clearance are given by:

$$W_c = \dot{m}\, w_{id} = \left(\eta_{V,cl} \frac{\dot{V}_{SW}}{v_e}\right) w_{id} \qquad \text{mep} = \eta_{V,cl} \frac{w_{id}}{v_e}$$

- Thus the power input to the compressor and mep decrease with clearance due to decrease in mass flow rate with clearance.
- If the process is reversible and adiabatic (i.e., n = k), then the power input to the compressor with clearance is given by:

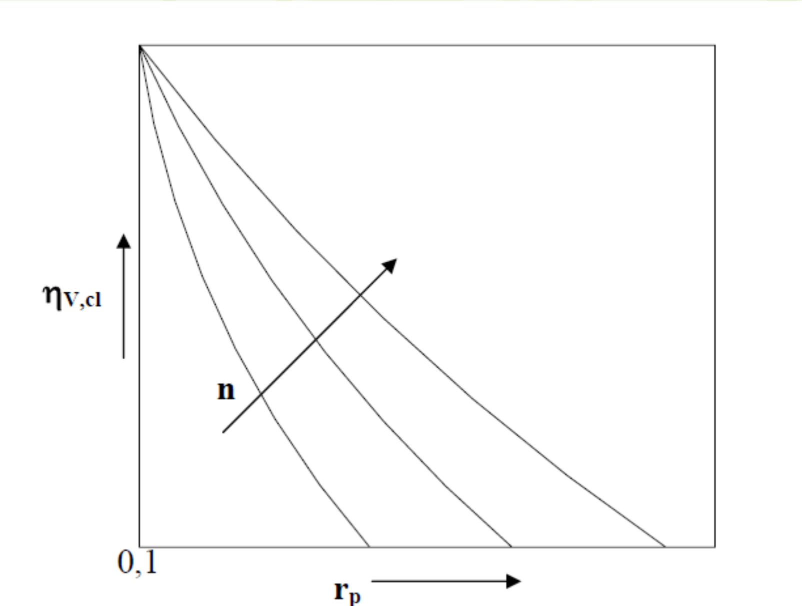
$$W_c = = \left(\eta_{V,cl} \frac{\dot{V}_{SW}}{v_e}\right)(h_B - h_A) = \left(\eta_{V,cl} \frac{\dot{V}_{SW}}{v_e}\right) \Delta h_{c,s}$$

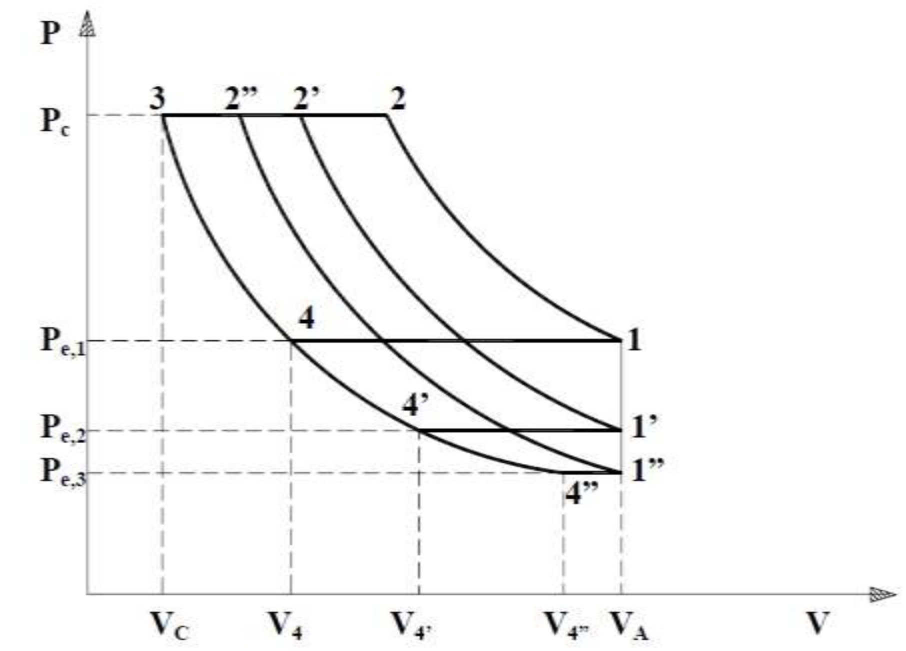
  - where Δh is the isentropic work of compression (kJ/kg)

- The above expression holds good for any reversible compression process with clearance. If the process is not reversible, adiabatic (i.e., non-isentropic) but a reversible polytropic process with an index of compression and expansion equal to n, then k in the above equation has to be replaced by n, i.e., in general for any reversible compression process;

$$\eta_{V,cl} = 1 + \varepsilon - \varepsilon\left(\frac{P_c}{P_e}\right)^{1/n} = 1 - \varepsilon\left[r_p^{\;1/n} - 1\right]$$

- The above expression shows that $\eta^{V,cl}$ ↓ as $r^p$ ↑ and ε ↑ as shown in the Figure.



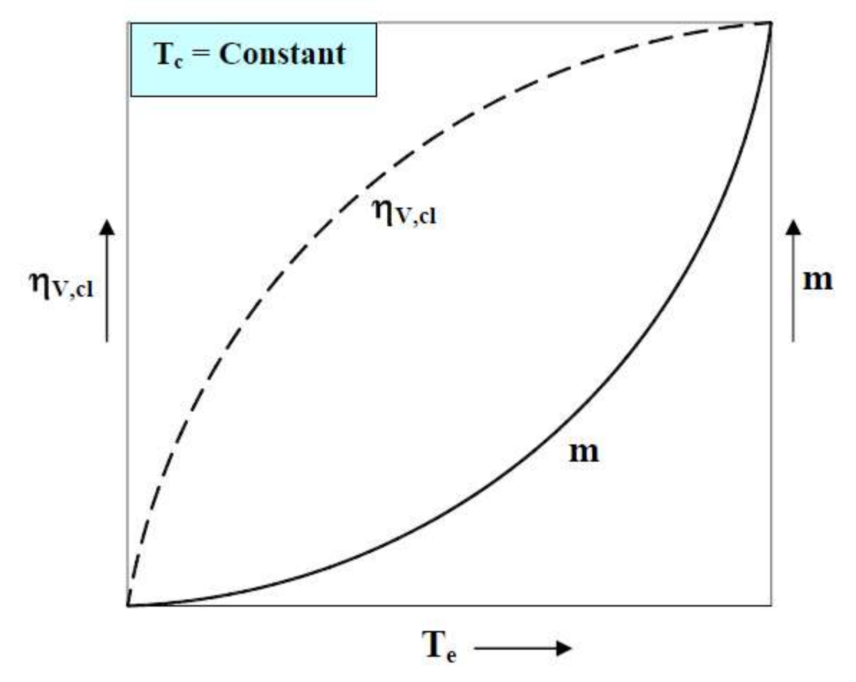


- For a given condensing temperature (or pressure), the pressure ratio $r_p$ increases as the evaporator temperature (or evaporator pressure) decreases. Hence, from the expression for clearance volumetric efficiency, it is obvious that the volumetric efficiency decreases as evaporator temperature decreases.
  - This is also explained with the figure, which shows the P-V diagram for different evaporator pressures. As shown, as the evaporator pressure decreases, the volume of refrigerant compressed decreases significantly, since the compressor displacement remains same the clearance volumetric efficiency decreases as evaporator temperature decreases.

- The above discussion shows that the performance of the system degrades as the evaporator temperature decreases and condensing temperature increases, i.e., the temperature lift increases. This is in line with the effect of these temperatures on reverse Carnot refrigeration system. It is seen that compared to the condensing temperature, the effect of evaporator temperature is quiet significant. When the heat sink temperature does not vary too much then the effect of condensing temperature may not be significant.

- As the evaporator temperature decreases the clearance volumetric efficiency decreases and the specific volume of refrigerant at compressor inlet $v_e$ increases. As a result of these two effects, the mass flow rate of refrigerant through the compressor decreases rapidly as the evaporator temperature decreases as shown in the following figure.
- The mass flow rate of refrigerant is given by

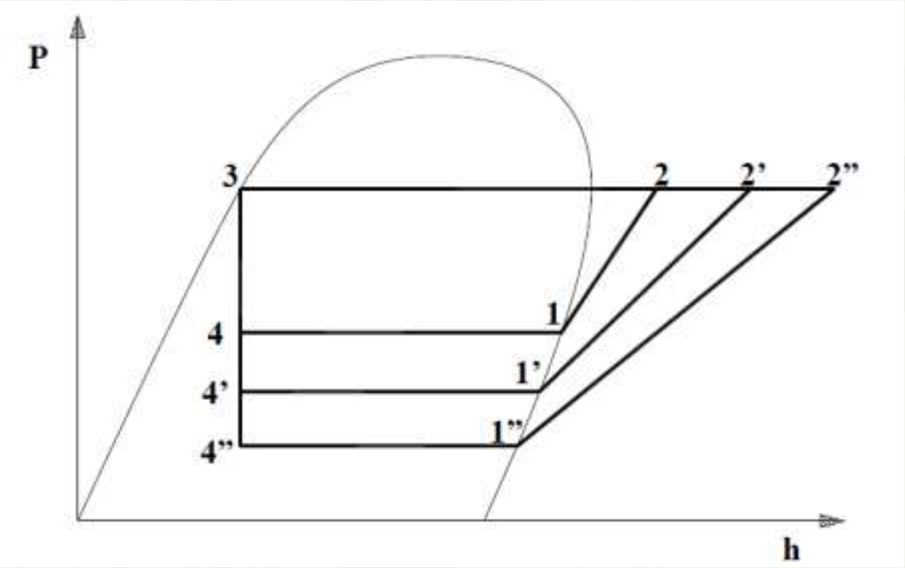
$$\dot{m} = \eta_{V,cl} \frac{\dot{V}_{sw}}{v_e}$$

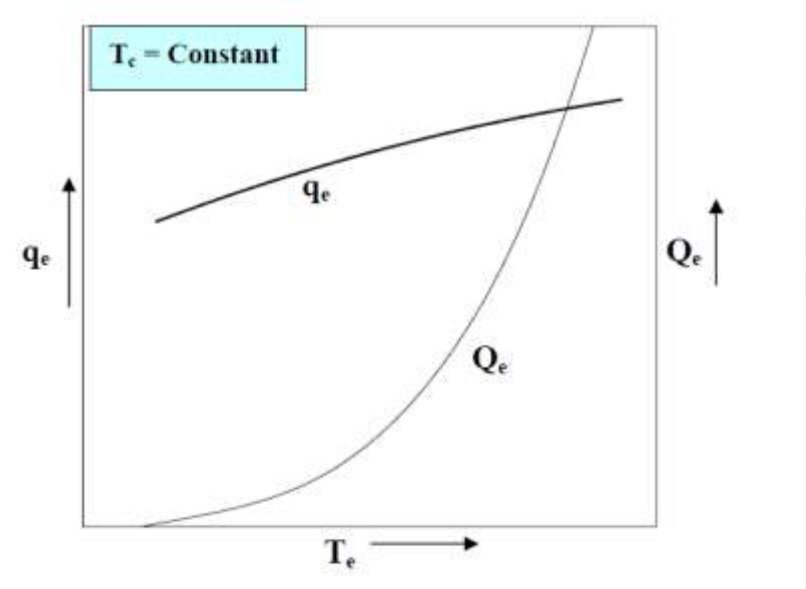


# On refrigeration effect and refrigeration capacity

- The refrigeration capacity of the compressor Q is given by: ($q_e = h_1 - h_4$)

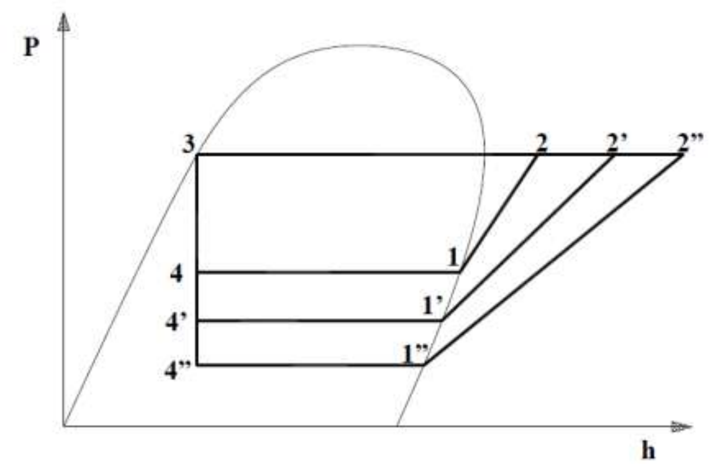
$$Q_e = \dot{m} . q_e$$

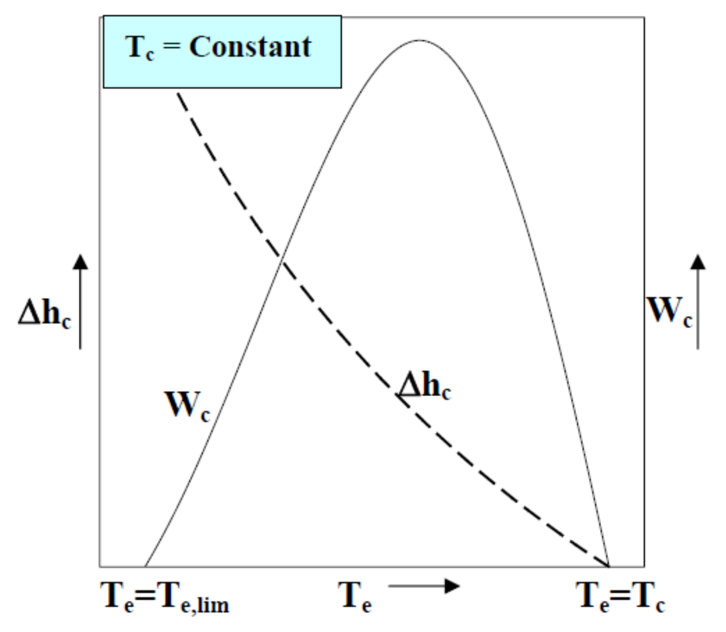




# On work of compression and power requirement

- The power input to the compressor is given by: $W_c = \dot{m}.\Delta h_c$
  - ($\Delta h_c = h_2 - h_1$)
- For a given clearance ratio and condenser temperature, the volumetric efficiency and hence the mass flow rate becomes zero at a lower limiting value of evaporator temperature ($T_e = T_{e,lim}$).

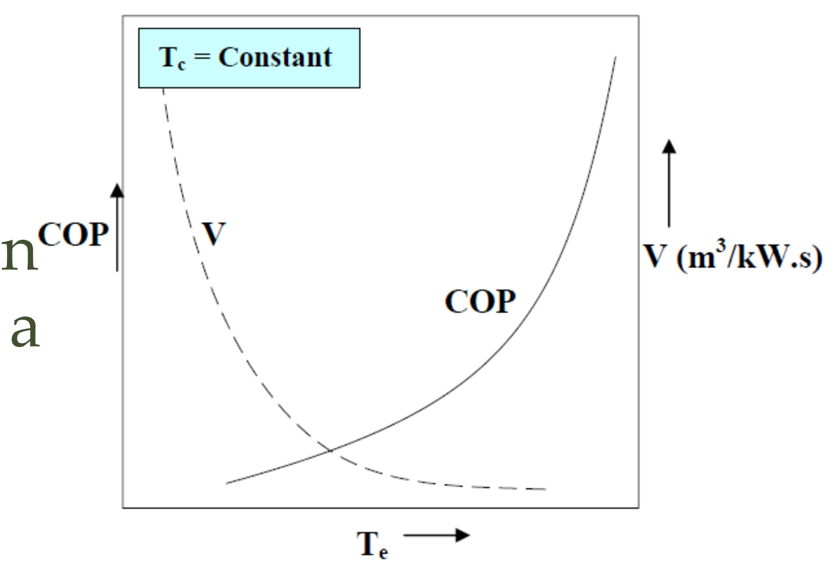




# On COP and volume flow rate per unit capacity

- As evaporator temperature increases the specific volume of the refrigerant at compressor inlet reduces rapidly and the refrigerant effect increases marginally. Due to the combined effect of these two (and volumetric efficiency), the volume flow rate of refrigerant per unit capacity reduces sharply with evaporator temperature as shown in the following. This implies that for a given refrigeration capacity, the required volumetric flow rate and hence the size of the compressor becomes very large at very low evaporator temperatures.

$$V = \frac{\eta_{v,cl} \cdot \dot{V}_{sw}}{Q_e} = \frac{v_e}{q_e}$$



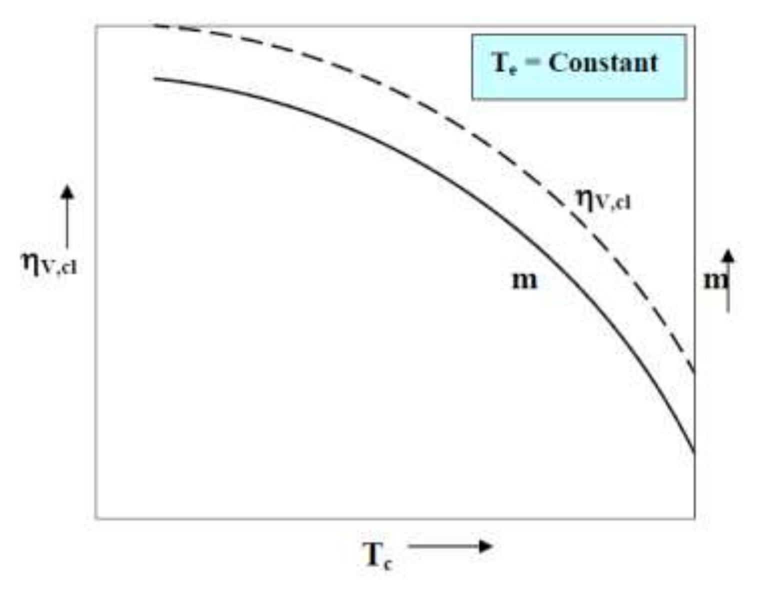
# Effect of condenser temperature

- Atmospheric air is the cooling medium for most of the refrigeration systems. Since the ambient temperature at a location can vary over a wide range, the heat rejection temperature (i.e., the condensing temperature) may also vary widely. This affects the performance of the compressor and hence the refrigeration system. The effect of condensing temperature on compressor performance can be studied by keeping evaporator temperature constant.

# On volumetric efficiency and refrigerant mass flow rate

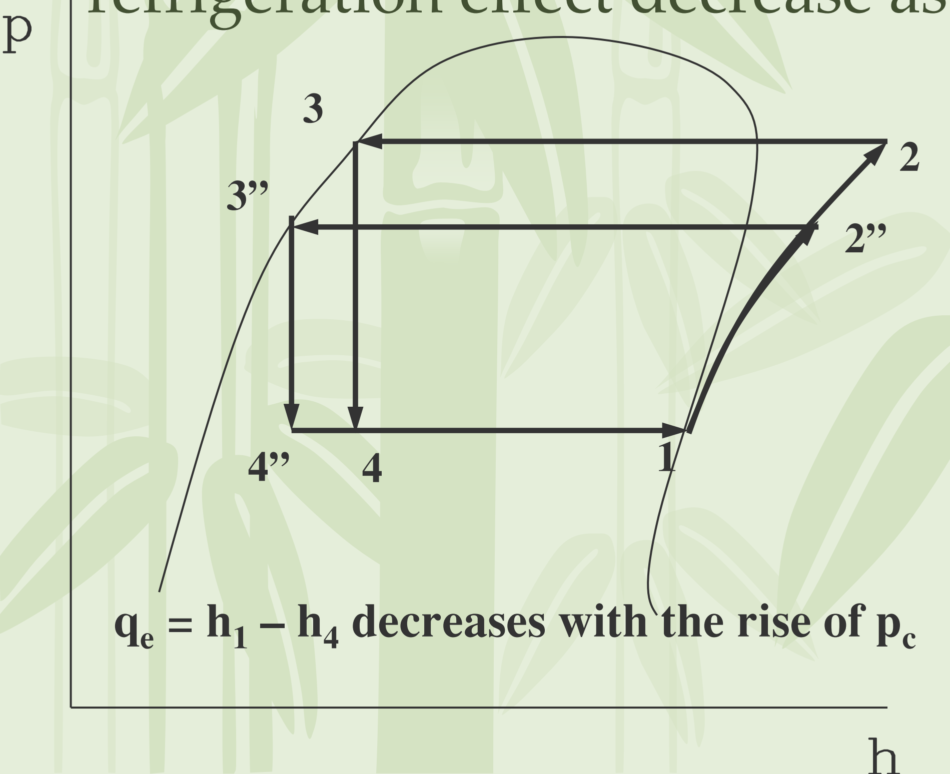
- At a constant evaporator temperature as the condensing temperature increases, the pressure ratio increases, hence, both the volumetric efficiency and mass flow rate decrease as shown in the figure. However, the effect of condensing temperature on mass flow rate is not as significant as the evaporator temperature as the specific volume of refrigerant at compressor inlet is independent of condensing temperature.

$$\eta_{V,cl} = 1+\varepsilon-\varepsilon\left(\frac{P_c}{P_e}\right)^{1/n} = 1-\varepsilon\left[r_p^{\,1/n}-1\right]$$

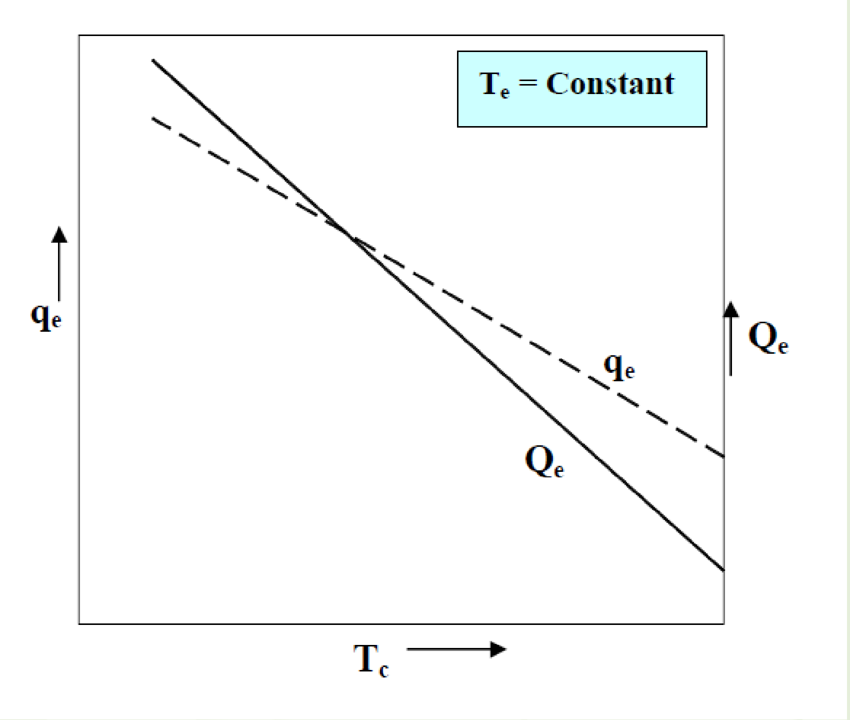


# On refrigeration effect and refrigeration capacity

- Since the evaporator enthalpy remains constant at a constant evaporator temperature, the refrigeration effect decreases with increase in condensing temperature as shown in Figure. The refrigeration capacity (Qe) also reduces with increase in condensing temperature as both the mass flow rate and refrigeration effect decrease as shown.

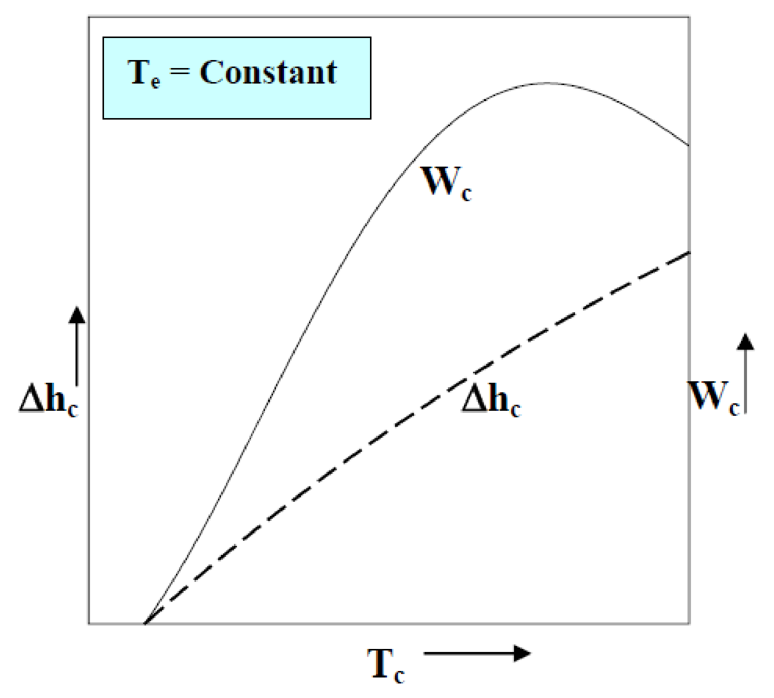


**$q_e = h_1 - h_4$ decreases with the rise of $p_c$**



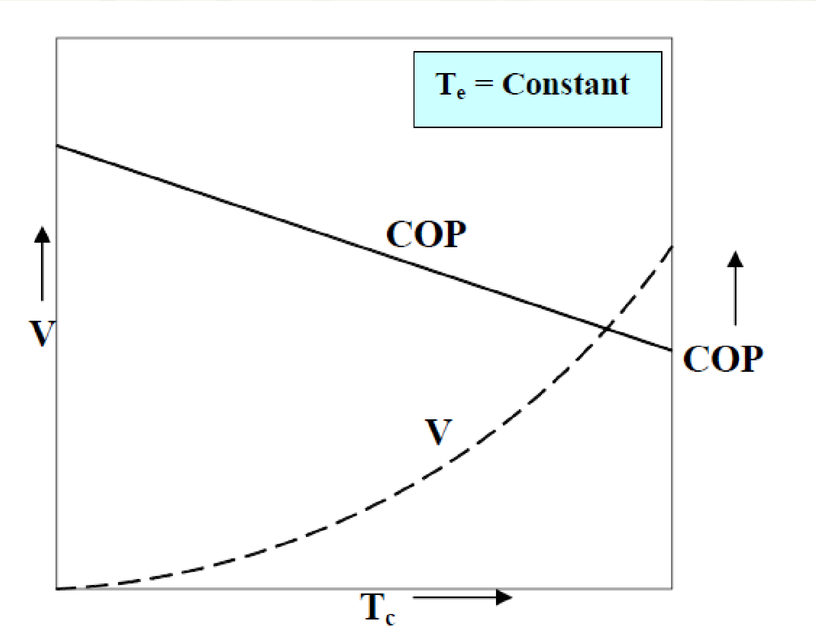
# On work of compression and power requirement

- The work of compression is zero when the condenser temperature is equal to the evaporator temperature, on the other hand at a limiting condensing temperature the mass flow rate of refrigerant becomes zero as the clearance volumetric efficiency becomes zero as explained before.



# On COP and volume flow rate per unit capacity

- As condensing temperature increases the refrigeration effect reduces marginally and work of compression increases, as a result the COP reduces as. Even though the specific volume at compressor inlet is independent of condensing temperature, since the refrigeration effect decreases with increase in condensing temperature, the volume flow rate of refrigerant per unit capacity increases as condenser temperature increases as shown.



# Compressor discharge temperature

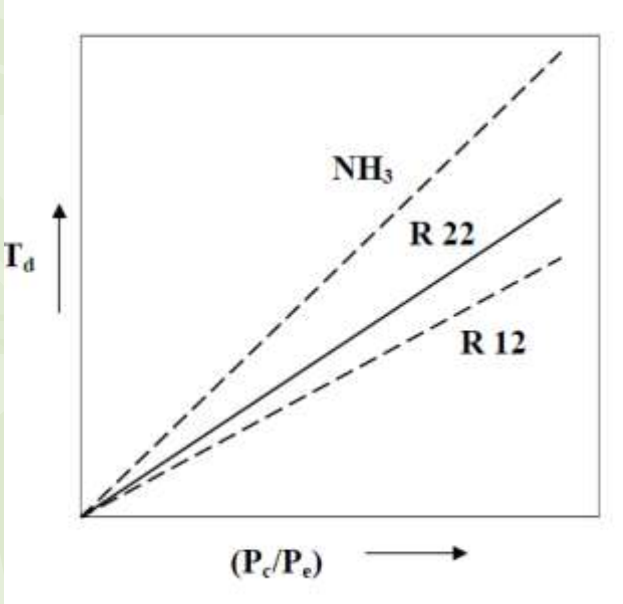
- If the compressor discharge temperature is very high then it may result in breakdown of the lubricating oil, causing excessive wear and reduced life of the compressor valves (mainly the discharge valve). In hermetic compressors, the high discharge temperature adversely affects the motor insulation (unless the insulation is designed for high temperatures).

$$Pv^{\gamma} = \text{constant} \quad \text{and } Pv = RT$$

- Then the discharge temperature,

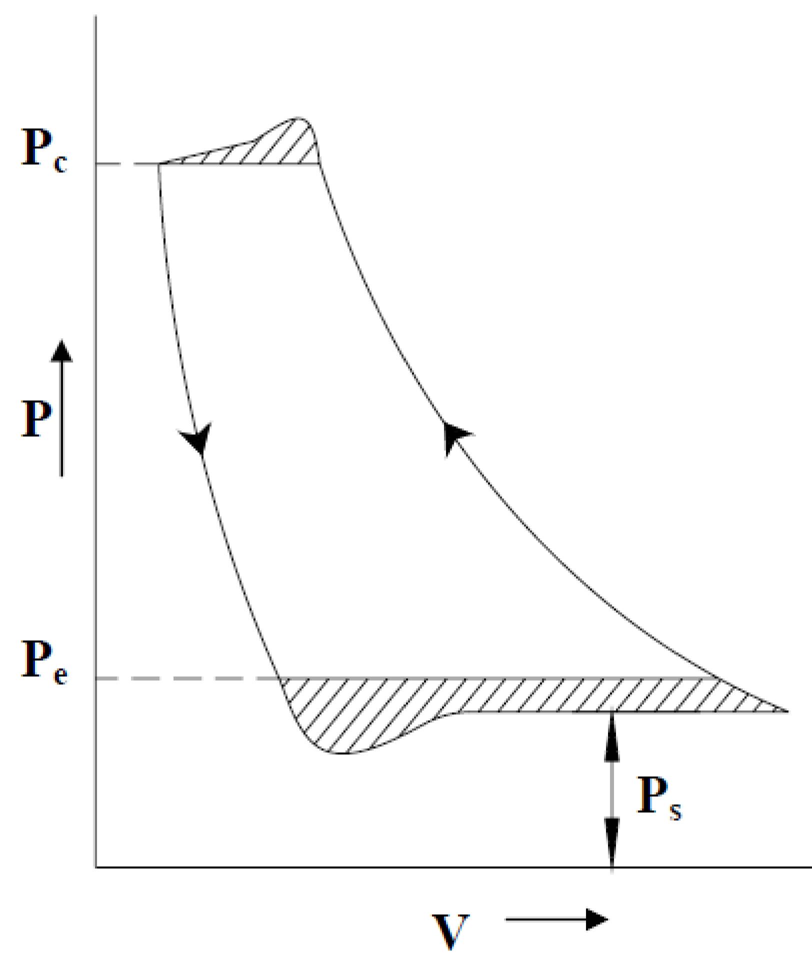
  T is given by:

$$T_d = T_e \left( \frac{P_c}{P_e} \right)^{\frac{\gamma - 1}{\gamma}}$$



# Actual compression process

- Actual compression processes deviate from ideal compression processes due to:
  - Heat transfer between the refrigerant and surroundings during compression and expansion, which makes these processes non-adiabatic
  - Frictional pressure drops in connecting lines and across suction and discharge valves
  - Losses due to leakage

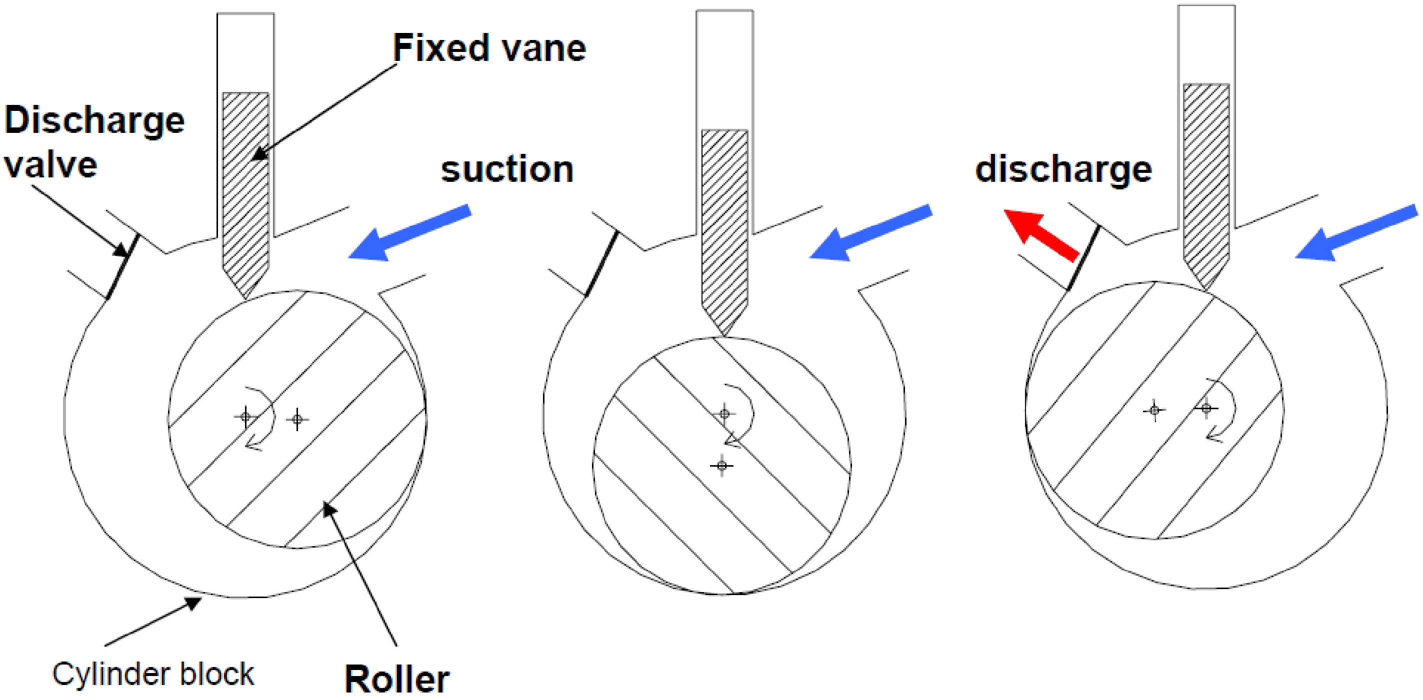


The mass flow rate decides the refrigeration capacity of the system and for a given compressor inlet condition, it depends on the volumetric efficiency of the compressor. The volumetric efficiency, $\eta_V$ is defined as the ratio of volumetric flow rate of refrigerant to the maximum possible volumetric flow rate, which is equal to the compressor displacement rate, i.e.,

$$\eta_V = \frac{\text{Volumetric flow rate}}{\text{Compressor Displacement rate}} = \frac{\dot{m}.v_e}{\dot{V}_{SW}} \qquad (18.1)$$

where $\dot{m}$ and $\dot{V}_{SW}$ are the mass flow rate of refrigerant (kg/s) and compressor displacement rate ($m^3/s$) respectively, and $v_i$ is the specific volume ($m^3/kg$) of the refrigerant at compressor inlet.

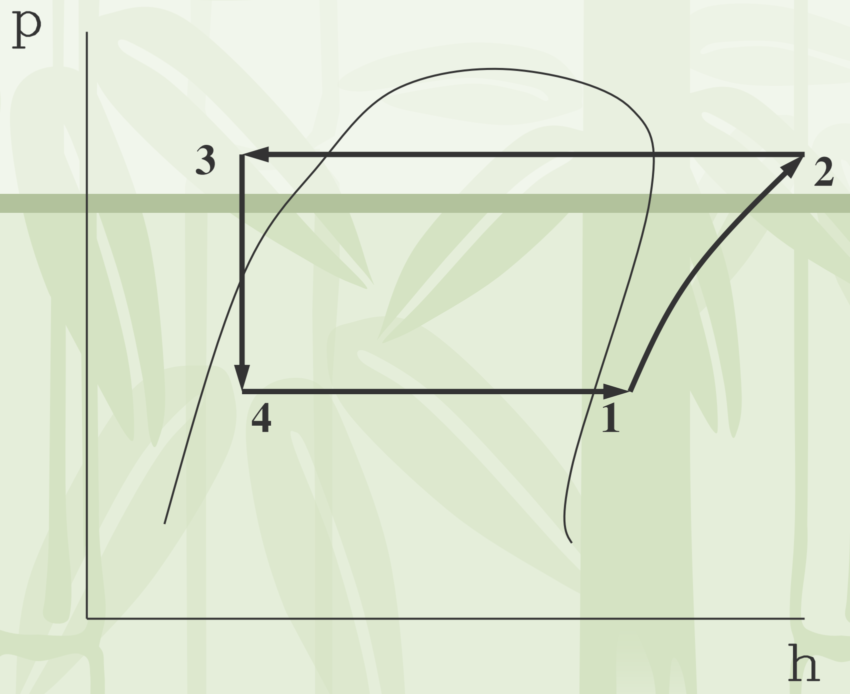
# Process 1-2 冷媒流量的計算



The mass flow rate of refrigerant through the compressor is given by:

$$\dot{m}=\eta_V\left(\frac{\dot{V}_{SW}}{v_e}\right)=\left(\frac{\eta_V}{v_e}\right)\left(\frac{\pi}{4}\right)\left(\frac{N}{60}\right)(A^2-B^2)L \tag{20.1}$$

where A = Inner diameter of the cylinder
B = Diameter of the roller
L = Length of the cylinder block
N = Rotation speed, RPM
$\eta_V$ = Volumetric efficiency
$v_e$ = specific volume of refrigerant at suction



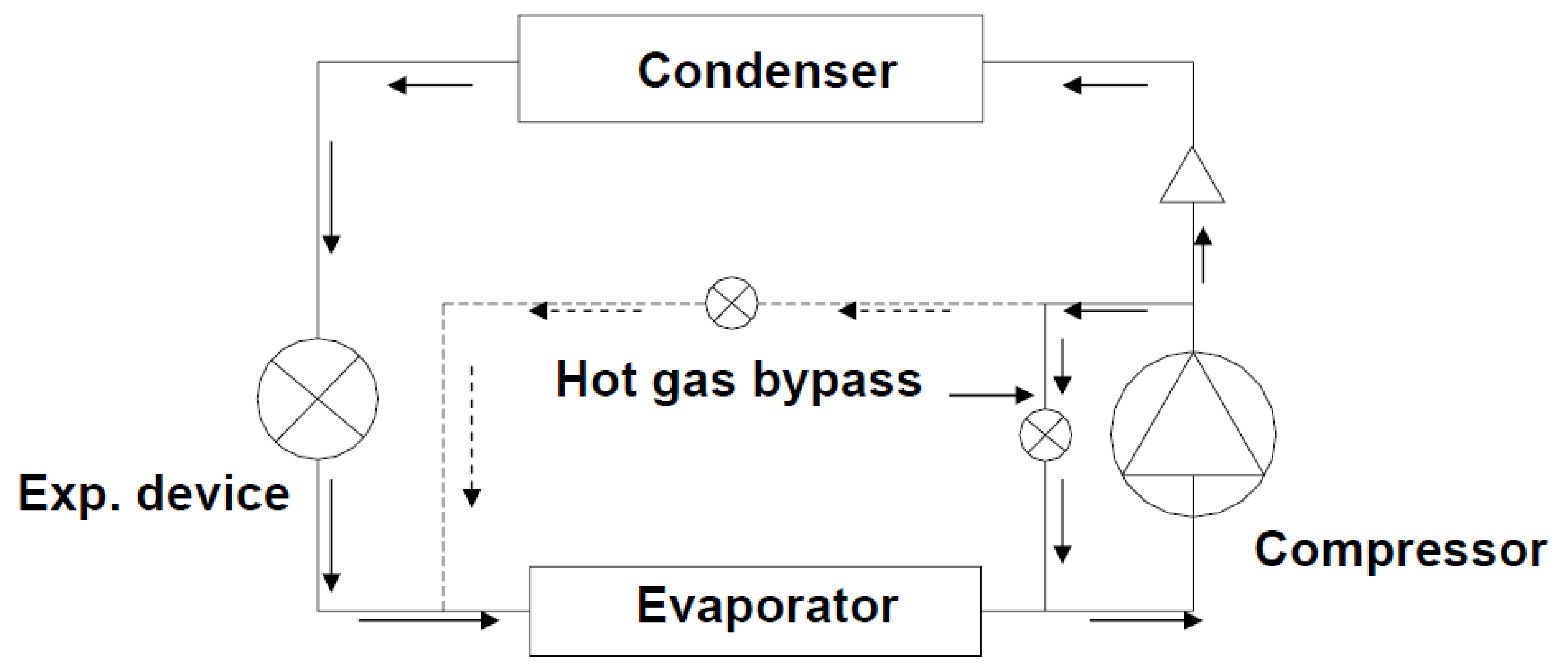
# Process 2-3

## 冷凝器設計流程

# Capacity control..

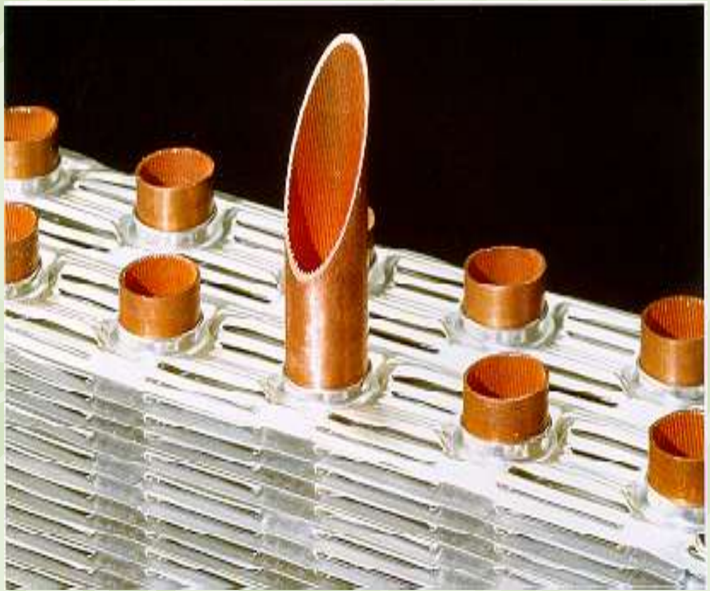
- Various methods available in practice for controlling the capacity of compressors are:
  - Cycling or on-off control
  - Back pressure regulation
  - Hot gas bypass
  - Unloading of cylinders in
  - Compressor speed control

# Hot gas bypass



# 鰭管式熱交換器介紹

- □熱交換器(heat exchanger)是一種熱能傳遞的裝置，藉由熱傳導（heat conduction），熱對流（heat convection）和熱輻射（heat radiation）等三種熱傳現象，使兩種（或多種）不同溫度的流體得以彼此交換能量，達到熱量傳遞的目的。一般空調設備有兩個重要的熱交換器：冷凝器（condenser）和蒸發器（evaporator），做為管內冷媒和盤管外面空氣的熱量傳遞工具。空調設備為了增加熱傳面積，在熱交換的設計上，往往有鰭片（fin）附著於銅管上，稱為鰭片(管)式熱交換器(fin-and-tube heat exchanger)。



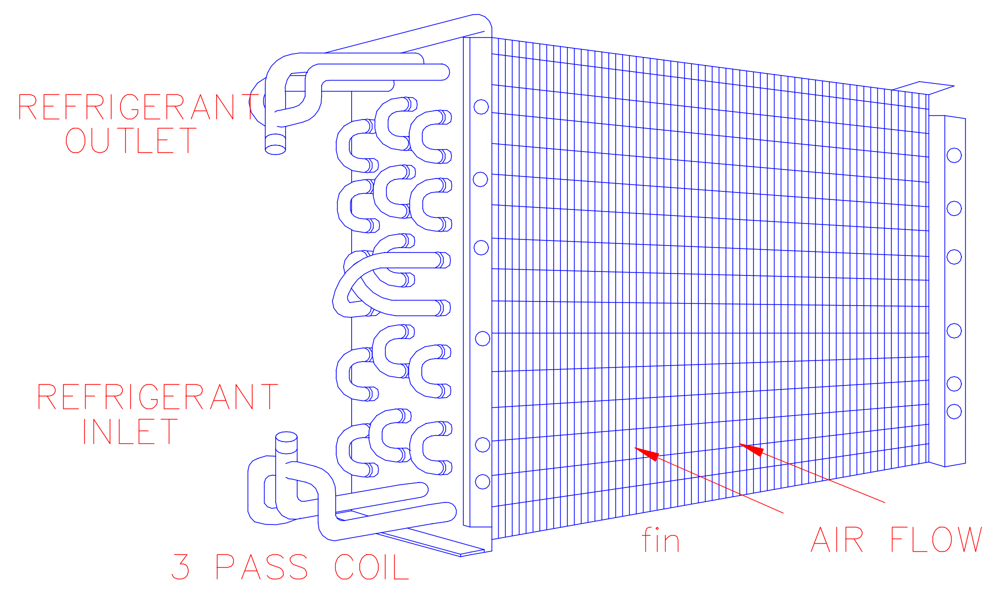


Fig.鰭管式熱交換器示意圖

# 鰭管式熱交換器幾何尺寸

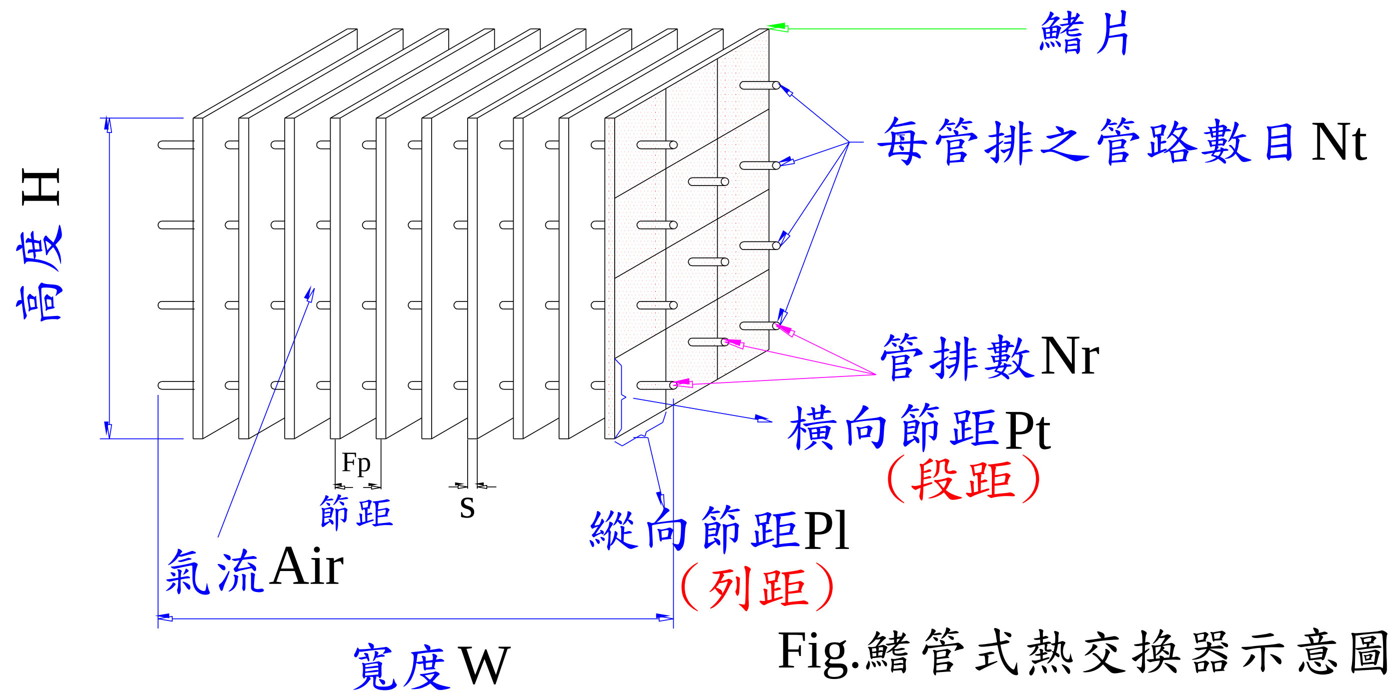
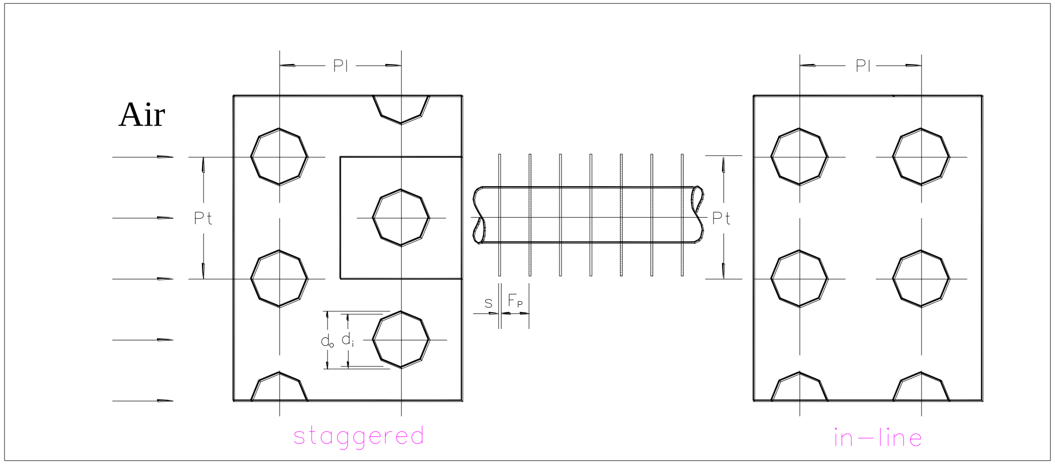


Fig.鰭管式熱交換器示意圖



# 鰭管式熱交換器幾何尺寸(Cont.)

$A_p$=管壁面積($m^2$)

$A_t$=裸管面積 ($m^2$)(primary surface)

$A_f$=鰭片面積($m^2$)(secondary surface)

$A_o$=空氣側熱傳面積 ($m^2$)

$=A_t+A_f$

$A_i$=內管壁面積($m^2$)

$W$=熱交換器有效寬度(m)

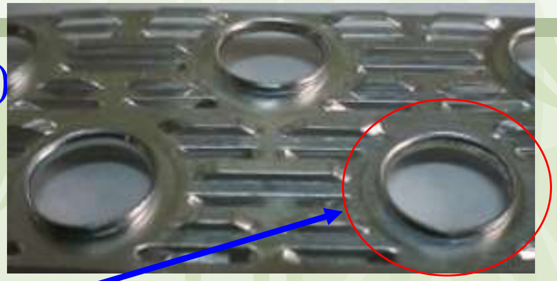
$H$=熱交換器高度(m)

$F_p$=鰭片節距(mm)

$\delta_f$=鰭片厚度(mm)

$P_t$ =段距

$P_l$ =列距



$d_c$=包含頸領(collar)之管徑(m)

$(d_c=d_o+2^*\ \delta_f)$

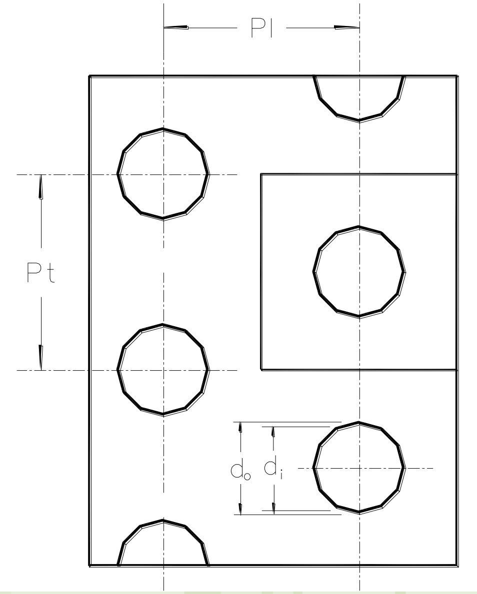
$N_F$=鰭片數目(=W/$F_p$)

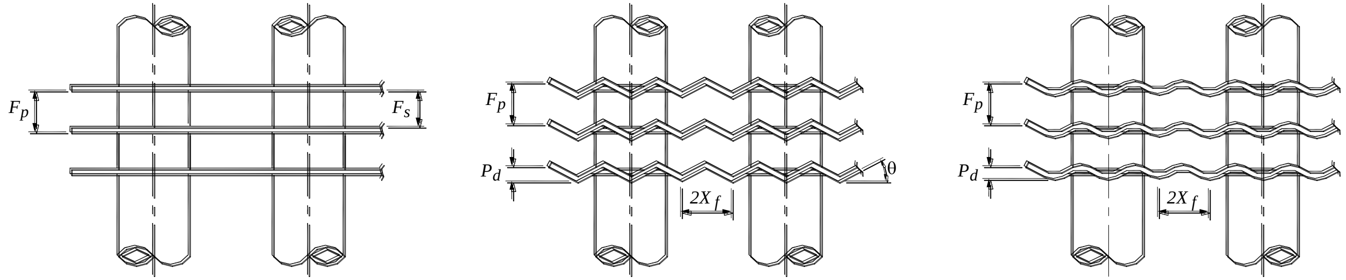
$N_T$=每排管之段數(tube number)

$N_R$=管排數(row number)

$A_c$=最小空氣流道面積 ($m^2$)(minimum free flow area)

$A_{fr}$=熱交換器正向截面積($m^2$)(=W*H)

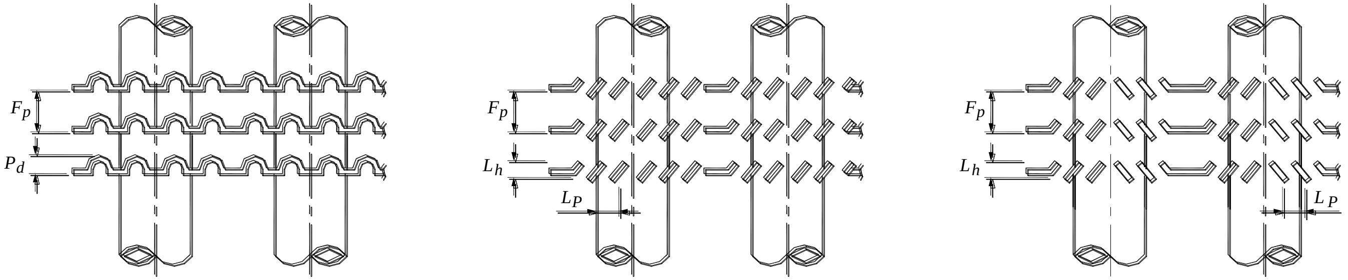




(a) 平板型 Plain fin

(b) 波浪型 Herringbone wavy fin

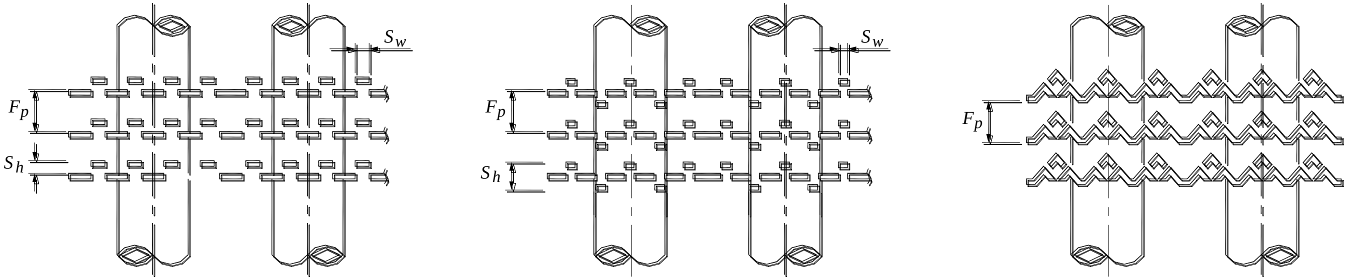
(c) 平滑波浪型 Smooth wavy fin, type (I)



(d) 平滑波浪+平板型 Smooth wavy fin, type (II)

(e) 單向百葉窗型 Louver fin, one-sided

(f) 雙向百葉窗型 Louver fin, with re-direction louver



(g) 單向裂口型 Slit fin, one-sided

(h) 雙向裂口型 Slit fin, double-sided

(i) 複合百葉窗型 Convex-louver fin

圖5-7 各式常見鰭片

# 鰭片幾何形狀

波浪鰭片
(wavy)

複合百葉窗鰭片－convex louver

百葉窗鰭片－單向開口
(louver)

百葉窗鰭片－雙向開口
(louver)

裂口式鰭片－單向開口
(slit)

裂口式鰭片－雙向開口
(slit)

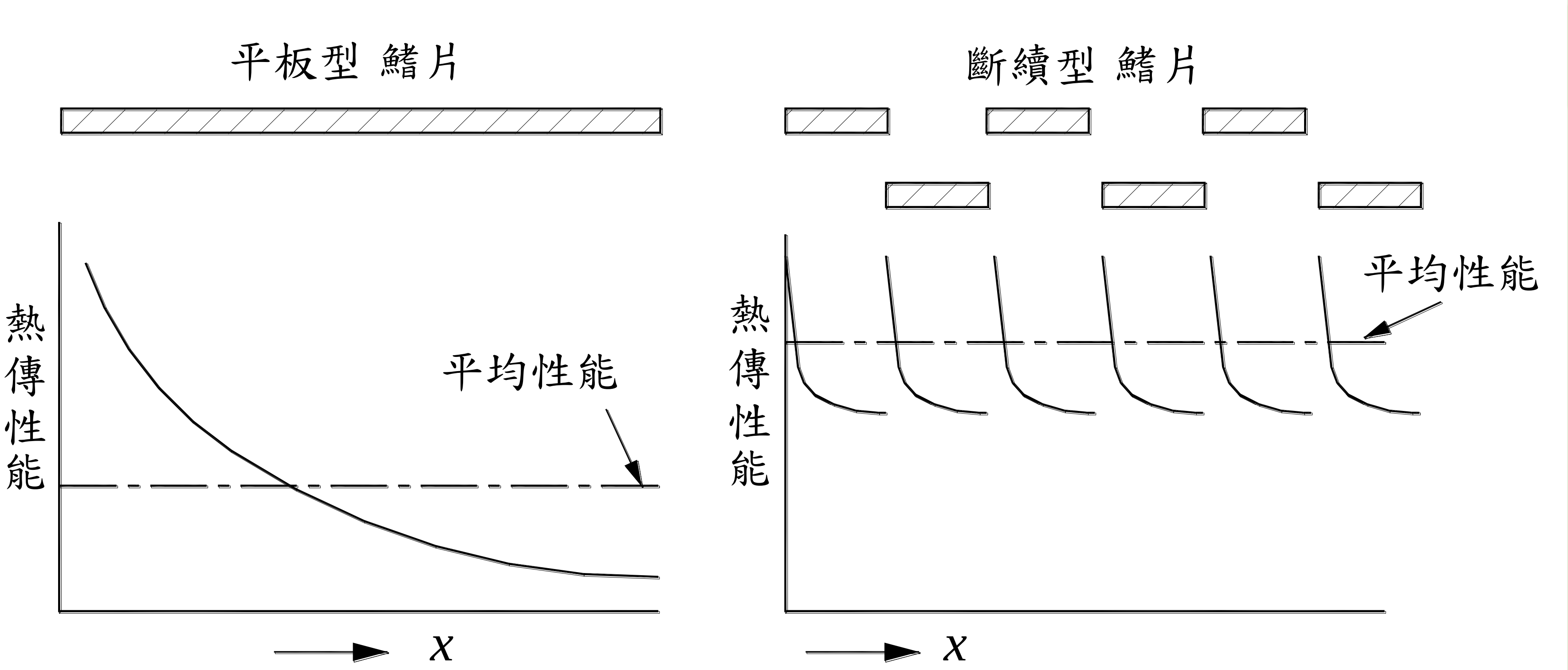


圖 5-12　斷續型鰭片與連續型鰭片之性能比較

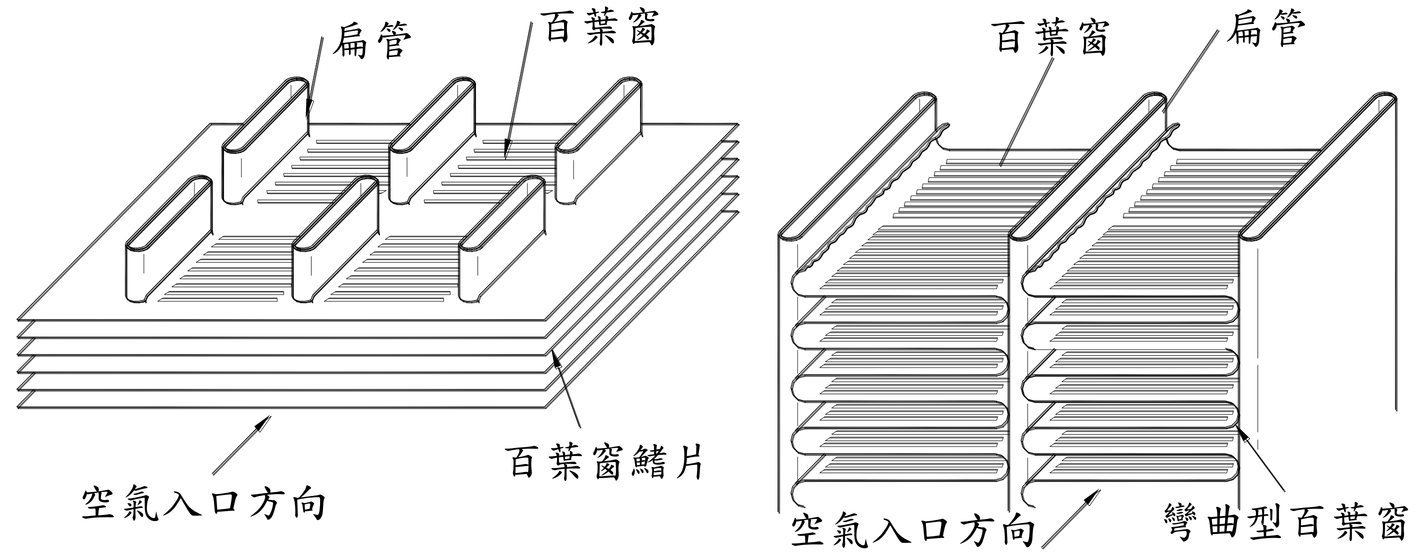


圖5-13　扁管式百葉窗型熱交換器

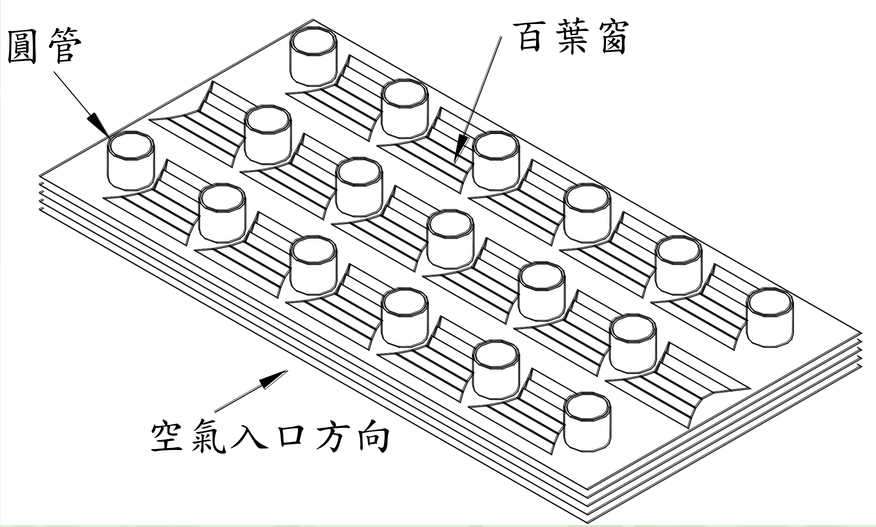


圖5-14　圓管式百葉窗型熱交換器

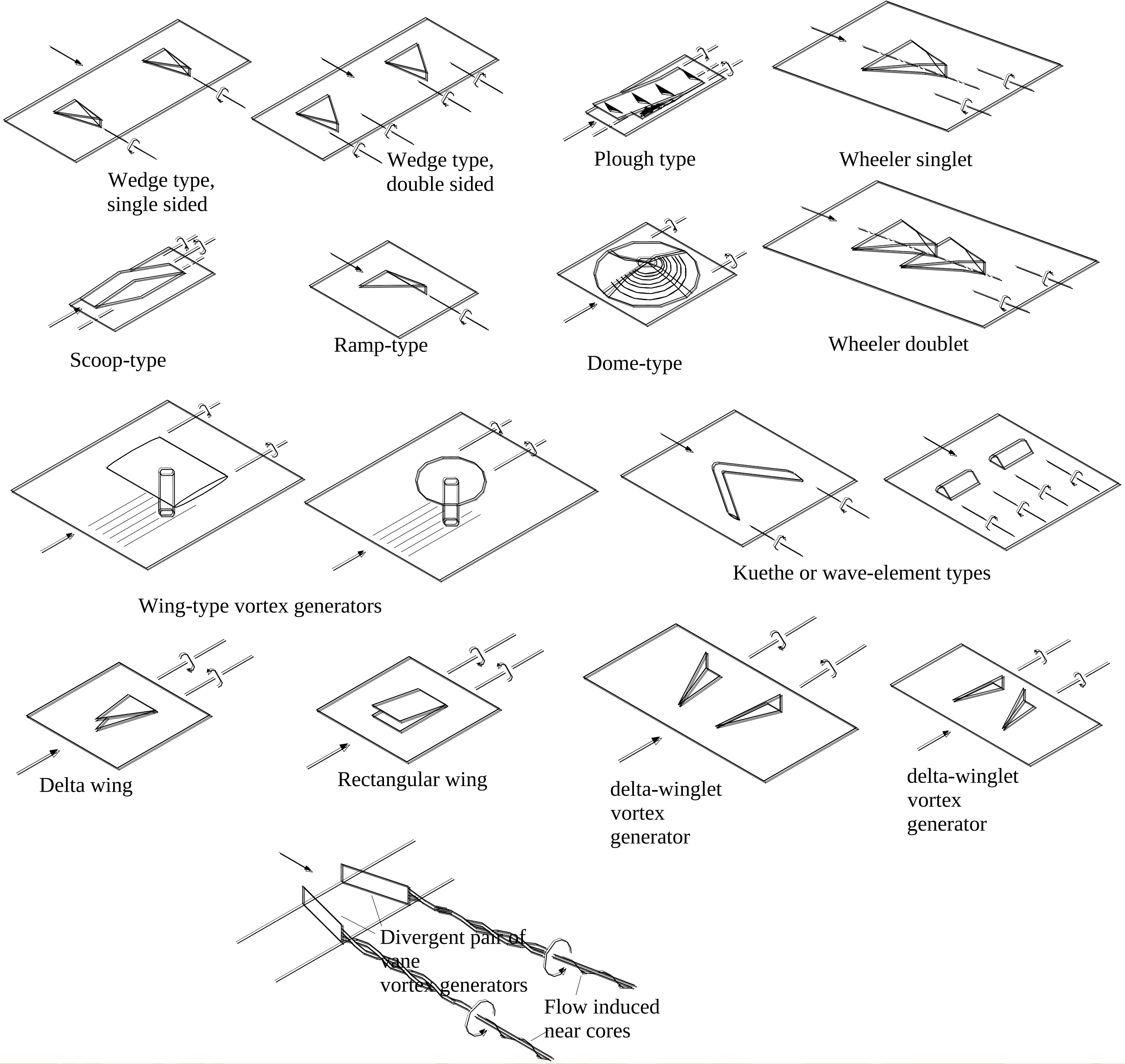
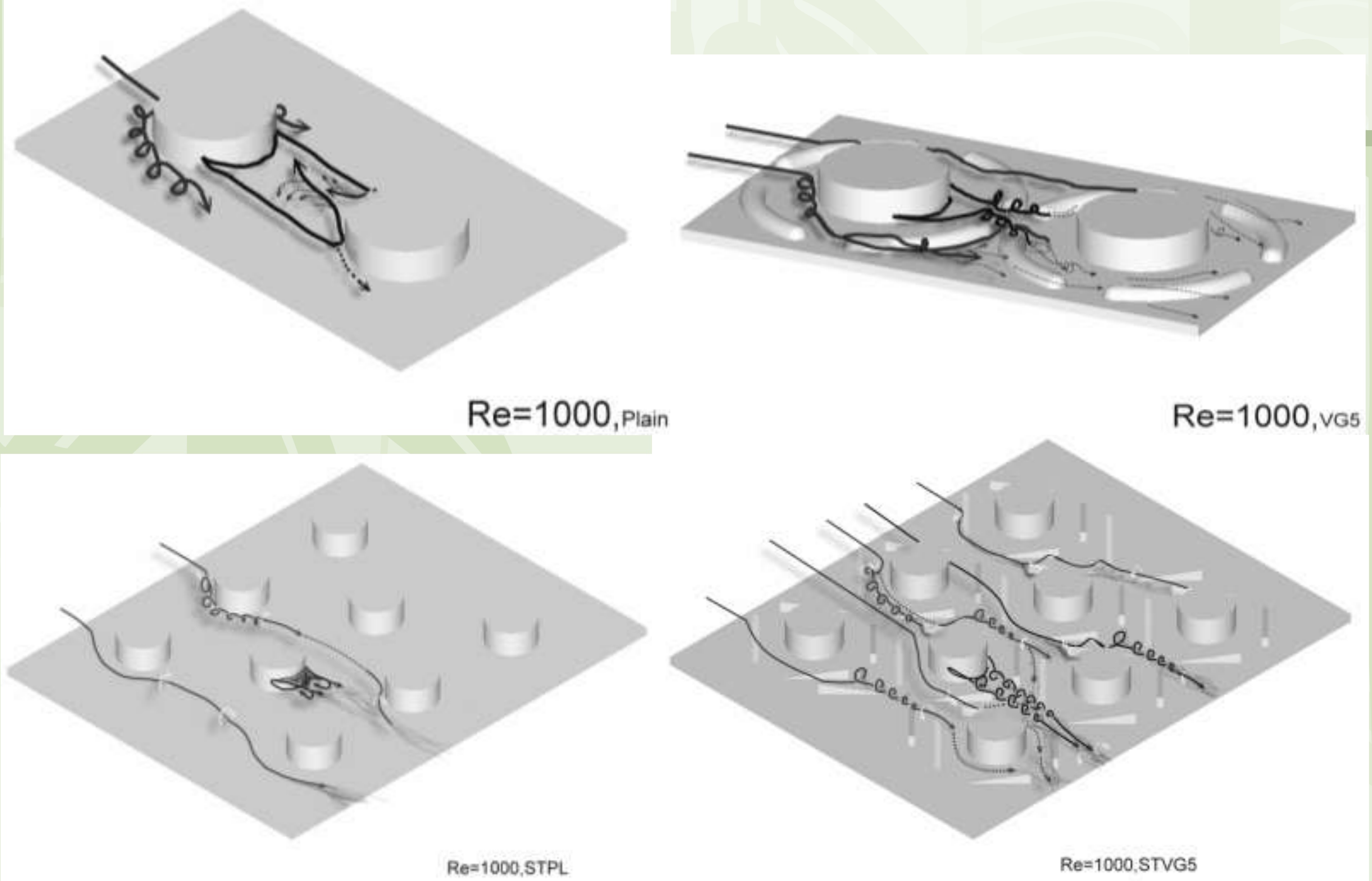


圖5-16　常見的渦流產生器



Re=1000,Plain

Re=1000,VG5

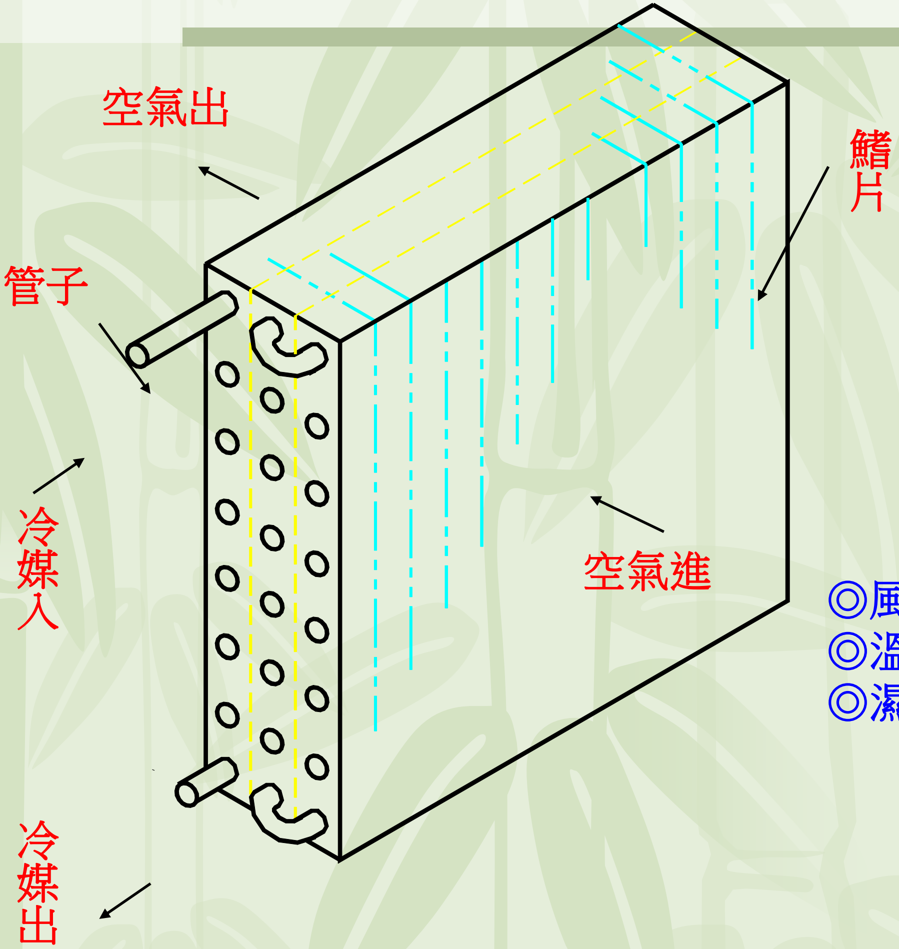
Re=1000,STPL

Re=1000,STVG5

圖5-17　環狀與三角翼渦流產生器在排列與交錯形式下對流動的影響

# 熱交換器性能參數

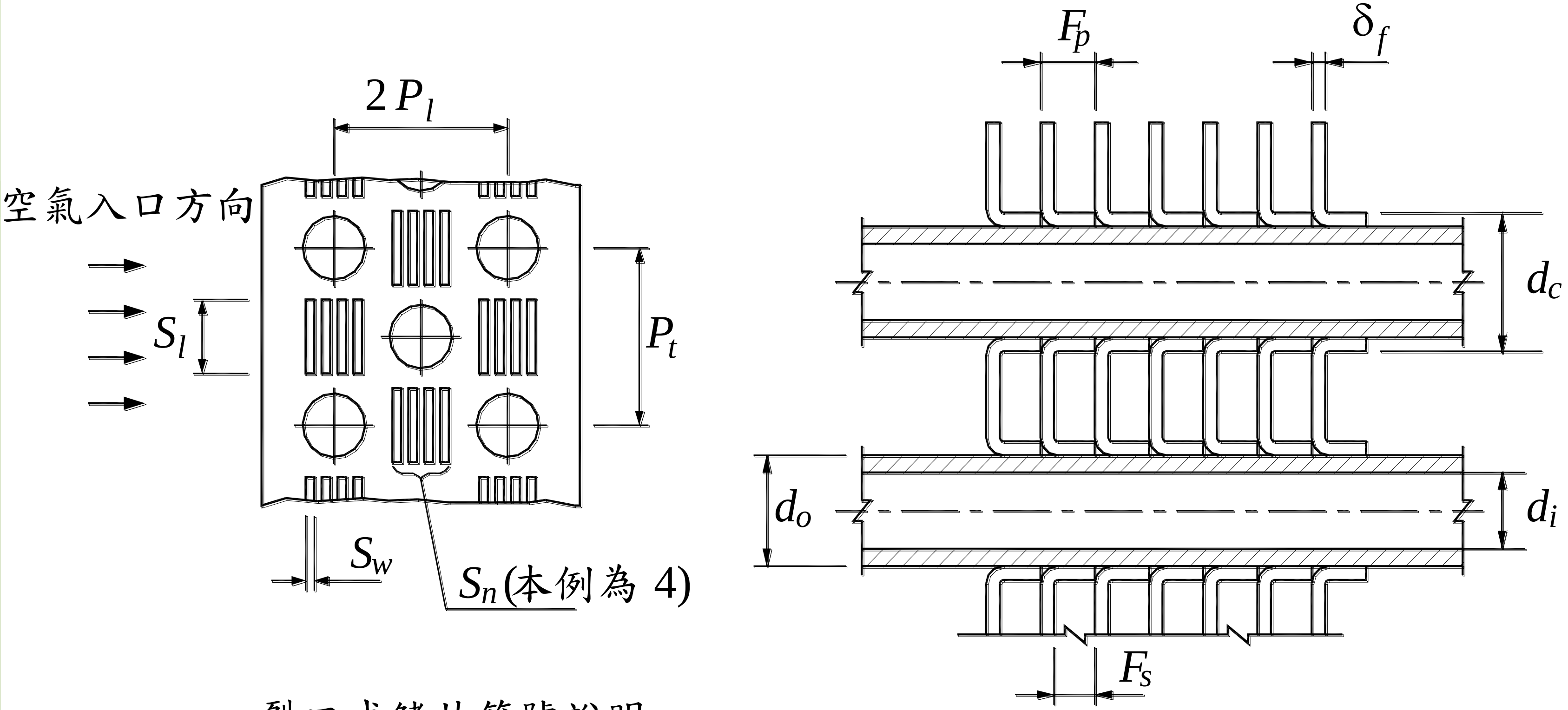
◎管子種類
◎管徑大小
◎管子材質
◎管排多寡
◎排列方式



◎鰭片型式
◎鰭片厚度
◎鰭片密度
◎鰭片材料
◎鰭片表面處理
◎鰭片和管子結合方式

◎冷媒種類
◎質量流率
◎壓力
◎溫度

◎風速大小
◎溫度
◎濕度

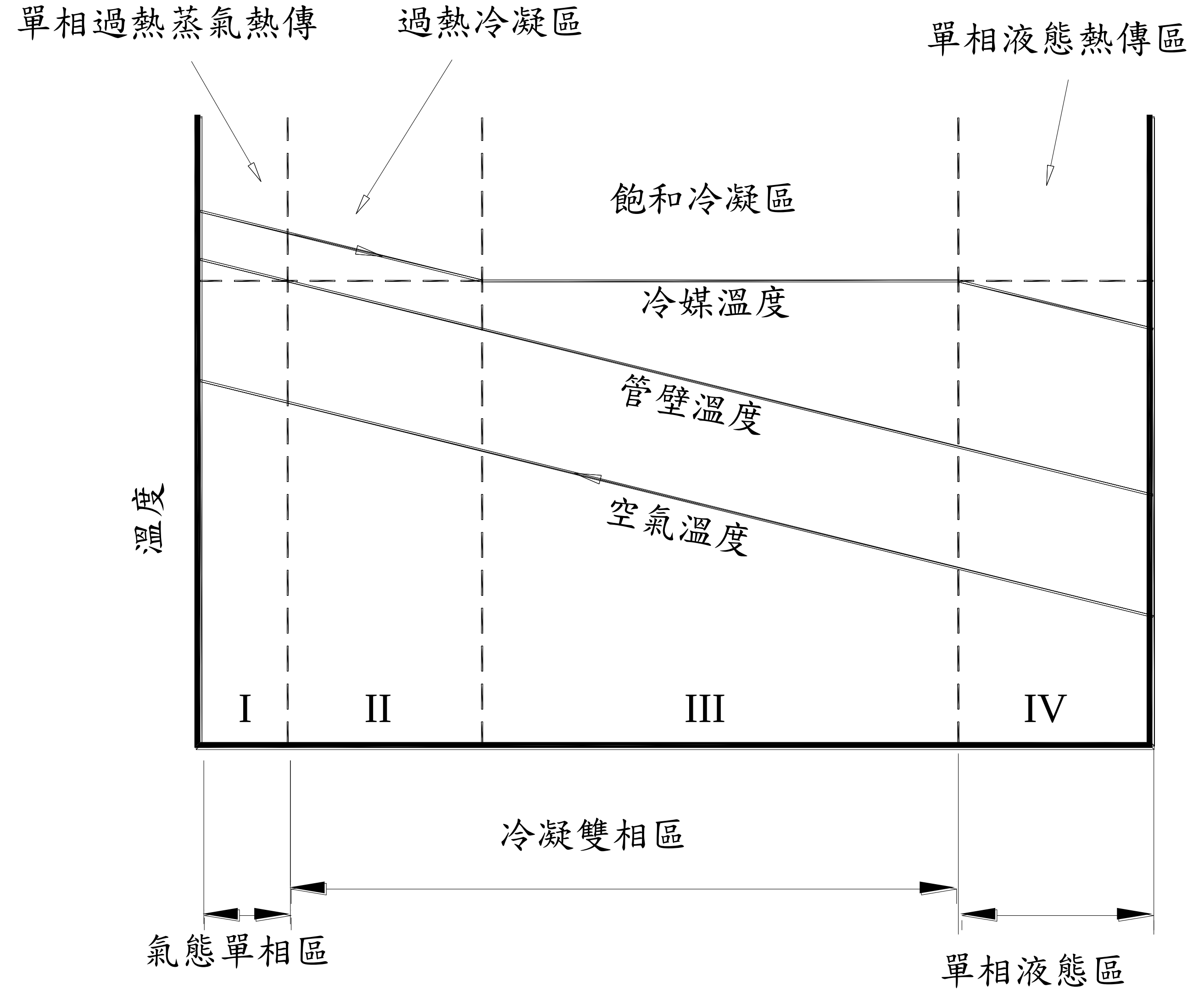


裂口式鰭片符號說明

鰭管式熱交換器幾何尺寸說明

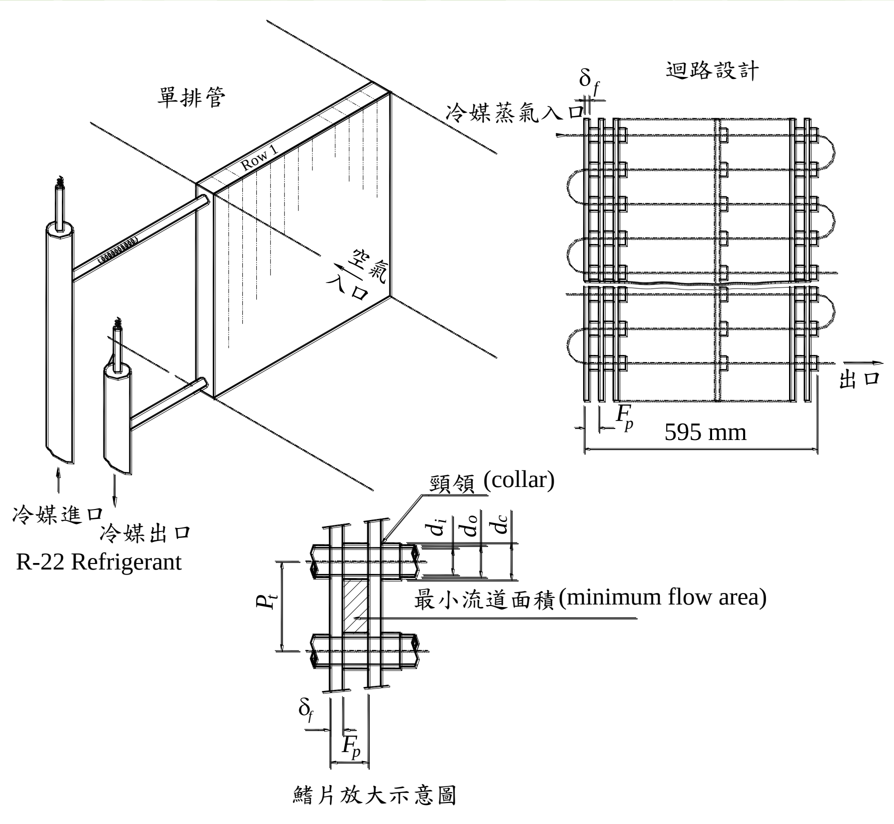
圖5-8　鰭管式熱交換器的一些符號定義

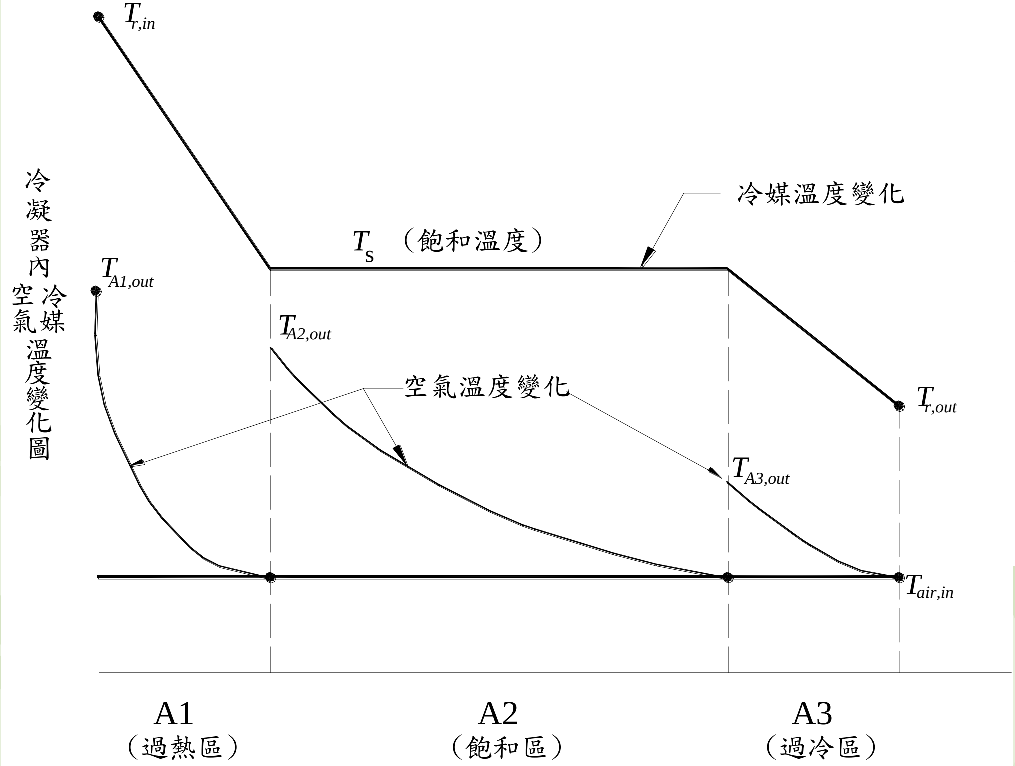
# 冷凝現象說明

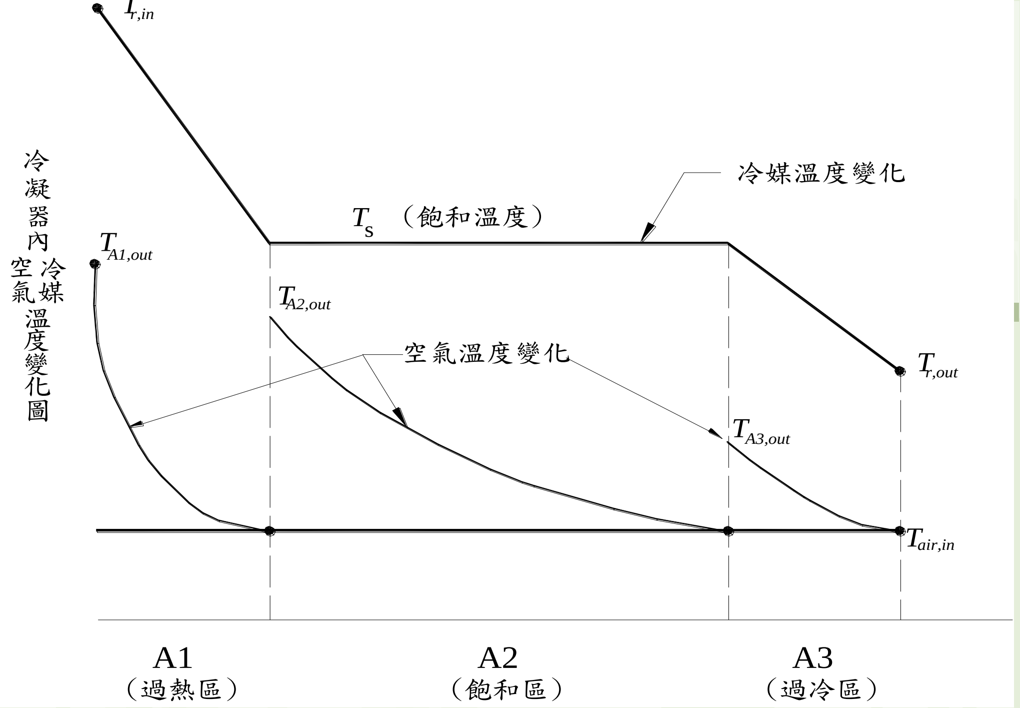


冷凝器管內熱傳型態

# 冷凝器計算流程


單排管
Row 1
空氣入口
冷媒進口
冷媒出口
R-22 Refrigerant
迴路設計
$\delta_f$
冷媒蒸氣入口
出口
$F_p$
595 mm
頸領 (collar)
$d_i$
$d_o$
$d_c$
$P_t$
最小流道面積(minimum flow area)
$\delta_f$
$F_p$
鰭片放大示意圖

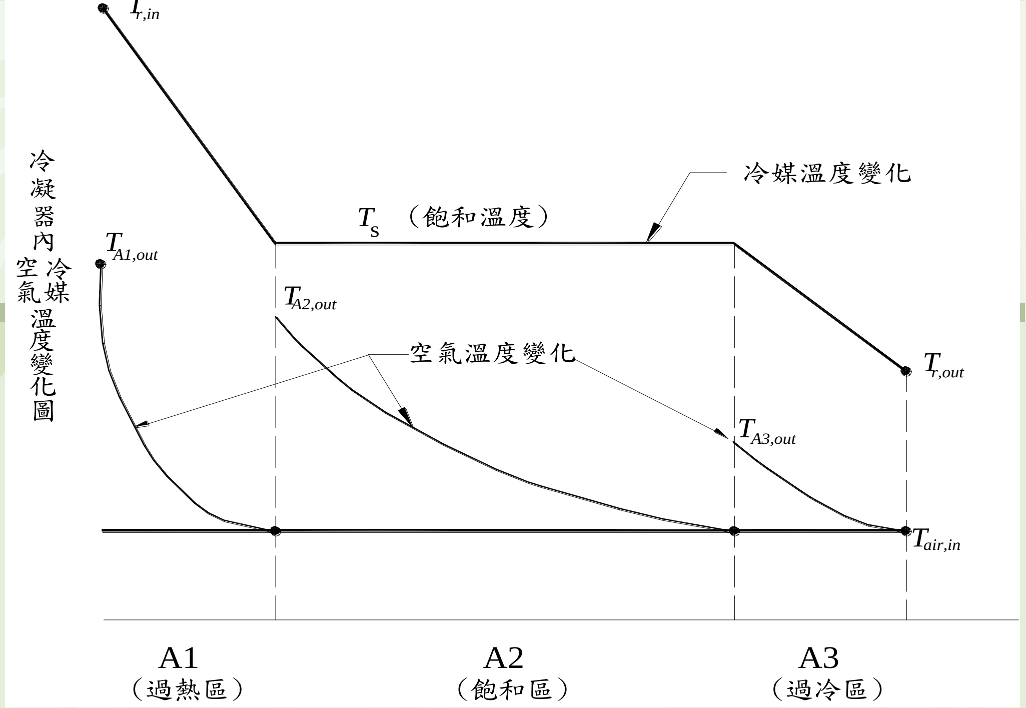

冷凝器內
冷媒
空氣
溫度變化圖
$T_{r,in}$
$T_{A1,out}$
$T_{A2,out}$
冷媒溫度變化
$T_s$ （飽和溫度）
空氣溫度變化
$T_{r,out}$
$T_{A3,out}$
$T_{air,in}$
A1
（過熱區）
A2
（飽和區）
A3
（過冷區）



# 過熱區計算

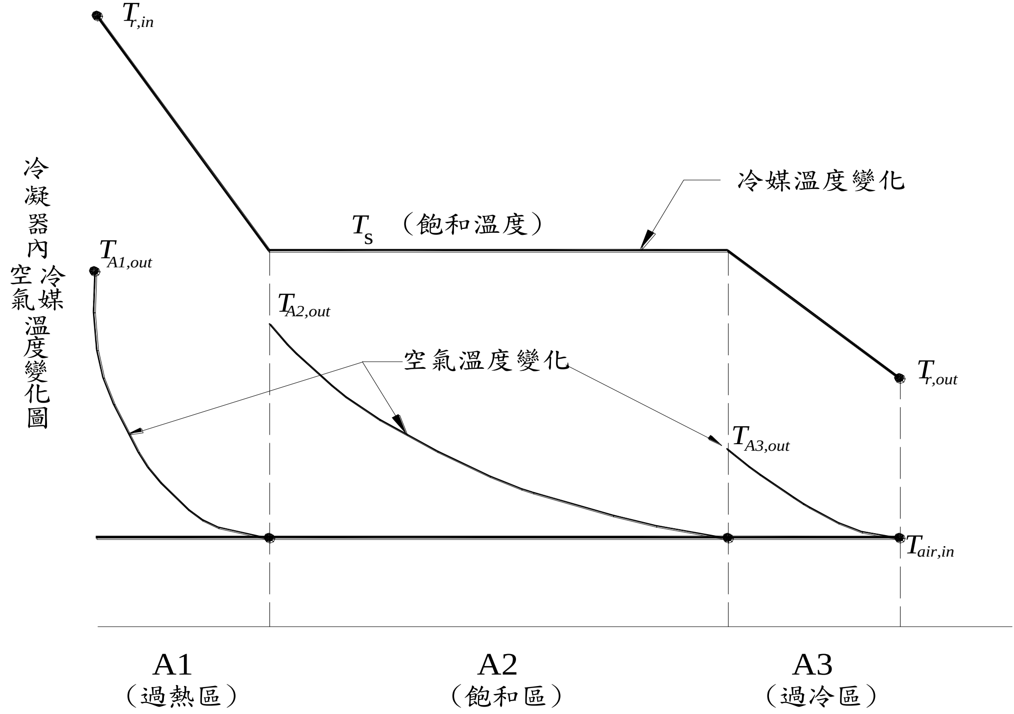
- (1) 算出管內單相熱傳係數 $h_i$
- (2) 算出管外空氣測熱傳係數 $h_o$
- (3) 第一區散熱量 $Q_1 = m_r cp_r (T_{r,in} - T_s)$，由能量平衡算出第一區的空氣出口溫度
- (4) 冷媒出口溫度為 $T_s$，故過熱區散熱量殼算出 → 以 sizing 方式算出過熱區面積$A_1$

# 飽和區計算



- (1) 假設第二段出口為飽和溫度,且算出管內冷凝熱傳係數 $h_c$ , 第二區熱傳量為 $m \times i_{fg}$
- (2) 算出管外空氣測熱傳係數 $h_o$
- (3) 以 sizing 方式算出飽和區面積 $A_2$ 若 $A_2 > A-A_1$, 代表出口並未完全冷凝，則冷媒出口溫度為$T_s$ 此時第二區面積為 $A-A_1$ ，再以此面積以Rating 方式算出第二區熱傳量$Q_2$與出口乾度x及空氣出口溫度

# 過冷區計算



- (1) 若過冷區存在，其面積為$A-A_1-A_2$
- (2)算出管內單相液體熱傳係數 $h_i$
- (2) 算出管外空氣測熱傳係數 $h_o$
- (3) 以 rating 方式算出過冷區熱傳量$Q_3$ 與出口溫度 $T_{sub}$。再由能量平衡算出空氣側出口溫度。

# 氣冷式鰭管式熱交換器性能計算

# 冷凝器熱傳熱傳路徑示意圖

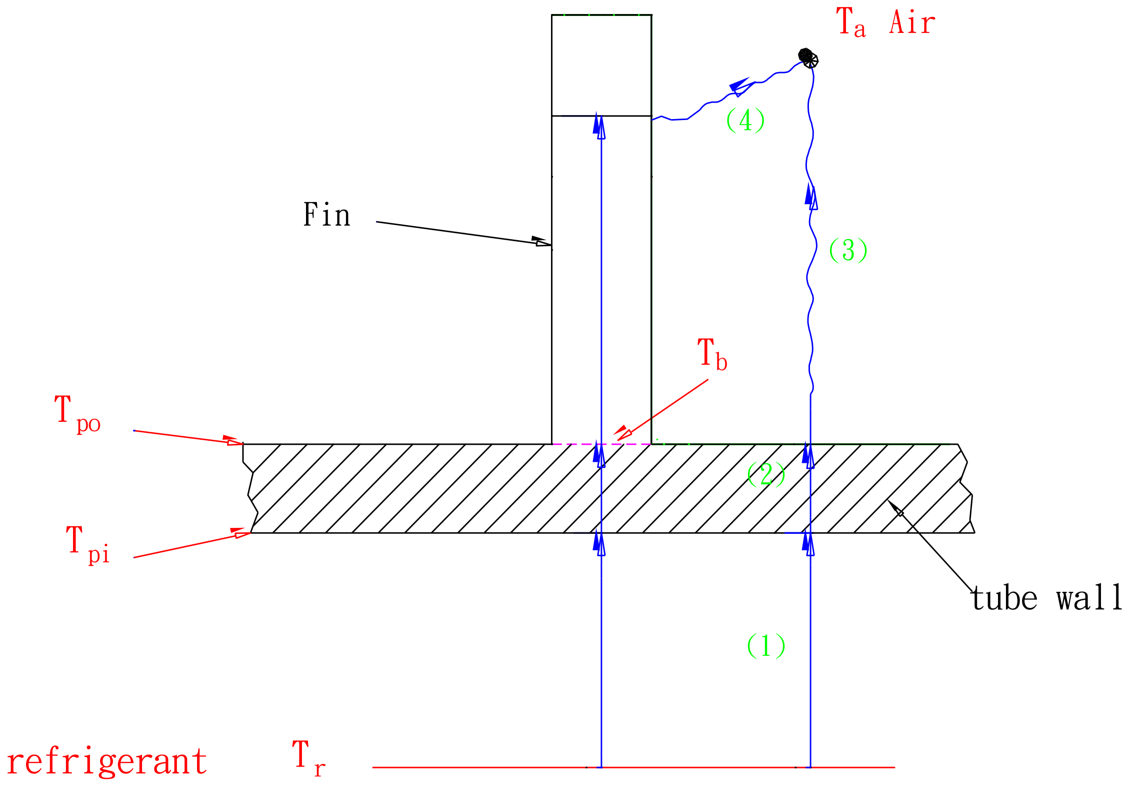


圖2.冷凝器熱傳路徑示意圖(冷媒側至空氣)

# 冷凝器熱傳計算(Cont.)

- 冷凝器的熱傳量可用ε-*NTU* 方法進行合理估算，熱傳量可表示：

$$\dot{Q}_{cond} = \varepsilon C_{\min}(T_{h1} - T_{c1})$$

其中 $C_{min}$ 為空氣與冷媒兩者之間熱容量(heat capacity)較小者，$T_{h1}$和$T_{c1}$ 分別為冷媒和空氣的進口溫度。ε稱為有效性(effectiveness)，可表示為 *NTU* (傳遞單位，number of transfer units)、和流體流動型態的函數。

$$NTU \equiv UA / C_{\min}$$

- 上式中*UA*為總熱傳係數與熱傳面積的乘積，可表示成：

$$\frac{1}{UA} = \frac{1}{h_o \eta_s A_o} + \frac{X_p}{k_p A_p} + \frac{1}{h_i A_i}$$

其中 $X_p / k_p A_p$ 表示管壁阻抗，其值僅佔總阻抗的1%左右，通常可忽略不計。$h_o$、$h_i$和 $\eta_s$分別為空氣側熱傳係數、管側熱傳係數和鰭片表面有效性。Wang(2000)針對不同鰭片型式、熱交換器幾何形狀及空氣雷諾數(Reynolds number)等參數關係，提出完整的$h_o$經驗公式。

**表5-1　交錯流動下$\varepsilon$-$NTU$與排數的關係式**

| $N$ 排數 | $C_{min}$ 較小的一側 | 關係式 |
|---|---|---|
| 1 | 空氣側 Air | $\varepsilon = \frac{1}{C^*}\left[1-e^{-C^*\left(1-e^{-NTU}\right)}\right]$ |
| | 管側 Tube | $\varepsilon = 1 - e^{-\frac{\left(1-e^{-NTU\cdot C^*}\right)}{C^*}}$ |
| 2 | 空氣側 Air | $\varepsilon = \frac{1}{C^*}\left[1-e^{-2KC^*}\left(1+C^*K^2\right)\right]$，$K = 1 - e^{-NTU/2}$ |
| | 管側 Tube | $\varepsilon = 1 - e^{-2K/C^*}\left(1+\frac{K^2}{C^*}\right)$，$K = 1 - e^{-NTU\cdot C^*/2}$ |
| 3 | 空氣側 Air | $\varepsilon = \frac{1}{C^*}\left[1-e^{-3KC^*}\left(1+C^*K^2(3-K)+\frac{3\left(C^*\right)^2K^4}{2}\right)\right]$，$K = 1 - e^{-NTU/3}$ |
| | 管側 Tube | $\varepsilon = 1 - e^{-3K/C^*}\left(1+\frac{K^2(3-K)}{C^*}+\frac{3K^4}{2(C^*)^2}\right)$，$K = 1 - e^{-NTU\cdot C^*/3}$ |
| 4 | 空氣側 Air | $\varepsilon = \frac{1}{C^*}\left[1-e^{-4KC^*}\left(1+C^*K^2\left(6-4K+K^2\right)+4\left(C^*\right)^2K^4(2-K)+\frac{8\left(C^*\right)^3K^6}{3}\right)\right]$<br>$K = 1 - e^{-NTU/4}$ |
| | 管側 Tube | $\varepsilon = 1 - e^{-4K/C^*}\left(1+\frac{K^2\left(6-4K+K^2\right)}{C^*}+\frac{4K^4(2-K)}{(C^*)^2}+\frac{8K^6}{3(C^*)^3}\right)$<br>$K = 1 - e^{-NTU\cdot C^*/4}$ |
| $\infty$ | - | $\varepsilon = 1 - \exp\left[NTU^{0.22}\cdot\left\{\exp\left(-C^*\cdot NTU^{0.78}\right)-1\right\}\Big/C^*\right]$<br>注意：本式為近似方程式 |

# 冷凝器熱傳計算(Cont.)

- 單相流體的熱傳係數計算採Gnielinski Correlation (1976) 經驗式：

$$Nu = \frac{(\mathrm{Re}_{D_i}-1000)\,\mathrm{Pr}_f\,(f/2)}{1+12.7\sqrt{f/2}\,(\mathrm{Pr}_f^{2/3}-1)}$$

其中 $f = \left(1.58\ln(\mathrm{Re}_{D_i})-3.28\right)^{-2}$

$$h_i = \frac{Nu \times k_i}{D_i}$$

其中$k_i$=冷媒熱傳導係數，$D_i$=熱傳管內直徑

- 上兩相流體的熱傳係數採Shah (1979) 經驗式（僅適用平滑管）：

$$h_i = h_l\left[(1-x)^{0.8} + \frac{3.8x^{0.76}(1-x)^{0.04}}{P_r^{0.38}}\right]$$

其中其中 $P_r$ 稱為減低壓力(reduced pressure)，為飽和壓力與臨界壓力之比值。$x$為氣體乾度(vapor quality)，而$h_l$ 為單相流體的熱傳係數，可由前述的Gnielinski correlation (1976)求得。

- 鰭片效率方法是以Schmidt (1949)方法來估算。

# 鰭片效率(fin efficiency)經驗式

- 鰭片效率方法是以Schmidt (1949)法為主，說明如下：

1.鰭片表面有效性$\eta_s$定義為 $$\eta_s = 1 - \frac{A_f}{A_o}(1-\eta_f)$$

2.鰭片效率 $\eta_f$為 $$\eta_f = \frac{\tanh(mr\phi)}{mr\phi}$$

其中 $m = \sqrt{\dfrac{2h_o}{k_f \delta_f}}$，$k_f$=鰭片熱傳導係數，$\delta_f$=鰭片厚度

$\phi = \left(\dfrac{R_{eq}}{r} - 1\right)\left[1 + 0.35\ln\left(R_{eq}/r\right)\right]$，$R_{eq}$=矩形鰭片等效半徑，$r$=圓管半徑

不同的排列方與$R_{eq}/r$的關係如下：

(i) 對齊排列(in-line) $$\frac{R_{eq}}{r} = 1.28\frac{X_M}{r}\left(\frac{X_L}{X_M} - 0.2\right)^{1/2}, \mathrm{X_M} = \frac{P_t}{2}, \mathrm{X_L} = \frac{P_\ell}{2}$$

(ii) 交錯排列(staggered) $$\frac{R_{eq}}{r} = 1.27\frac{X_M}{r}\left(\frac{X_L}{X_M} - 0.3\right)^{1/2}，\quad \mathrm{X_M} = \frac{P_t}{2},\quad \mathrm{X_L} = \frac{1}{2}\sqrt{\left(\frac{P_t}{2}\right)^2 + P_l^2}$$

# 氣冷式冷凝器熱傳計算流程(尺寸設計)

條件：已知HX能力及部份幾何尺寸，流體進口條件及風量，求*Nr*=?

(1)HX幾何尺寸計算

(2)管內熱傳係數 $h_i$ 計算

$$h_i = \frac{Nu \times k_i}{D_i}$$

(3) 空氣側熱傳係數 $h_o$

$$\therefore h_o = j_s \times G_c \times C_{pa} \times \Pr_a^{-2/3}$$

(4) 鰭片效率算

$$\eta_f = \frac{\tanh(mr\phi)}{mr\phi}$$

(5) 總熱傳係數*U*

$$U_o = \frac{1}{\left(\frac{A_o}{A_i}\right)\frac{1}{h_i} + \frac{1}{h_o \eta_s}}$$

(6)對數平均溫差 *LMTD*

$$LMTD = \frac{(\Delta T_1 - \Delta T_2)}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

(7)由 *Q=Uo xAoxF*LMTD* 可算出*Ao* ⇨ 求得管排數Nr

(8)計算空氣側和管側壓損

# 空氣側壓降之計算與分析(Cont.)

- 流經熱交換器的壓降可分為如下4個部分：

$$\Delta P = \Delta P_i + \Delta P_f + \Delta P_a + \Delta P_e$$

其中：(a) $\Delta P_i$ 為流入熱交換器時因流道變小所造成的壓降。

(b) $\Delta P_f$ 為流體經過熱交換器的摩擦壓降。

(c) $\Delta P_a$ 為流體因密度變化引起速度改變所造成的壓降。

(d) $\Delta P_e$ 為流出熱交換器時因流道變大所造成的壓降。

(此段因流道變大，速度變小，故(d)項部分的壓降為一壓升)

Kays and London (1984)提出的壓降關係式如下：

$$\Delta P = \frac{G_c^2}{2}\left[\frac{(1-\sigma^2+K_c)}{\rho_1} + \frac{f}{\rho_m}\frac{A_o}{A_c} + 2\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right) - \frac{(1-\sigma^2-K_e)}{\rho_2}\right]$$

其中$\sigma$為截縮比率 (contraction ratio)，為最小空氣流量面積 (minimum free flow area)，$\rho_1$、$\rho_2$分別為空氣進口和出口的密度，$\rho_m$為空氣平均密度，$K_c$為進口壓降係數 (entrance pressure loss coefficient)，$K_e$為出口壓降係數 (exit pressure loss coefficient)，$f$為空氣的摩擦因子 (friction factor)。

# 案例計算

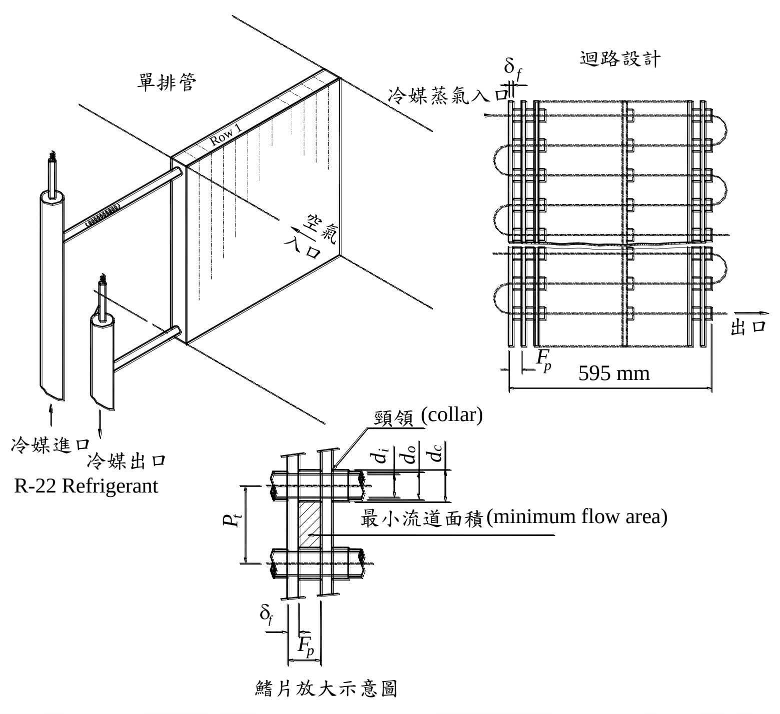


圖5-18　平板型冷凝器熱交換器之示意圖

- 如圖5-18所示，一連續型平板鰭片的冷凝器的相關資料
- 冷媒測：

  R-22冷媒：

  $T_{rin}$ = 92 ºC；$c_{p,r,in}$ = 910 J/kg·K；$\rho_{r,in}$ = 73.46 kg/m$^3$；

  $k_{r,in}$ = 0.0161 W/m·K；$\mu_{r,in}$ = 16.08×10$^{-6}$ N·s/m$^2$；Pr$_{r,in}$ = 0.909

  Pr$_{s,G}$ = 1.121；Pr$_{s,L}$ = 2.426；　$P_r$ = 0.437 (reduced pressure)

  $\rho_{s,L}$ = 1062 kg/m$^3$；　$\rho_{s,G}$ = 95.4 kg/m$^3$；$i_{s,LG}$ = 148.3 kJ/kg；$T_s$ = 54ºC

  $\mu_{s,L}$ = 116.9×10$^{-6}$ N·s/m$^2$；$\mu_{s,G}$ = 14.47×10$^{-6}$ N·s/m$^2$；$c_{p,s,L}$ = 1461 J/kg·K

  $c_{p,s,G}$ = 1173 J/kg·K；$k_{s,L}$ = 0.0704 W/m·K；$k_{s,G}$ = 0.01514 W/m·K
- 空氣側：

  空氣的入口溫度為35°C，$\rho_a$ = 1.145 kg/m$^3$；$V_{fr}$ = 1.5 m/s；

  $\mu_a$ = 188.7×10$^{-7}$ N·s/m$^2$；$c_{p,a}$ = 1007 J/kg·K；Pr$_a$ = 0.71

  熱交換器幾何尺寸：

  $W$ = 595 mm，$H$ = 355 mm，$N$ = 1，$\delta_f$ = 0.12 mm，$F_p$ = 1.28 mm，$P_t$ =25.4 mm，$P_l$ = 22 mm，$d_c$ = 10.34 mm，$\delta_w$ = 0.3 mm，$d_i$ = 9.5 mm

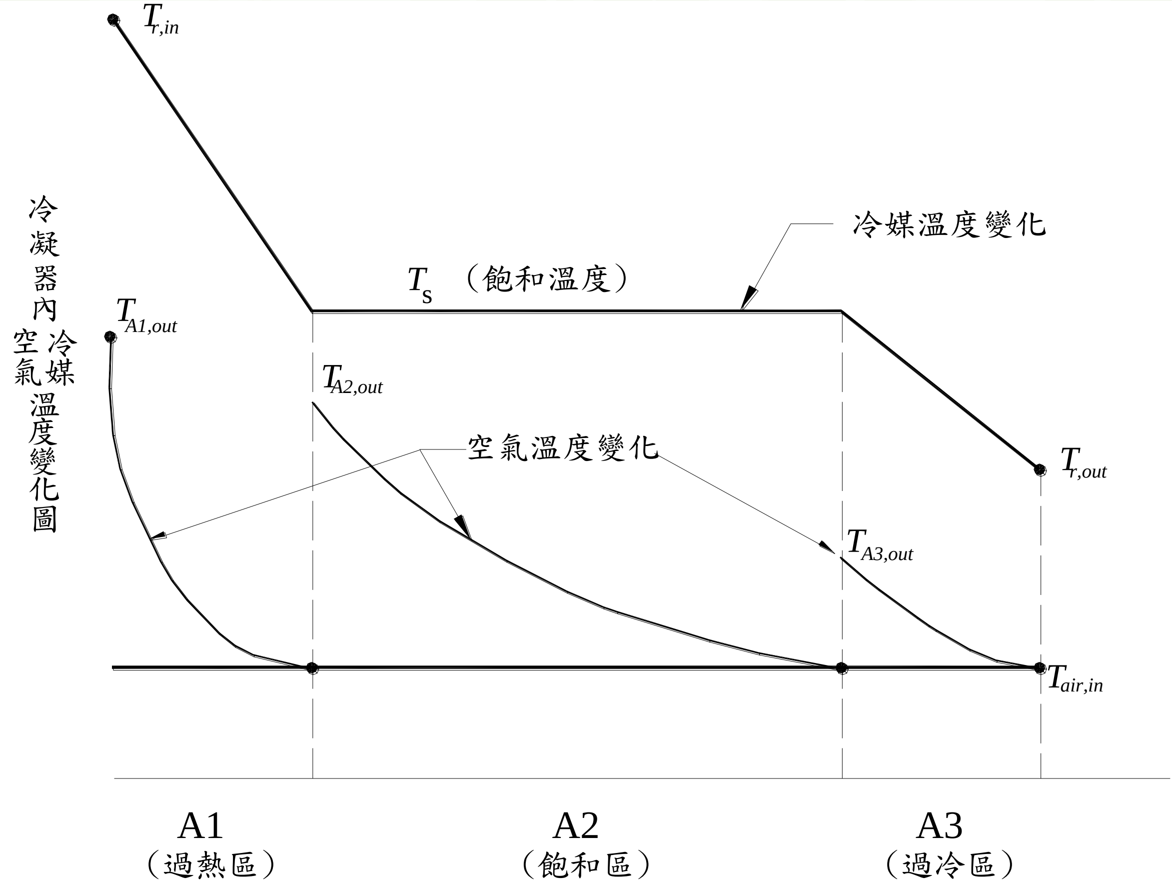


圖5-19　冷凝器內溫度變化示意圖 (請注意！每一區的出口溫度都不盡相同，這是因為交錯流動安排方式所造成，若熱交換器採用Counter-Cross 安排時，空氣側的溫度變化可參考圖5-21)

管外空氣側方面：

熱交換器正向面積： $A_{fr} = 0.595\times0.355 = 0.2112\ \mathrm{m}^2$

(1) 收縮比：$\sigma = A_c/A_{fr}$

所以必須先算$A_c$

由於熱交換器高度 = 0.355 m

所以熱交換器每列的管數$N_T$為(0.355/0.0254) = 14支

由於熱交換器鰭片間距為 = 0.00128 m

所以熱交換器鰭片片數($N_F$)為(0.595/0.00128) ≈ 465 片

$A_c = A_{fr} - N_T\times(d_c\times W + N_F\times\delta_f\times(P_t - d_c))$

{請參考第三章的圖例以瞭解公式的由來！！}

$= 0.2112 - 14\times(0.01034\times0.595+465\times0.00012\times(0.0254-0.01034))$

$= 0.1143\ \mathrm{m}^2$

$\sigma = A_c/A_{fr} = 0.1143/0.2112 = 0.541$

(2) 熱交換器之總面積：總面積 $A_o$ = 鰭片面積 ($A_f$) +管子面積($A_t$)

$A_f = 2\times N_F\times(P_l\times H - \pi/4\times d_c^2\times N_T)\times N + 2\times\delta_f\times N_F\times(H + P_l\times N)$

{同樣請參考第三章的圖例以瞭解公式的由來！！}

$= 2\times465\times(0.022\times0.355 - \pi/4\times0.01034^2\times14)\times1$

$+ 2\times0.00012\times465\times(0.355 + 0.022\times1) = 6.21\ \mathrm{m}^2$

$A_t = \pi\times d_c\times(W - N_F\times\delta_f)\times N_T\times N$

$= \pi\times0.01034\times(0.595-465\times0.00012)\times14\times1 = 0.245\ \mathrm{m}^2$

$A_o = A_f + A_t = 6.21+0.245 = 6.455\ \mathrm{m}^2$

注意本例中，鰭片的面積佔總面積的 96.2%！

$D_h = 4A_cL/A_o = 4\times0.1143\times0.022/6.455 = 0.001558\ \mathrm{m}$

管內面積 = $A_i = \pi\times d_i\times W\times N_T\times N = 0.2486\ \mathrm{m}^2$

$A_o/A_i = 25.96$ (此值不會隨著熱交換器的管支數增加而改變，稍後會用到這個比值的資料)。

(1) $V_{fr}$ = 1.5 m/s下的熱傳係數

由式5-41的1排平板型鰭片 方程式

$$j = 0.108\mathrm{Re}_{d_c}^{-0.29}\left(\frac{P_t}{P_l}\right)^{P1}\left(\frac{F_p}{d_c}\right)^{-1.084}\left(\frac{F_p}{D_h}\right)^{-0.786}\left(\frac{F_p}{P_t}\right)^{P2}$$

$$P1 = 1.9 - 0.23\log_e\left(\mathrm{Re}_{d_c}\right)$$

$$P2 = -0.236 + 0.126\log_e\left(\mathrm{Re}_{d_c}\right)$$

首先要算出該條件下的雷諾數，由於本例的雷諾數是以$d_c$為準，

$\therefore \mathrm{Re}_{dc} = \rho V_c d_c/\mu = \rho V_{fr} d_c/\mu/\sigma$

$= 1.145\times1.5\times0.01034/(188.7\times10^{-7})/0.541 \approx 1740$

$$P1 = 1.9 - 0.23\log_e\left(\mathrm{Re}_{d_c}\right) = 0.1839$$

$$P2 = -0.236 + 0.126\log_e\left(\mathrm{Re}_{d_c}\right) = 0.7042$$

$$j = 0.108\mathrm{Re}_{d_c}^{-0.29}\left(\frac{P_t}{P_l}\right)^{P1}\left(\frac{F_p}{d_c}\right)^{-1.084}\left(\frac{F_p}{D_h}\right)^{-0.786}\left(\frac{F_p}{P_t}\right)^{P2}$$

$$= 0.108\times1740^{-0.29}\left(\frac{0.0254}{0.022}\right)^{0.1839}\left(\frac{0.00128}{0.01034}\right)^{-1.084}\left(\frac{0.00128}{0.001558}\right)^{-0.786}\times$$

$$\left(\frac{0.00128}{0.0254}\right)^{0.7042} = 0.01746$$

$V_c$ =1.5/0.541 = 2.773 m/s

$j$ = 0.01746 = $h_o/(\rho V_c c_{p,a})\times\mathrm{Pr}_a^{2/3}$ =$(h_o/1.145/2.773/1007)\times(0.71)^{2/3}$

$\therefore h_o \approx 70\ \mathrm{W/m^2\cdot K}$

接下來要算表面效率 $\eta_o$，以獲得空氣側阻抗，由於管排數為1排，所以可視為排列型式(inline)，過程如下(請參考式5-20 ~ 5-25)：

$$m=\sqrt{\frac{2h_o}{k_f\delta_f}}=\sqrt{\frac{2\times70}{204\times0.00012}}=75.62\ (\mathrm{m}^{-1})$$

$X_L = P_l/2 = 0.022/2 = 0.011$ m (排列型式算法，若是交錯型式需用 $X_L=\sqrt{(P_t/2)^2+P_l^2}/2$)

$X_M = P_t/2 = 0.0254/2 = 0.0127$ m

$r = d_c/2 = 0.01034/2 = 0.00517$ m

$$\frac{r_{eq}}{r}=1.28\frac{X_M}{r}\left(\frac{X_L}{X_M}-0.2\right)^{1/2}=1.28\frac{0.0127}{0.00517}\left(\frac{0.011}{0.0127}-0.2\right)^{1/2}=2.566$$

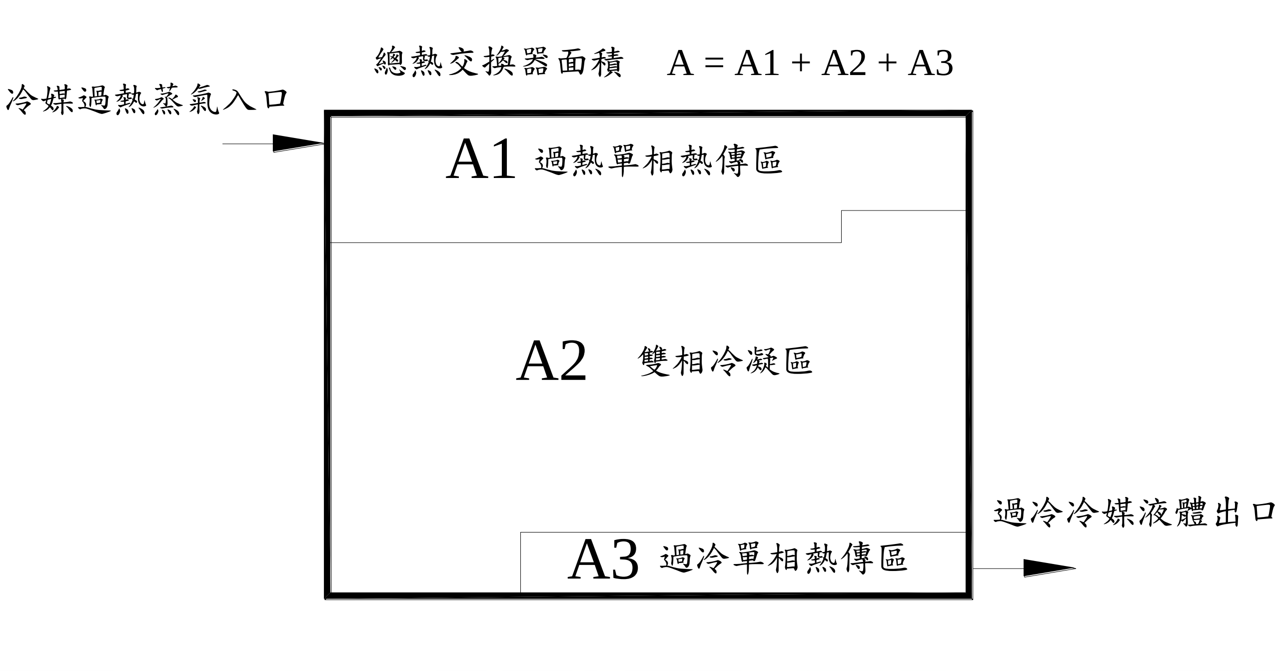
$$\phi=\left(\frac{r_{eq}}{r}-1\right)\left[1+0.35\ln\left(r_{eq}/r\right)\right]=\left(2.566-1\right)\left[1+0.35\ln\left(2.566\right)\right]=2.083$$

$$\eta_f=\frac{\tanh(mr\phi)}{mr\phi}=\frac{\tanh(75.62\times0.00517\times2.083)}{75.62\times0.00517\times2.083}=0.825$$

$$\eta_o=1-\frac{A_f}{A_o}(1-\eta_f)=1-\frac{6.21}{6.455}(1-0.825)=0.8316$$

$\therefore \eta_o h_o = 0.8316\times70 = 58.21\ \mathrm{W/m^2\cdot K}$

接下來我們要分三段A1、A2、A3來分別計算各段的$UA$值以獲得各段的性能。



總熱交換器面積　A = A1 + A2 + A3

冷媒過熱蒸氣入口

A1 過熱單相熱傳區

A2 雙相冷凝區

A3 過冷單相熱傳區

過冷冷媒液體出口

圖5-20　冷凝器內單相及雙相熱傳區示意圖

## 『A1部份』

A1部份，管內為單相氣態熱傳，所以管內的熱傳係數需使用單相熱傳係數，過程如下：

首先將資料變為標準 SI單位， $\dot{m}_r = 75\,\text{kg/hr} = 0.02083\,\text{kg/s}$

A1部份的散熱量均為單相熱傳

即 $Q_{A1} = \dot{m}_r c_{p,r,G}(T_{r,in} - T_s)$

$c_{p,r,G}$ 的值可以單相過熱進口值與飽和氣態部份的值來估算

$\therefore c_{p,r,G} = (c_{p,r,in} + c_{p,s,G})/2 = (910 + 1173)/2 = 1041.5\ \text{J/kg·K}$

$$Q_{A1} = \dot{m}_r c_{p,r,G}(T_{r,in} - T_s) = 0.02083 \times 1041.5 \times (92 - 54) = 824.4\ \text{W}$$

$$C_r = \dot{m}_r c_{p,r} = 0.02083 \times 1041.5 = 21.7\ \text{J/K}$$

空氣側的 $\dot{m}_a = \rho_a V_{fr} A_{fr} = 1.145 \times 1.5 \times 0.2112 = 0.363\ \text{kg/s}$

$$C_A = \dot{m}_a c_{p,a} = 0.363 \times 1007 = 365.3\ \text{J/K}$$

但是如圖5-20所示，通過A1部份的空氣量僅佔$A_1/A$，所以A1區的有效熱容量$C_{A1} = A_1/A \times C_A$，由於$C_r$與$C_A$相差甚多，所以我們暫時假設$C_{min} = C_r$

$$Q_{max,A1} = \dot{m}_r c_{p,r,G}(T_{r,in} - T_{a,in}) = 0.02083 \times 1041.5 \times (92 - 35) = 1236.6\ \text{W}$$

$$\therefore \varepsilon_{A1} = Q_{A1}/Q_{max,A1} = 824.4/1236.6 = 0.667$$

管內的$h_i$ 算出來如下：

管內氣態的平均雷諾數 $= Gd_i/\mu_G$

$\mu_G$、$k_G$ 的值，同樣可以單相過熱進口值與飽和氣態部份的平均值來估算

$\mu_G = (\mu_{G,in}+\mu_{s,G})/2=(16.08\times10^{-6}+14.47\times10^{-6})/2 = 15.28\times10^{-6}\ \mathrm{N\cdot s/m^2}$

$k_G = (0.0161+0.01514)/2 = 0.01562\ \mathrm{W/m\cdot K}$

平均的$\mathrm{Pr}_G$ 數 $= (\mathrm{Pr}_{r,i}+\mathrm{Pr}_{s,G})/2 = (0.909+1.121)/2 = 1.015$

$$G=\frac{\dot{m}_r}{\frac{\pi}{4}d_i^2}=\frac{0.02083}{\frac{\pi}{4}0.0095^2}=293.9\ \mathrm{kg/m^2\cdot s}$$

$$\mathrm{Re}_G = Gd_i/\mu_G = 182726$$

$$f=\left(1.58\ln\mathrm{Re}_G-3.28\right)^{-2}=0.003974$$

$$Nu=\frac{\left(\frac{f}{2}\right)\left(\mathrm{Re}_G-1000\right)\mathrm{Pr}_b}{1.07+12.7\sqrt{\frac{f}{2}}\left(\mathrm{Pr}_b^{2/3}-1\right)}=\frac{\left(\frac{0.003974}{2}\right)\left(182726-1000\right)\times1.015}{1.07+12.7\sqrt{\frac{0.003974}{2}}\left(1.015^{2/3}-1\right)}=342.3$$

$$\therefore h_i = Nu\times k_G/d_i = 342.3\times0.01562/0.0095 = 563.2\ \mathrm{W/m^2\cdot K}$$

接下來，開始進行疊代，我們首先假設 $A_1^* = A_1/A_o = 0.2$

$\therefore A_1 = 0.2\times 6.455 = 1.291\ \mathrm{m}^2$

管內面積$A_{1,i} = A_1/25.96 = 0.0497\ \mathrm{m}^2$

$C_{A1} = 0.2\times C_A = 0.2\times 365.3 = 73.1$ W/K

$C_{A1}^* = C_r / C_{A1} = 21.7/73.1 = 0.297$

由表5-1的1排管$\varepsilon$-$NTU$方程式，當$C_{min}$ 於管側時：

$$\varepsilon_{A1} = 1 - e^{-\frac{\left(1-e^{-NTU\cdot C_{A1}^*}\right)}{C^*}}$$

即 $$NTU = \frac{-\log_e\left(C_{A1}^*\log_e\left(1-\varepsilon_{A1}\right)+1\right)}{C_{A1}^*}$$

而$\varepsilon_{A1} = 0.667$，$C_{A1}^* = 0.297$；帶入上式後可得$NTU_{A1} = 1.33$

$NTU_{A1} = (UA)_{A1}/C_r \quad \Rightarrow (UA)_{A1} = 28.89$ W/K

但若是由下面公式來計算$(UA)_{A1}$(暫且忽略管壁阻抗)

$$\frac{1}{(UA)_{A1}} = \frac{1}{\eta_o h_o A_1} + \frac{1}{h_i A_{1,i}}$$

可算出$(UA)_{A1} = 20.4$ W/K

所以假設值太小，要繼續進行疊代，經過數次疊代後，大致可以得到
$(UA)_{A1} = 27.34$ W/K 而 $A_1^* = A_1/A_o = 0.2681$，即$A_1 = 1.731\ \mathrm{m}^2$

『**A2部份**』

A2部份牽涉到管內冷凝熱傳的計算，這裡，我們使用第四章兩相流章節介紹的Shah (1979)方程式，我們以整段冷凝的熱傳係數的平均值來估算：

$$h_L = \frac{k_L}{d_i} 0.023 \left( \frac{G d_i}{\mu_L} \right)^{0.8} \mathrm{Pr}^{0.4}$$

$$h_{c,m} = h_L \left( 0.55 + \frac{2.09}{P_r^{0.38}} \right)$$

$$h_L = \frac{k_L}{d_i} 0.023 \left( \frac{G d_i}{\mu_L} \right)^{0.8} \mathrm{Pr}^{0.4} = \frac{0.0704}{0.0095} 0.023 \left( \frac{293.9 \times 0.0095}{116.9 \times 10^{-6}} \right)^{0.8} 2.426^{0.4}$$

$$= 772.7\ \mathrm{W/m^2 \cdot K}$$

$$h_{c,m} = h_L \left( 0.55 + \frac{2.09}{P_r^{0.38}} \right) = 772.7 \left( 0.55 + \frac{2.09}{0.437^{0.38}} \right) = 2637\ \mathrm{W/m^2 \cdot K}$$

同樣的，我們假設可以完全冷凝，所以在A2區的熱傳量為

$$Q_{A2} = \dot{m}_r i_{s,LG} (1 - x) = 0.02083 \times 148.3 \times (1 - 0) 1000 = 3089\ \mathrm{W}\ (\text{先假設完全冷凝})$$

在冷凝段A2有兩件事要注意(這在前面的章節已有完整說明)，即

(1) $C_{min} = C_A$而$C_{max} = C_r \to \infty$

(2) $C^* = 0$，$\varepsilon$-$NTU$ 關係式簡化成$\varepsilon = 1 - e^{-NTU}$

接下來，開始進行疊代，如同A1部份的計算，我們首先假設

$$A_2^* = A_2 / A_o = 0.6$$

$$Q_{max,A2} = \dot{m}_{a,A2} c_{p,a} (T_{r,s} - T_{a,in}) = 0.363 \times 1007 \times 0.6 \times (54 - 35) = 4167.2\ \mathrm{W}$$

$$\varepsilon_{A2} = Q_{A2} / Q_{max,A2} = 3089 / 4167.2 = 0.741$$

由$\varepsilon = 1 - e^{-NTU}$，可算出$NTU_{A2}$ = 1.351 W/K

$\therefore A_2 = 0.6 \times 6.455 = 3.873\ \mathrm{m}^2$

管內面積$A_{2,i} = A_2/25.96 = 0.1492\ \mathrm{m}^2$

$C_{A2} = 0.6 \times C_A = 0.6 \times 365.3 = 219.2$ W/K

$NTU_{A2} = (UA)_{A2}/C_{a,A2} \quad \Rightarrow (UA)_{A2} = 296.1$ W/K

但若是由下式來計算$(UA)_{A2}$(同樣暫且忽略管壁阻抗)

$$\frac{1}{(UA)_{A2}} = \frac{1}{\eta_o h_o A_2} + \frac{1}{h_i A_{2,i}} \tag{5-148}$$

由式5-148，可算出$(UA)_{A2}$ = 143.32 W/K

這個答案已經暗示A2區將無法完全冷凝，

如果再次將A2區的面積增加到最大，即：

$$A_2^* = 1 - A_1^* = 1 - 0.2681 = 0.7319$$

在A2最大面積下時的最大可能熱傳量可估算如下：

$$\therefore Q_{max,A2} = \dot{m}_{a,A2} c_{p,a} (T_{r,s} - T_{a,in}) = 0.363 \times 1007 \times 0.7319 \times (54 - 35) = 5083.2 \text{ W}$$

$C_{A2}$ = 0.7319×$C_A$ = 0.6×365.3 = 267.36 W/K

$\therefore A_2$ = 0.7319×6.455 = 4.724 $\text{m}^2$

管內面積$A_{2,i}$ = $A_2$/25.96 = 0.18197 $\text{m}^2$

再由式5-148，可得$(UA)_{A2}$ = 174.8

另外，由於此時沒有完全冷凝，因此必須先假設出口的乾度，才能算出真正的熱傳量，然後再由有效度反算*NTU*(即*UA*)，最後檢查*UA*是否滿足5-148的計算結果已決定是否需要再次疊代；因此，我們先假設出口乾度為$x$ = 0.3，所以：

$$\therefore Q_{A2} = \dot{m}_r i_{s,LG} (1 - x) = 0.02083 \times 148.3 \times 1000 \times (1 - 0.3) = 2162 \text{ W}$$

$$\therefore \varepsilon_{A2} = Q_{A2} / Q_{max,A2} = 2162 / 5083.2 = 0.4253$$

由$\varepsilon = 1 - e^{-NTU}$，可算出 $NTU_{A2}$ = 0.5539 W/K

$NTU_{A2} = (UA)_{A2}/C_{a,A2} \Rightarrow (UA)_{A2}$ = 148.1 W/K

而利用式5-148的計算法結果則為174.82 W/K，大於由式5-148算出的148.1 W/K；因此乾度應較大，如此經過反覆數次疊代後可以得到出口的乾度為0.2102。

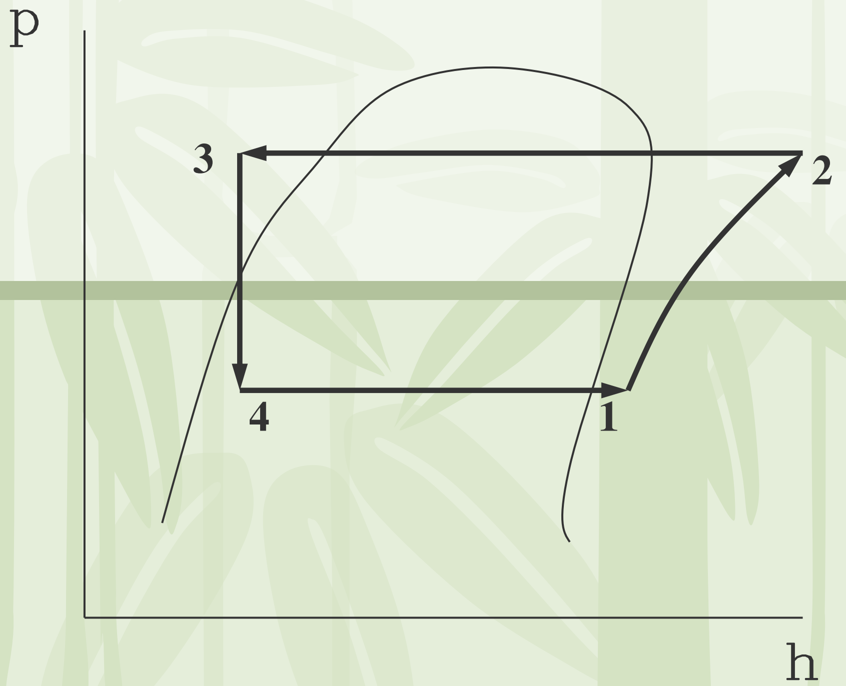
A2區的總熱傳量為 $3089 \times 0.7898 = 2439.8$ W

總熱傳量為 A1+A2 = 824.4+2439.8 = 3264.2 W

A2的面積為4.758 $m^2$

本計算例中，如果還需要計算A3區時，算法基本上與A1區類似，所不同的地方在於(1) 要將氣體部份的資料換成液態來計算；(2) A3區的面積為已知 = A – A1 –A2，所以要利用此給定的A3值來計算熱傳量 (及空氣側與冷媒側的出口溫度)。

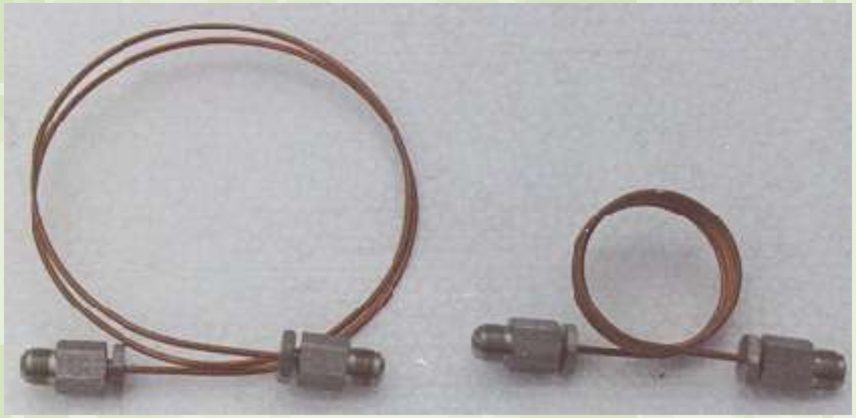
另外，在許多氣冷式熱交換器性能的測試標準中(冷凝器)，多以固定進口風速下(例如$V_{fr}$ = 1.5 m/s)、固定冷媒進口過熱度(例如過熱25°C)、固定冷媒冷凝溫度與固定冷媒出口過冷度(例如過冷5°C)時的能力為標準能力；讀者若碰到類似的問題，則可以本例題來估算；這類問題最大的困難在於冷媒流量的假設，讀者必須假設一個冷媒流量，然後以本例的計算過程去檢查冷媒出口狀態是否滿足過冷度的需求，若否，則需繼續假設冷媒流量重複疊代。



# 毛細管之計算分析
# process 3→4

# 毛細管介紹

- □膨脹裝置(expansion device)必須具備兩個基本功能：
  - ◆提供減壓功能，使高壓液態冷媒降壓至蒸發壓力。
  - ◆調節蒸發器所需之冷媒流量。
- □冷凍空調設備常用的膨脹裝置主要有：毛細管、感溫式膨脹閥(thermostatic expansion valve)和電子式膨脹閥(electric expansion valve)
- □應用在小型空調的毛細管內徑 一般約從1.0到2.0 mm ，長度約為400到2500 mm 。
  - ◆優點：價廉、 無運動元件、構造簡單；壓縮機停止運轉時，冷媒尚能緩慢流入低壓側，平衡系統壓力，減低壓縮機啟動時的轉矩 。
  - ◆缺點：冷凍負荷改變時，無法隨負載來變化冷媒流量；易受外物阻塞而失去效應；冷媒充填量保持在精密的範圍內，因此只能使用在封閉系統中，這種系統比較不會洩漏。



# 毛細管介紹(Cont.)

- 毛細管幾何尺寸決定並予安裝後，毛細管對於系統的吐出壓力、吸入壓力，及冷凍負荷等變化，將無法自行調整。一旦系統的高低壓力改變時，壓縮機與毛細管會暫處於不平衡狀況，然後再逐漸趨向另一個平衡點而穩定下來。
- 壓縮機與毛細管須達成平衡關係，使得在某一個吐出和吸入壓力條件下，由壓縮機吐出的冷媒流量，與通過毛細管至蒸發器的流量一致。

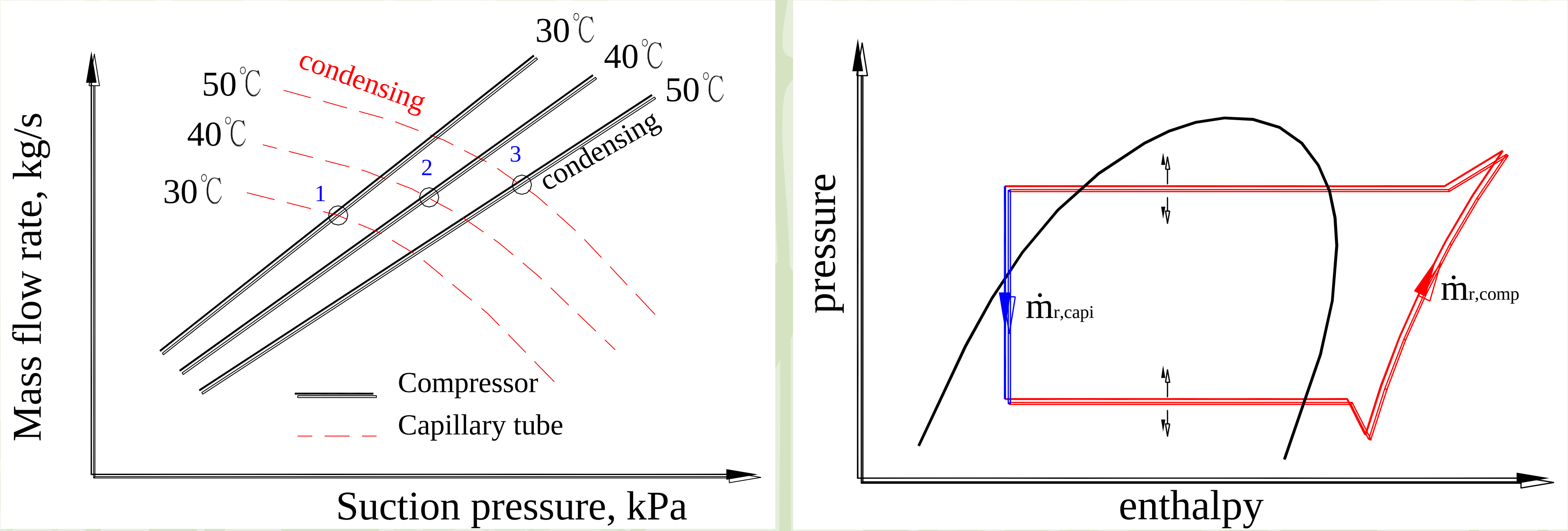


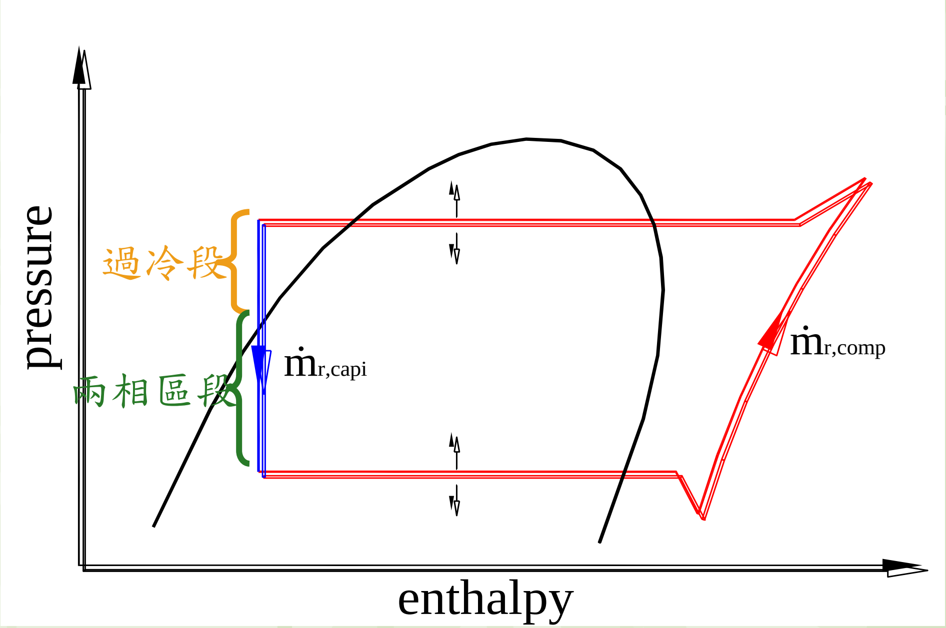
Fig. 壓縮機與毛細管之平衡點

# 毛細管介紹(Cont.)

- 毛細管依靠其流動阻力沿長度方向產生壓損，來控制冷媒流量和維持冷凝器和蒸發器的壓差。

(1)當有一定過冷度的冷媒流量進入毛細管後，會沿著流動方向產生壓力和狀態變化，先是過冷液體隨壓力的逐步降低，先變爲相應壓力下的飽和液體，這一段稱液相段，其壓力降不大，且呈線性變化。

(2)從出現第一個氣泡開始至毛細管末端，均爲氣液並存段，也稱兩相流動段，該段內飽和蒸汽含量沿流動方向逐漸增加(液態冷媒量逐漸減少)，因此壓力降呈非線性變化，愈到毛細管的末端，其單位長度上的壓損愈大。



# 毛細管冷媒壓力－溫度分佈曲線

❑絕熱狀態下，自冷凝器出口的高壓過冷液體進入毛細管後，壓力與溫度的變化情況：

◆1-2部分為管徑不同造成的收縮壓損。

◆2-3部分為過冷液態冷媒，受管壁摩擦作用，壓力呈線性遞減，而溫度則保持不變，點3狀態為熱力平衡的飽和壓力和溫度，此時第1顆氣泡開始產生。

◆隨後的3-4部分為氣液共存的兩相區，壓力伴隨著溫度呈非線性遞減，而且沿著流動方向氣泡逐漸增加，亦即乾度(quality)增加，因此每單位長度的壓損也隨之增加。

◆第4點為毛細管出口，此時冷媒為低壓兩相狀態，最後由點5進入蒸發器。

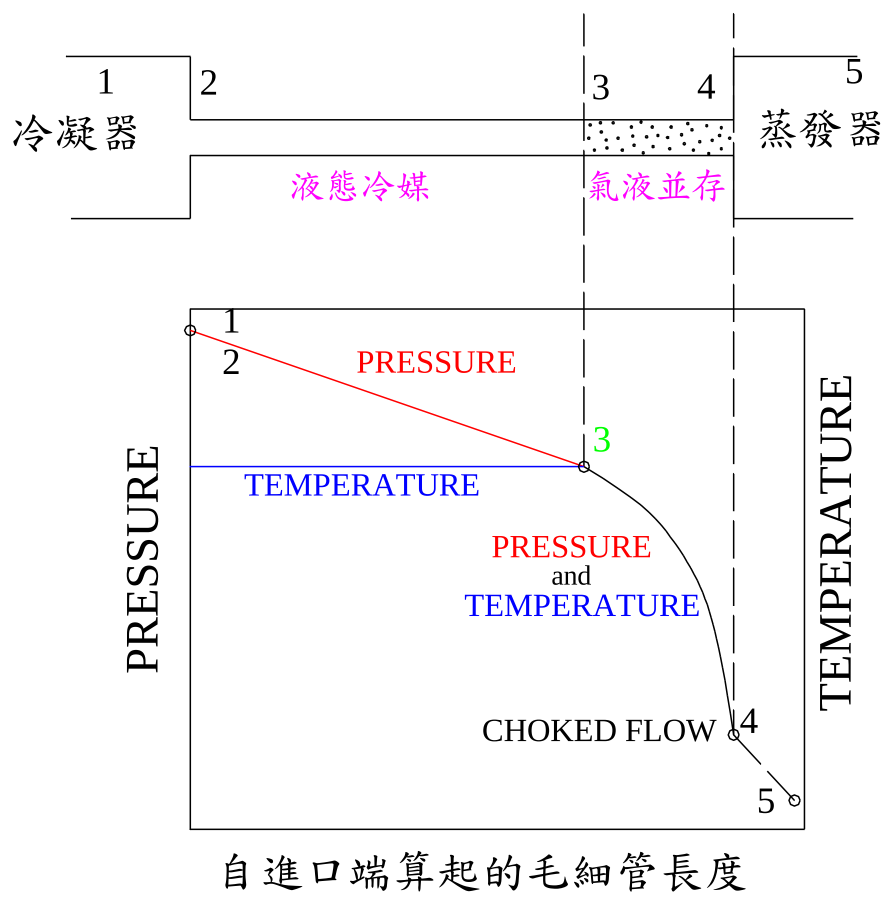


Fig.毛細管內冷媒之壓力與溫度分佈
(摘自Bolstad and Jordan(1948))

# 暫穩態現象(metastable phenomena)

- 實際上，毛細管內冷媒的蒸發閃化點並不是發生在熱力飽和狀態點，而是發生在該點下游的某一位置。亦即冷媒繼續保持液態，溫度不變， 而壓力持續下降，直到下游某處b點， 第一顆氣泡產生(實際閃化點)。之後溫度下降率大於壓力下降率，到c點兩條曲線才合併。
  - ◆ a點(理論閃化點)到b點(實際閃化點)的距離稱為「延遲長度」(delay length) 。
- 毛細管內溫度、壓力變化分為四段：

(Ⅰ)過冷單相液體---入口點o至a

(Ⅱ)暫穩態單相液體---a點至bb′

(Ⅲ)暫穩態氣液並存兩相---bb′至c點

(Ⅳ)熱力平衡氣液並存兩相---c點至入口端

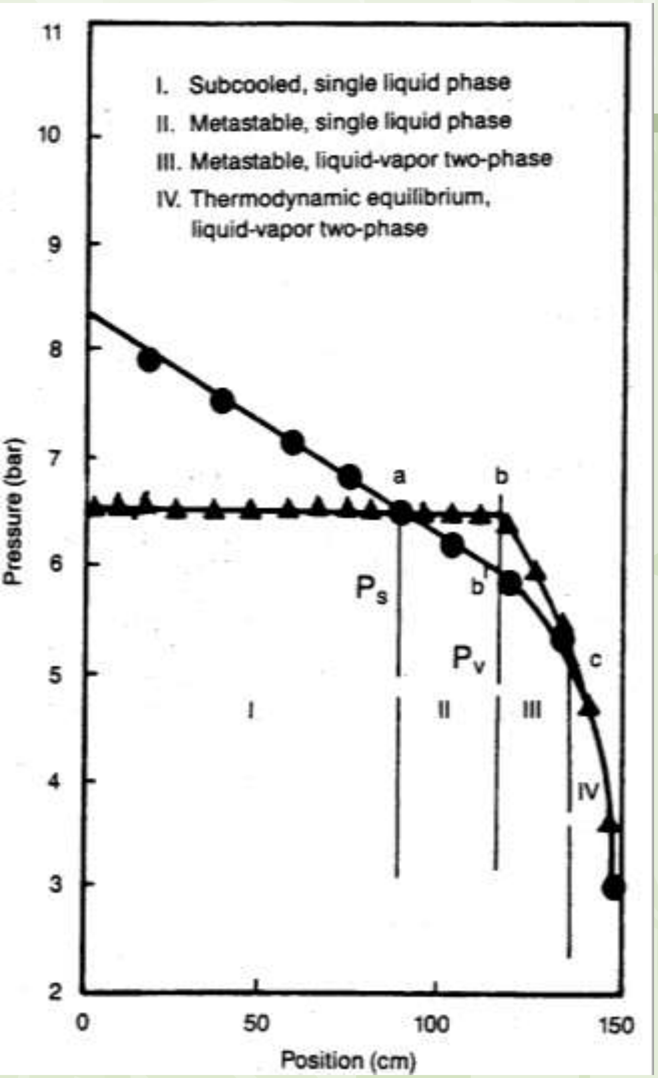
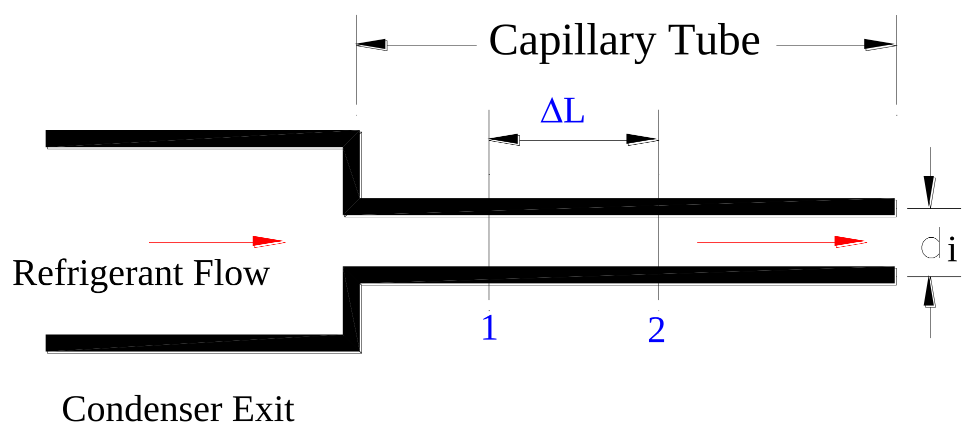


Fig.毛細管內飽和溫度與量測壓力之變化情形(摘自Chen et al.(1990))

# 毛細管流量計算

- 毛細管冷媒流量計算，假設管內流體為穩態流動(steady state)且為絕熱過程(adiabatic process)，則冷媒在毛細管內流動任意兩點1，2之間的平衡關係表示如下：



連續方程式(Continuity equation)：

$$\frac{\dot{m}_{r,capi}}{A_{capi}} = \frac{V_1}{\upsilon_1} = \frac{V_2}{\upsilon_2} = G \qquad (1)$$

能量方程式(Energy equation)：

$$h_1 + \frac{V_1^{\,2}}{2} = h_2 + \frac{V_2^{\,2}}{2} \qquad (2)$$

動量方程式(Momentum equation)：

$$\left[(P_1 - P_2) - f\,\frac{\Delta L}{d_i}\frac{V^2}{2\upsilon}\right]A_{capi} = \dot{m}_{r,capi}(V_2 - V_1) \qquad (3)$$

# 阻塞流(choked flow)

- 在冷凝壓力固定情況下，壓縮機吸入壓力下降，可使毛細管允許通過的冷媒量增加；當吸入壓力持續下降至某一程度，即使壓力再降低，流量仍不會再增加，表示此時毛細管已到達阻塞流(choked flow) 。
  - 阻塞流發生時，毛細管內冷媒流速已達到音速，由於無法突破音速極限，因此流量被限制住。
- 實際上毛細管壓損計算，除了管內冷媒流動所造成的摩擦及加速度阻抗外，尚須考慮毛細管進出端的因管徑改變所造成的突然收縮(sudden contraction)和突然膨脹(sudden expansion)的壓損效應。

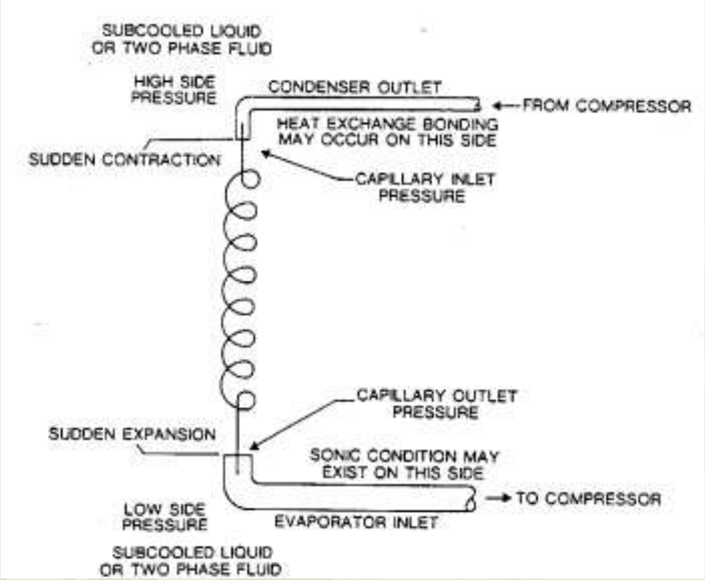
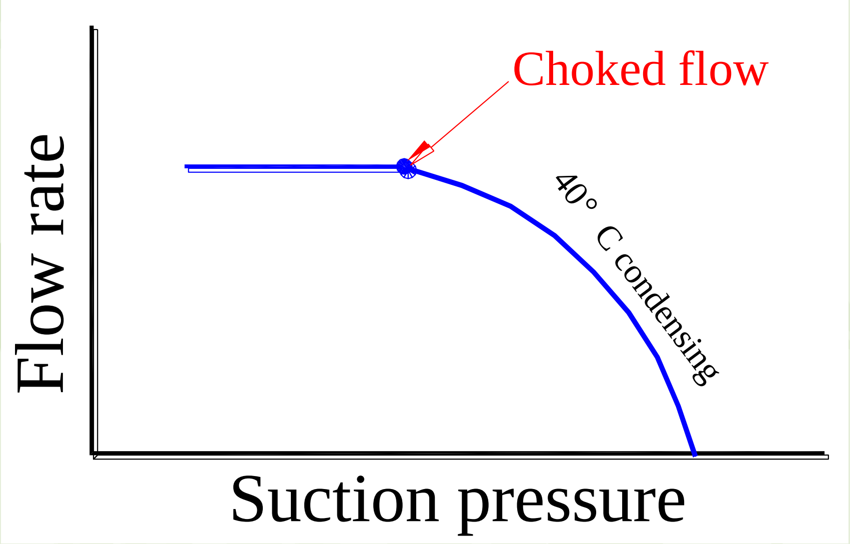


Fig.毛細管進出口端壓損示意圖



# 過冷度(subcooling)對流量的影響

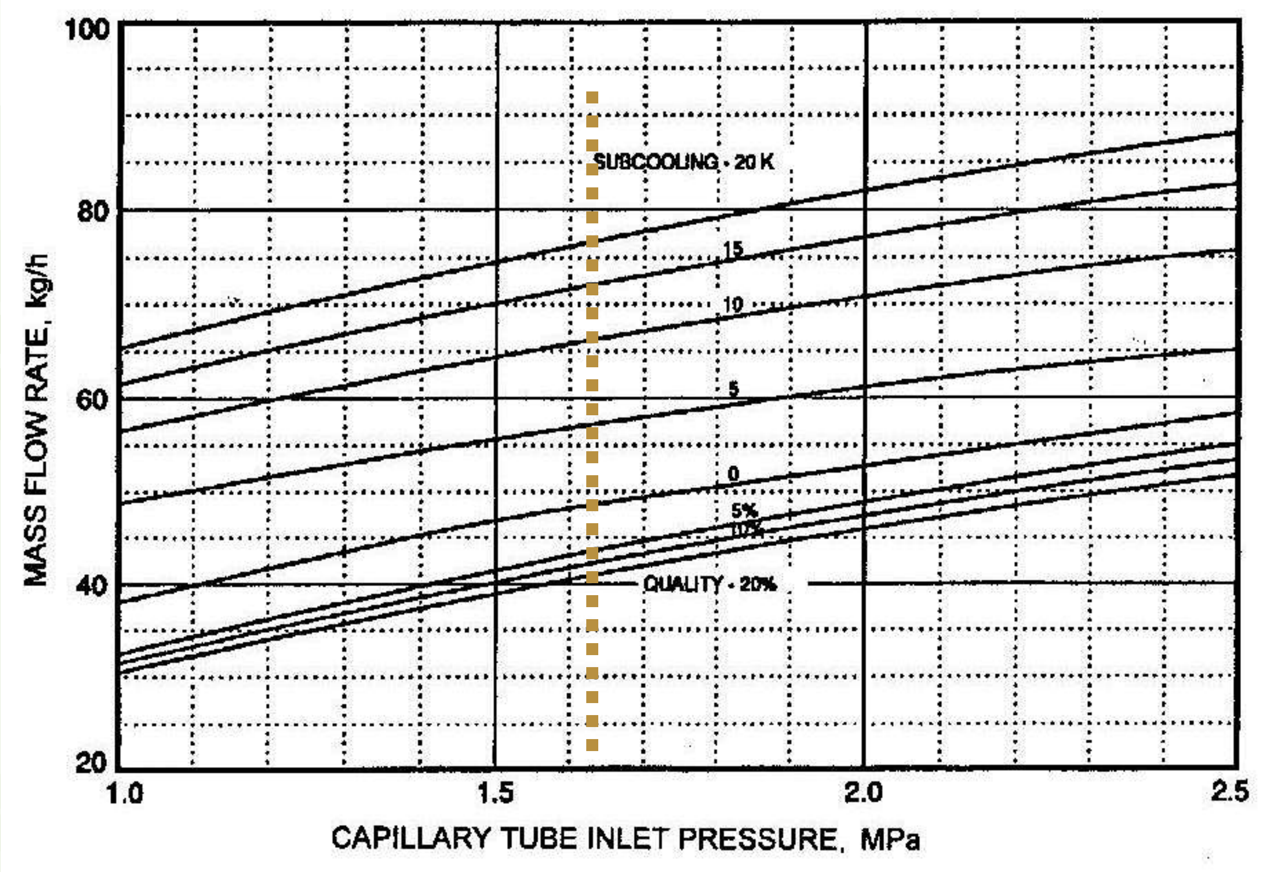


**Fig. 46 Mass Flow Rate of R-22 Through Capillary Tube**
(Capillary tube diameter is 1.68 mm I.D. and length is 1520 mm.)

Fig. 不同過冷度，冷媒流量對毛細管
進口壓力之變化情形
(摘自 ASHRAE HANDBOOK 1998)

# 毛細管流量影響

❑影響毛細管通過的冷媒流量因素主要有：流體性質(密度、黏滯度)、毛細管幾何尺寸(長度、內徑、粗糙度)和毛細管進口狀態(過冷度、入口壓力)。

◆ 毛細管的允許通過的冷媒流量，隨管內徑增加而增加；但隨著毛細管長度增加而減少。

◆ 對不同的毛細管內徑而言，每一個管徑相對於系統的能力與EER值均有一個最佳的毛細管長度，而最佳長度的發生位置值隨管徑不同而不同。

◆ 對同一個過冷度而言，冷媒流量隨入口壓力增加而明顯增加。

◆對同一個入口壓力而言，冷媒流量隨過冷度增加而增加；因為過冷度增加，縮短了毛細管中兩相區的長度，減少冷媒在管內的加速度阻抗

❑對冷凍空調系統而言，應有一個相對的毛細管設計最佳幾何尺寸。一旦尺寸決定後，若是持續增加毛細管長度，不但使阻抗增加，減少允許通過毛細管的流量，而且能力值下降。為達成系統流量平衡，將使系統壓力升高，而耗功因此增加，EER值降低。

# 系統平衡(Cont.)

❑壓縮機與毛細管在系統運轉中，無法將吸入端壓力(suction pressure)固定下來，因為必需滿足蒸發器的熱傳關係。一旦在壓縮機－毛細管的平衡點下，蒸發器的熱傳無法滿足，將發生蒸發器出口過熱度太高(冷媒短缺(starving))或者未完全蒸發(冷媒過量餵入(overfeeding))的不平衡現象。

❑蒸發器負載增加→吸入壓力溫度升高至B點→壓縮機供應冷媒量增加，大於毛細管允許通過的量→造成蒸發器冷媒量不足(starving) 。

◆冷凝器屯積過多液態冷媒， 有效冷凝面積減少→冷凝壓力升高，及過冷度增加→壓縮機之壓縮降低，毛細管通過的冷媒量上升→過冷度減少→回復到平衡流量 。

◆由於蒸發器內的冷媒與被冷卻流體須維持較大的溫差，系統吸入壓力將回至A點，回復平衡流量 。

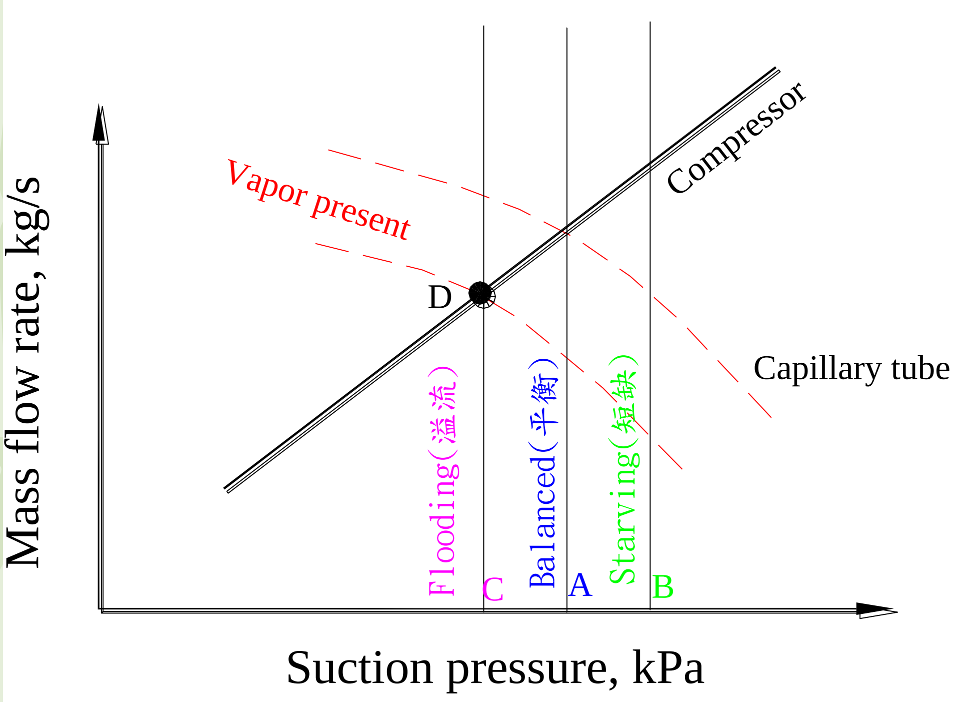


Fig.蒸發器負載改變造成之不平衡狀況

# 冷媒流量變化與壓縮機及毛細管間的關聯

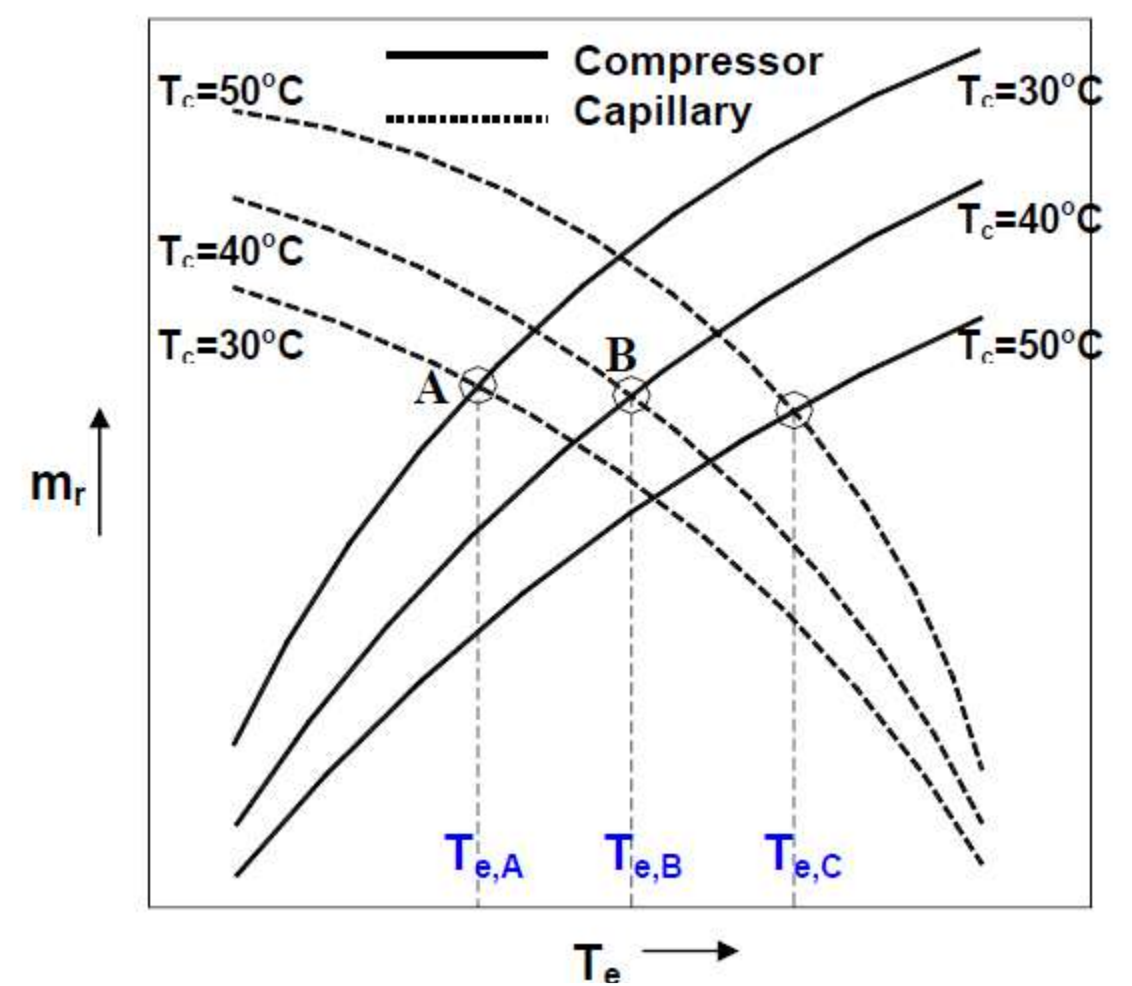


**Fig.24.1**: *Variation of refrigerant mass flow rate through compressor and capillary tube with evaporator and condenser temperatures (A,B & C are the balance points)*

## Increase in refrigeration Load:

If the refrigeration load increases, there is a tendency for the evaporator temperature to increase due to higher rate of evaporation. This situation is shown in Figure 24.2 for a condenser temperature of 40°C. The balance point for design load is shown by point B. As the load increases, the evaporator temperature rises to C. At point C the mass flow rate through compressor is more than the mass flow rate through the capillary tube. In such a situation, the compressor will draw more refrigerant through the evaporator than the capillary tube can supply to it. This will lead to *starving* of the evaporator. However, emptying of evaporator cannot continue indefinitely. The system will take some corrective action since changes are occurring in the condenser also. Since the capillary tube feeds less refrigerant to the evaporator, the refrigerant accumulates in the condenser. The accumulation of refrigerant in the condenser reduces the effective area of the condenser that is available for heat transfer. The condenser heat transfer rate is given by, $Q_c = U_c A_c ( T_c - T_\infty )$. If heat transfer coefficient $U_c$ and $T_\infty$ are constant, then for same heat transfer rate a decrease in area $A_c$ will lead to a higher condenser temperature $T_c$. It is observed from Figure 24.1 that an increase in condenser temperature leads to a decrease in compressor mass flow rate and an increase in capillary mass flow rate. Hence, the system will find a new balance point at higher condenser temperature.

## Decrease In refrigeration Load

If the refrigeration load decreases, there is a tendency for the evaporator temperature to decrease, say to state *A as* shown in Figure 28.2. In this condition the capillary tube feeds more refrigerant to the evaporator than the compressor can remove. This leads to accumulation of liquid refrigerant in the evaporator causing *flooding* of the evaporator. This may lead to dangerous consequences if the liquid refrigerant overflows to the compressor causing *slugging* of the compressor. This has to be avoided at all costs; hence the capillary tube based refrigeration systems use *critical charge* as a safety measure. Critical charge is a definite amount of refrigerant that is put into the refrigeration system so that in the eventuality of all of it accumulating in the evaporator, it will just fill the evaporator up to its brim and never overflow from the evaporator to compressor. The flooding of the evaporator is also a transient phenomenon, it cannot continue indefinitely. The system has to take some corrective action. Since the capillary tube feeds more refrigerant from the condenser, the liquid seal at the condenser-exit breaks and some vapour enters the capillary tube. The vapour has a very small density compared to the liquid; as a result the mass flow rate through the capillary tube decreases drastically. This situation is shown in Figure 28.2. This is not desirable since the refrigeration effect decreases and the COP also decreases. Hence, attempts are made in all the refrigeration plants to subcool the refrigerant before entry to the expansion device. A vapour to liquid subcooling heat exchanger is usually employed, wherein the low temperature refrigerant vapour leaving the evaporator subcools the liquid leaving the condenser.

# 系統平衡

- 蒸發器負載降低→吸入壓力溫度降至至C點→毛細管供給的冷媒量，大於壓縮機吐出的流量→系統處於不平衡狀態→造成蒸發器液態冷媒量過多，可能溢流(flooding)至壓縮機，壓縮機若無適當保護，可能有液壓縮危險 。(系統必須精確的充填冷媒量)

- 蒸發器發生溢流現象時， 部分冷媒氣體將回流至毛細管→冷媒氣體具有較大比容，增加毛細管阻抗而降低冷媒流量→回復新的流量平衡點D 。

- 由於新的流量平衡點D為氣液並存的兩相狀態 →毛細管入口冷媒狀態為氣液並存，減少冷凍效果。

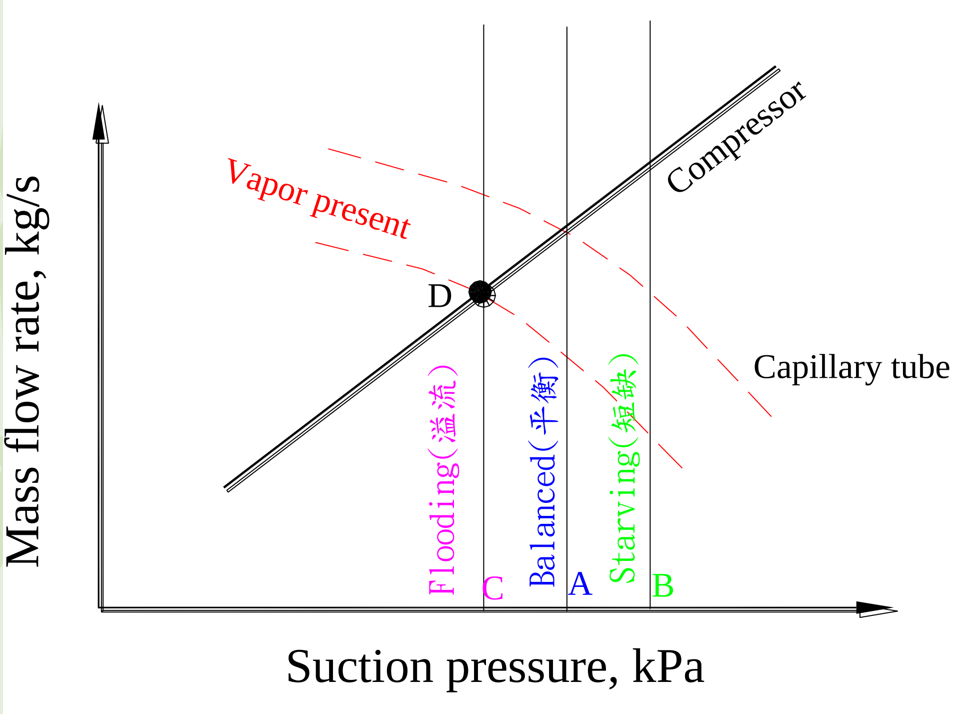


Fig.蒸發器負載改變造成之不平衡狀況

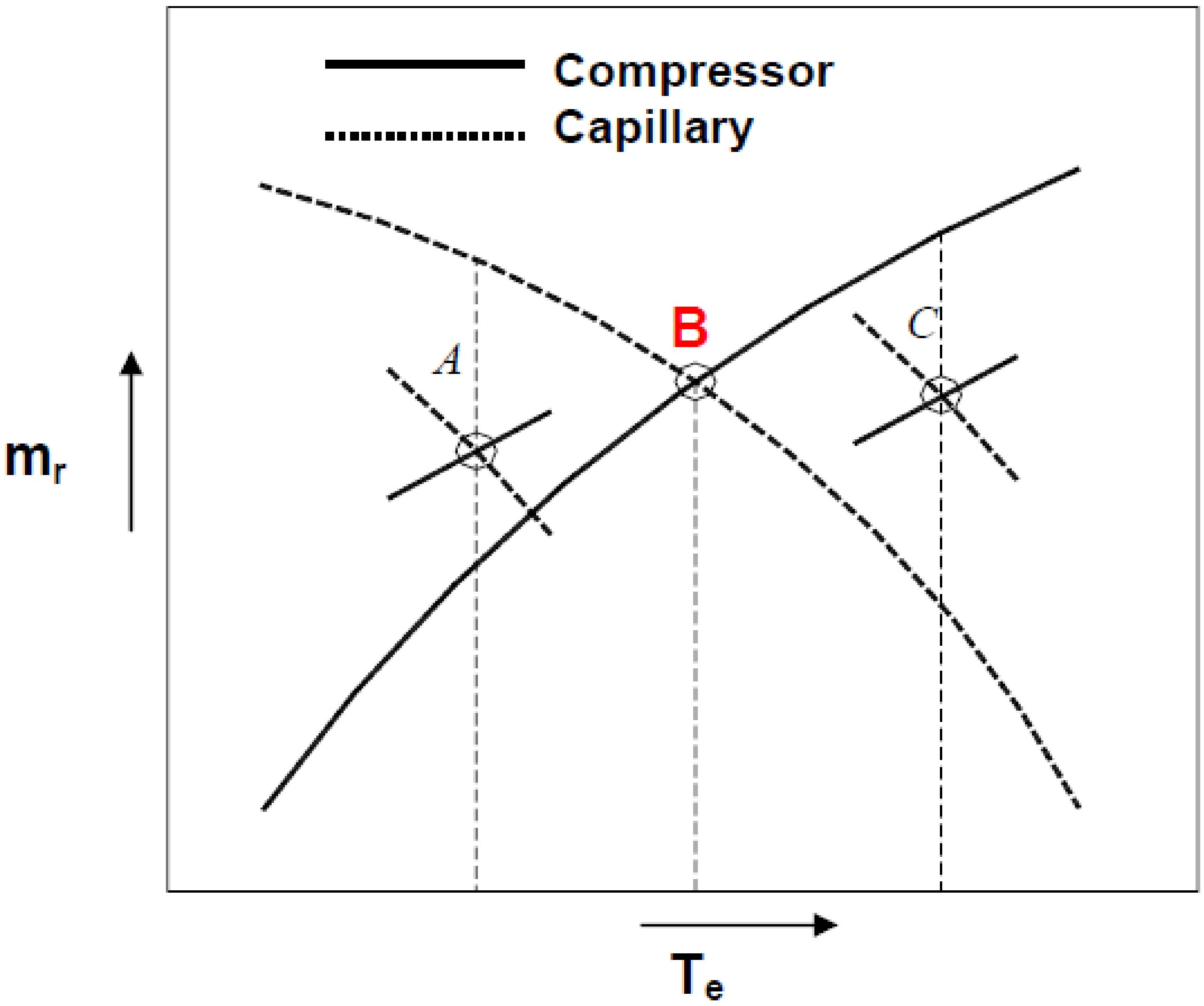


***Fig.24.2**: Effect of load variation on capillary tube based refrigeration systems. B: Design point; A: At low load; C: At high load*

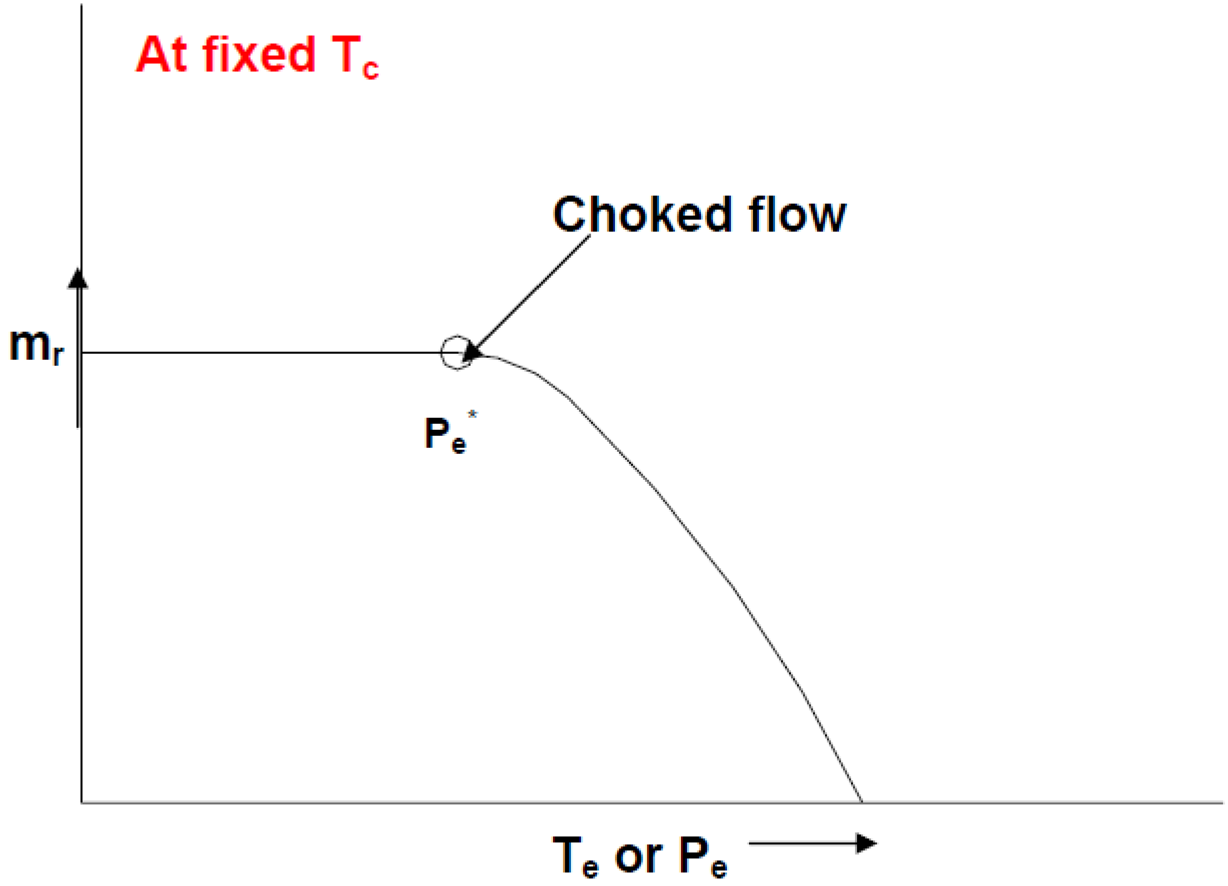
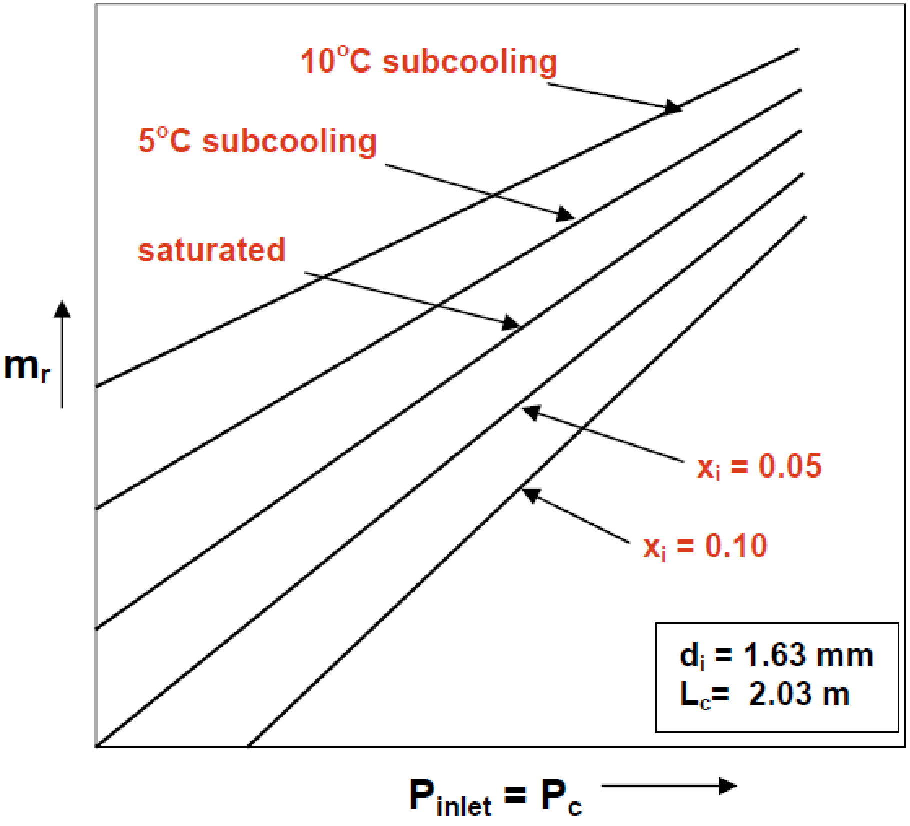
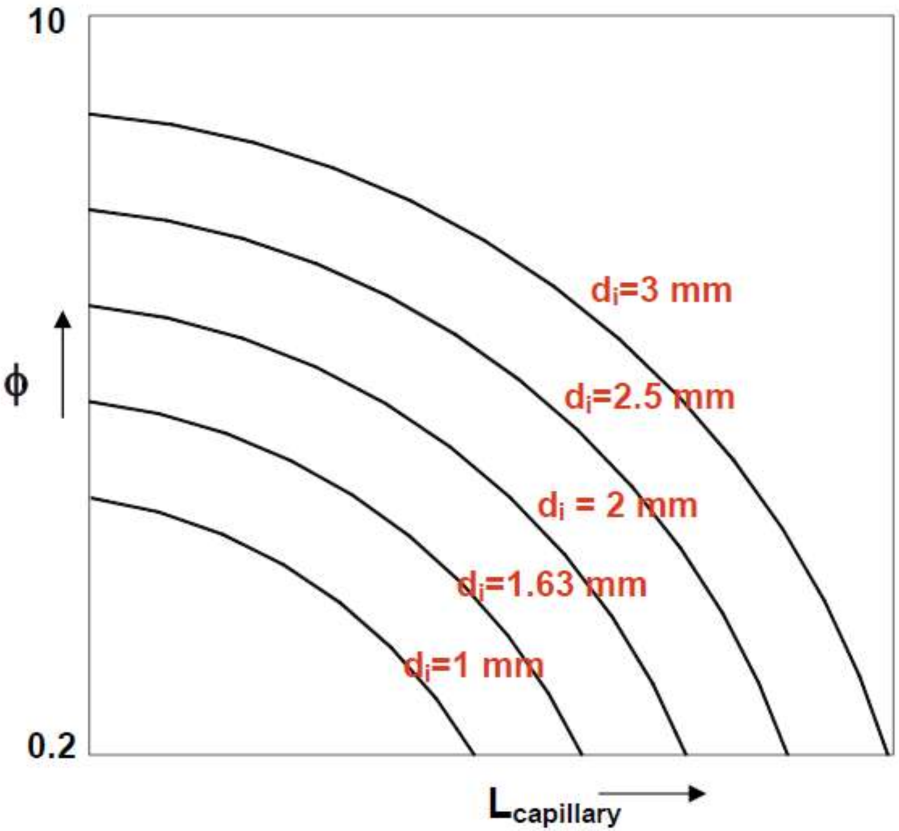


***Fig.24.5**: Variation mass flow rate with suction pressure for fixed condenser pressure*

# Rating a capillary tube ASHRAE (graph method) choked flow condtion

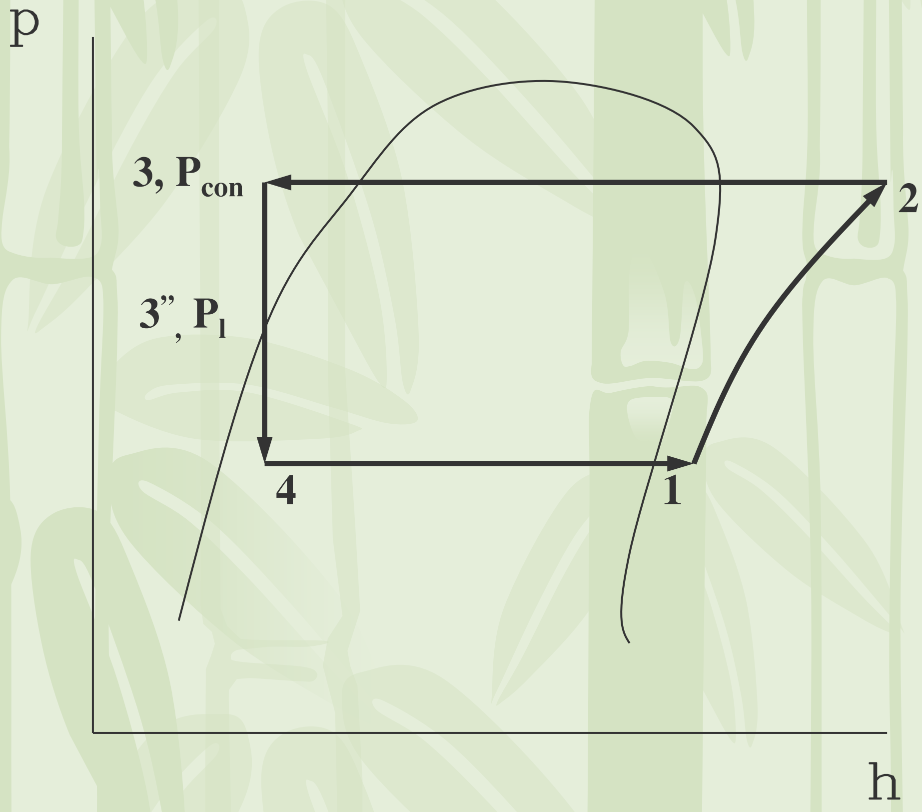
$$m_{d_i,L_c} = m_{1.63\ \text{mm},\ 2.03\ \text{m}}\cdot\phi$$



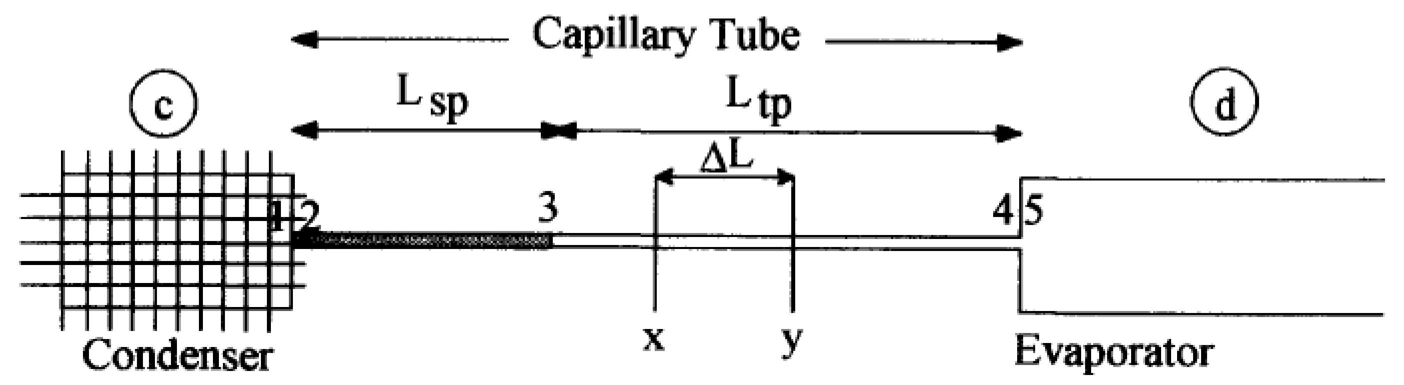


# 毛細管計算流程 (simplified version $h_3 = h_4$), Process 3→4

- ◆由給定之毛細管幾何尺寸與入口流量及溫度計算沿著毛細管長度增加時壓力會逐漸見下降 ，以單相流體模式來計算沿管路的壓降，當壓力到達飽和液體時流體將氣化(3 → 3”)。
- ◆3”→4，以兩相模式(例如假設homogeneous flow model)計算壓力變化，並同時計算逐漸增加的平均流速，如果到達毛細管出口端之流速尚未達音速，則毛細管出可的壓力與乾度以計算值，但如果已達音速，代表此一毛細管已達限流條件。即此一毛細管不可能達到此一流量，因此必須將蒸發壓力調低，讓1→2的流量下降。



# Typical modeling for 3→4 Process



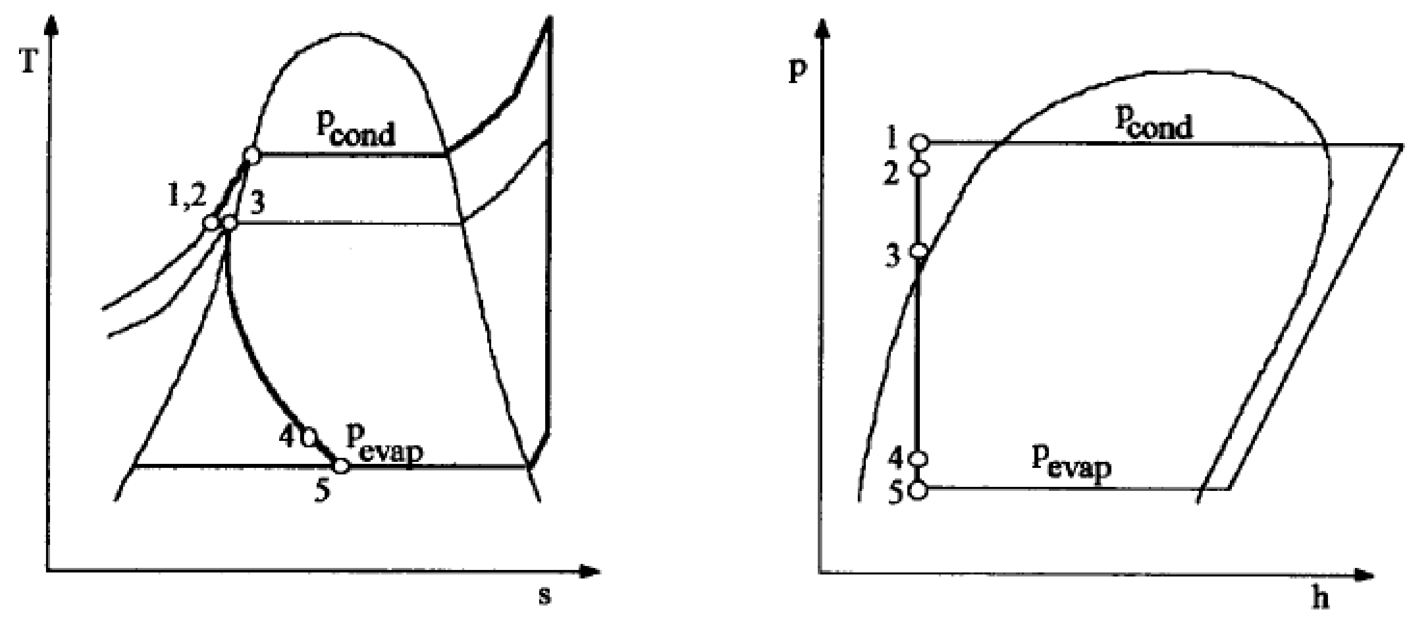
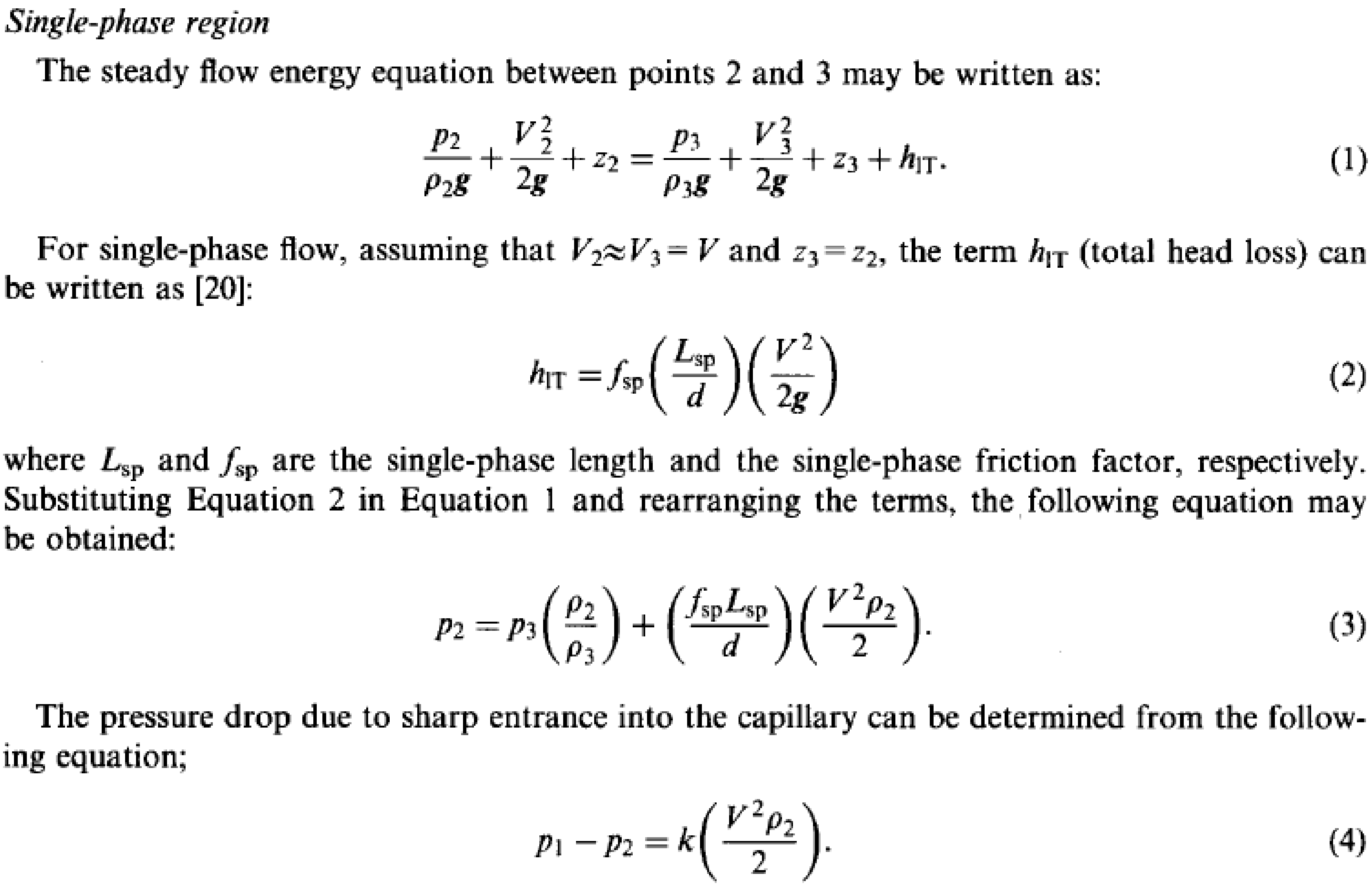


Fig. 3. Location of state points 1–5 in *T*–*s* and *p*–*h* diagrams.

# Single phase calculation



*Single-phase region*

The steady flow energy equation between points 2 and 3 may be written as:

$$\frac{p_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2=\frac{p_3}{\rho_3 g}+\frac{V_3^2}{2g}+z_3+h_{lT}. \tag{1}$$

For single-phase flow, assuming that $V_2 \approx V_3 = V$ and $z_3 = z_2$, the term $h_{lT}$ (total head loss) can be written as [20]:

$$h_{lT}=f_{sp}\left(\frac{L_{sp}}{d}\right)\left(\frac{V^2}{2g}\right) \tag{2}$$

where $L_{sp}$ and $f_{sp}$ are the single-phase length and the single-phase friction factor, respectively. Substituting Equation 2 in Equation 1 and rearranging the terms, the following equation may be obtained:

$$p_2=p_3\left(\frac{\rho_2}{\rho_3}\right)+\left(\frac{f_{sp}L_{sp}}{d}\right)\left(\frac{V^2\rho_2}{2}\right). \tag{3}$$

The pressure drop due to sharp entrance into the capillary can be determined from the following equation;

$$p_1-p_2=k\left(\frac{V^2\rho_2}{2}\right). \tag{4}$$

Saturation pressure at 3, $p_3$, can be determined by knowing the level of subcooling at the capillary entrance i.e. $T_3 = T_1 - \Delta T_{\mathrm{sub}}$. Therefore, adding Equations 3 and 4, and assuming $\rho_2 = \rho_3$, the single-phase length, $L_{\mathrm{sp}}$, of the capillary tube can be determined from:

$$L_{\mathrm{sp}} = \left\{ (p_1 - p_3)\frac{2\rho_2}{G^2} - k \right\} \frac{d}{f_{\mathrm{sp}}} \tag{5}$$

where G ($\dot{m}/A$) is the refrigerant mass flow rate per unit area, and the single-phase friction factor, $f_{\mathrm{sp}}$, can be calculated from the Churchill [21] correlation, as shown below:

$$f_{\mathrm{sp}} = 8\left[ \left(\frac{8}{Re}\right)^{12} + \frac{1}{(A+B)^{3/2}} \right]^{1/12} \tag{6}$$

where

$$A = \left\{ 2.457 \ln\left[ \frac{1}{\left(\frac{7}{Re}\right)^{0.9} + 0.27\varepsilon} \right] \right\}^{16} \tag{6a}$$

$$B = \left(\frac{37530}{Re}\right)^{16} \tag{6b}$$

and

$$Re = \frac{\dot{m}d}{\mu A} \tag{6c}$$

where $\epsilon$ is relative roughness of the tube, $d$ is the tube internal diameter and $\mu$ is the dynamic viscosity of the liquid refrigerant.

### Two-phase region

The fundamental equations applicable to the control volume bounded by points $x$ and $y$ (see Fig. 2) in the two-phase region are the conservation of mass, the conservation of energy, and the conservation of momentum. The equation for conservation of mass states that

$$\dot{m} = \frac{AV_3}{v_3} = \frac{AV_4}{v_4}. \tag{7}$$

Neglecting the elevation difference and the heat transfer in and out of the tube, the equation for energy conservation may be written as:

$$h_x + \frac{V_x^2}{2} = h + \frac{V^2}{2} \tag{8}$$

where $h$ and $V$ are, respectively, the enthalpy and the velocity at any point in the two-phase region.

From the conservation of momentum equation, the difference in forces applied to the element due to drag and pressure difference on opposite ends of the element should be equal to that needed to accelerate the fluid, i.e.

$$\left[(p_x - p_y) - \left(\frac{f_{\mathrm{tp}}\Delta L}{d}\right)\left(\frac{V^2}{2g}\right)\left(\frac{\rho_x + \rho_y}{2}\right)g\right]A = \dot{m}(V_x - V_y). \tag{9}$$

# Two-phase Viscosity Model

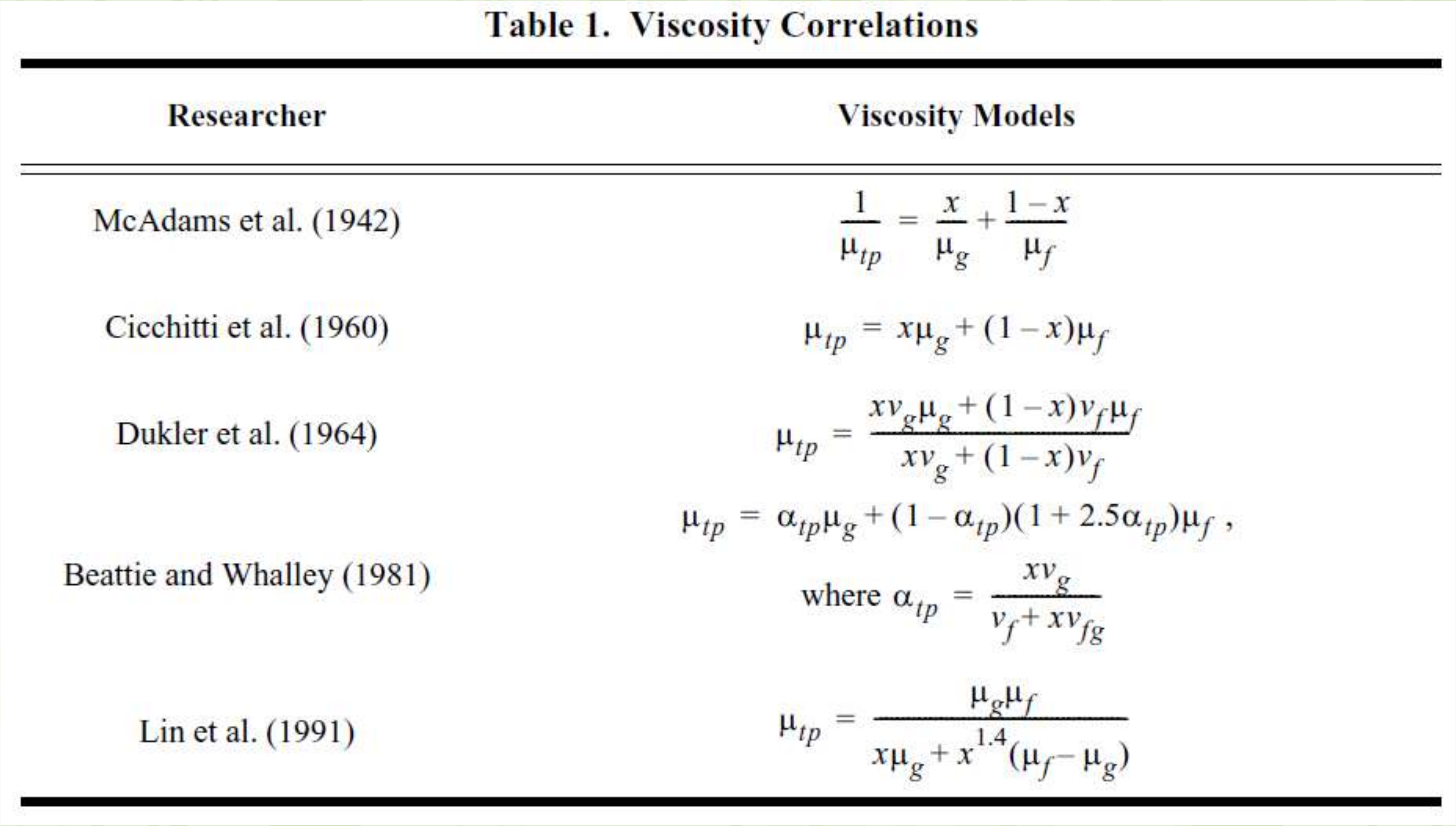


Table 1. Viscosity Correlations

| Researcher | Viscosity Models |
|---|---|
| McAdams et al. (1942) | $\frac{1}{\mu_{tp}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f}$ |
| Cicchitti et al. (1960) | $\mu_{tp} = x\mu_g + (1-x)\mu_f$ |
| Dukler et al. (1964) | $\mu_{tp} = \frac{x v_g \mu_g + (1-x) v_f \mu_f}{x v_g + (1-x) v_f}$ |
| Beattie and Whalley (1981) | $\mu_{tp} = \alpha_{tp}\mu_g + (1-\alpha_{tp})(1+2.5\alpha_{tp})\mu_f$ ,<br>where $\alpha_{tp} = \frac{x v_g}{v_f + x v_{fg}}$ |
| Lin et al. (1991) | $\mu_{tp} = \frac{\mu_g \mu_f}{x\mu_g + x^{1.4}(\mu_f - \mu_g)}$ |

# Calculation..

Require step-by-step calculation to ensure whether choked phenomenon will take place

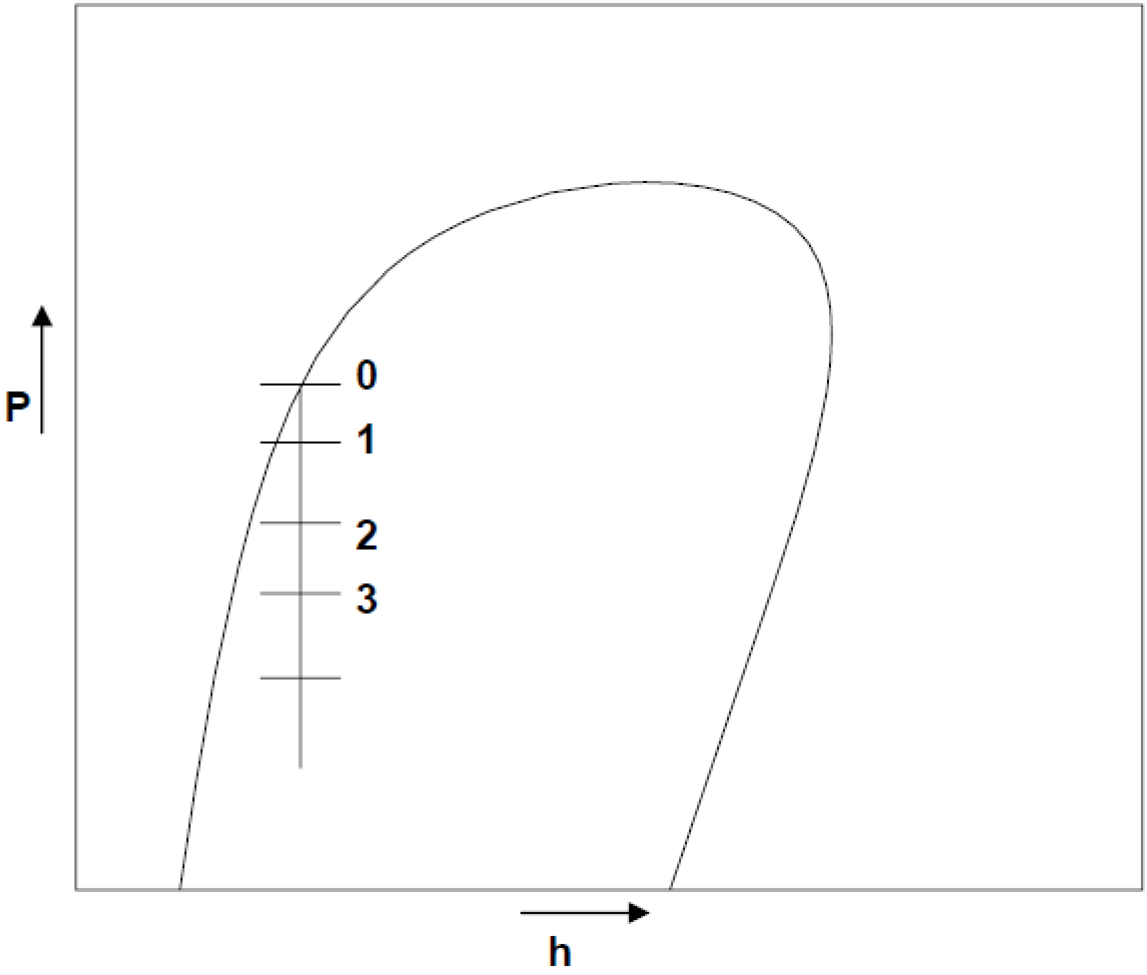
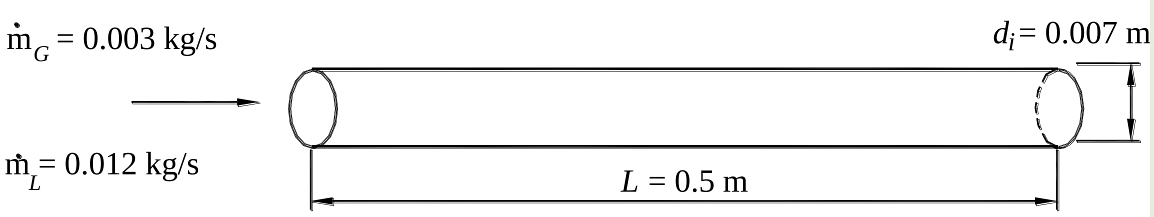


***Fig.24.4***: *Step-wise calculation procedure for capillary tube length on p-h diagram*

# Example: Two-phase pressure drop calculation 如下圖，空氣與水的兩相流流入一內徑 7 mm管，長度 0.5m的圓管，試以homogeneous model方程式來計算壓降，$\rho_L = 998.3\ \text{kg/m}^3$，$\rho_G = 1.098\ \text{kg/m}^3$，$\sigma_L = 0.0661\ \text{N/m}$，$\mu_L = 0.00046\ \text{Pa·s}$，$\mu_G = 0.0000203\ \text{Pa·s}$。



$$\dot{m} = \dot{m}_G + \dot{m}_L = 0.003 + 0.012 = 0.015\ \text{kg/s}$$

$$x = 0.003/(0.003 + 0.012) = 0.2$$

$$A_c = \pi \times d_i^2/4 = 3.848 \times 10^{-5}\ \text{m}^2$$

$$G = \dot{m}/A_c = 390\ \text{kg/m}^2\cdot\text{s}$$

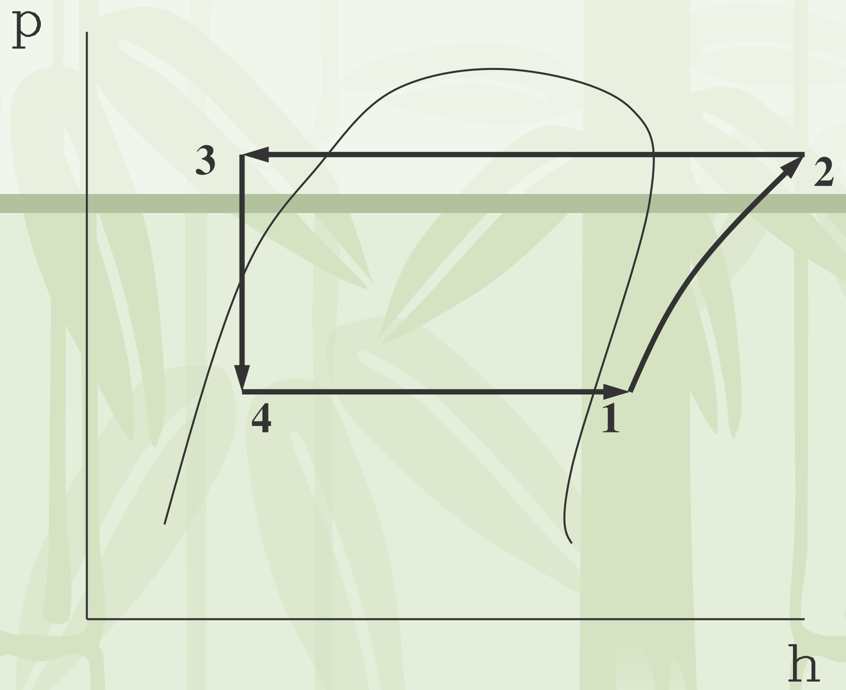
$$\bar{\rho} = \frac{1}{\left(\dfrac{x}{\rho_G} + \dfrac{(1-x)}{\rho_L}\right)} = \frac{1}{\left(\dfrac{0.2}{1.098} + \dfrac{(1-0.2)}{998.3}\right)} = 5.47\ \text{kg/m}^3$$

$$\mu_{TP} = \frac{1}{\left(\dfrac{x}{\mu_G} + \dfrac{(1-x)}{\mu_L}\right)} = \frac{1}{\left(\dfrac{0.2}{0.0000203} + \dfrac{(1-0.2)}{0.00046}\right)} = 0.0000863\ \text{Pa}\cdot\text{s}$$

$$\text{Re}_{TP} = \frac{GD_i}{\mu_{TP}} = \frac{390 \times 0.007}{0.0000863} = 31633.84 \Rightarrow \text{ 紊流，turbulent}$$

$$f_m = 0.079\text{Re}_{TP}^{-0.25} = 0.00593$$

$$dP_{\text{hom}} = \frac{4Lf_m}{D}\frac{G^2}{2\bar{\rho}} = \frac{4 \times 0.5 \times 0.00593}{0.007} \times \frac{390^2}{2 \times 5.47} = 23.56\ \text{kPa}$$



# 蒸發器之基本設計流程 4→1

# 蒸發器熱傳計算

- 蒸發器主要的作用是使低溫的冷媒氣體，吸收外界空間的熱量，達到造冷的效果，管內流體的流動狀態如圖 所示。
- 冷媒經過膨脹裝置的降溫降壓過程後，成為氣液兩相狀態，蒸發器入口處乾度通常是介於0.15～0.3 之間。當管內冷媒吸收周圍環境的熱量，使得冷媒逐漸汽化，最後達到飽和氣體狀態。設計上，通常是以過熱的狀態離開蒸發器，然後吸入壓縮機，完成一個冷凍循環。
- 冷媒的吸熱過程中，除了室內溫度下降外，若是盤管的表面溫度低於空氣的露點濕度（dew point temperature），當空氣流過盤管表面時，會有水份凝結下來，達到冷卻和除濕的目的。
- 此一熱傳現象包括顯熱 (sensible heat) 和潛熱 (latent heat) 兩種變化，因此一旦凝結開始，總熱傳為顯熱和潛熱兩者的結合，而代表溫差和相變化的驅動能力，是以焓勢 (enthalpy potential) 來表示。

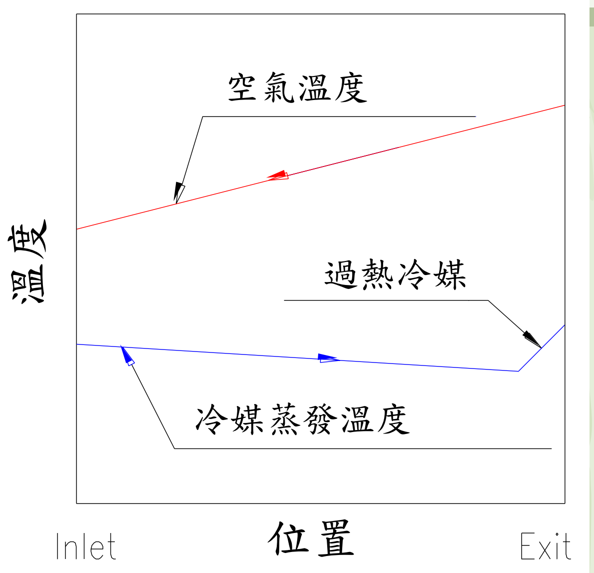


圖2. 蒸發器內冷媒與空氣的溫度變化

# 基本介紹

□蒸發器的熱傳分析，可以由熱交換器表面溫度判斷來區分為三種情況(見圖1)：

- ●熱交換器空氣入口鰭片表面溫度低於空氣露點溫度，則盤管**全濕**。
- ●熱交換器空氣出口鰭片表面溫度高於空氣露點溫度，則盤管**全乾**。
- ●熱交換器入口鰭片溫度高於露點溫度，而出口低於露點溫度，顯示空氣通過盤管當中，溫度降至某點而發生凝結現象，則盤管為**部份乾、部份濕**。

□目前所建立的熱交換器熱傳分析模式，以全乾的理論模式最為完整，而全濕次之； 至於半乾濕，尚無完整的系統理論基礎，但仍有一些研究將全乾與全濕的分析方法合成後來模擬。

# 何謂乾、濕盤管？

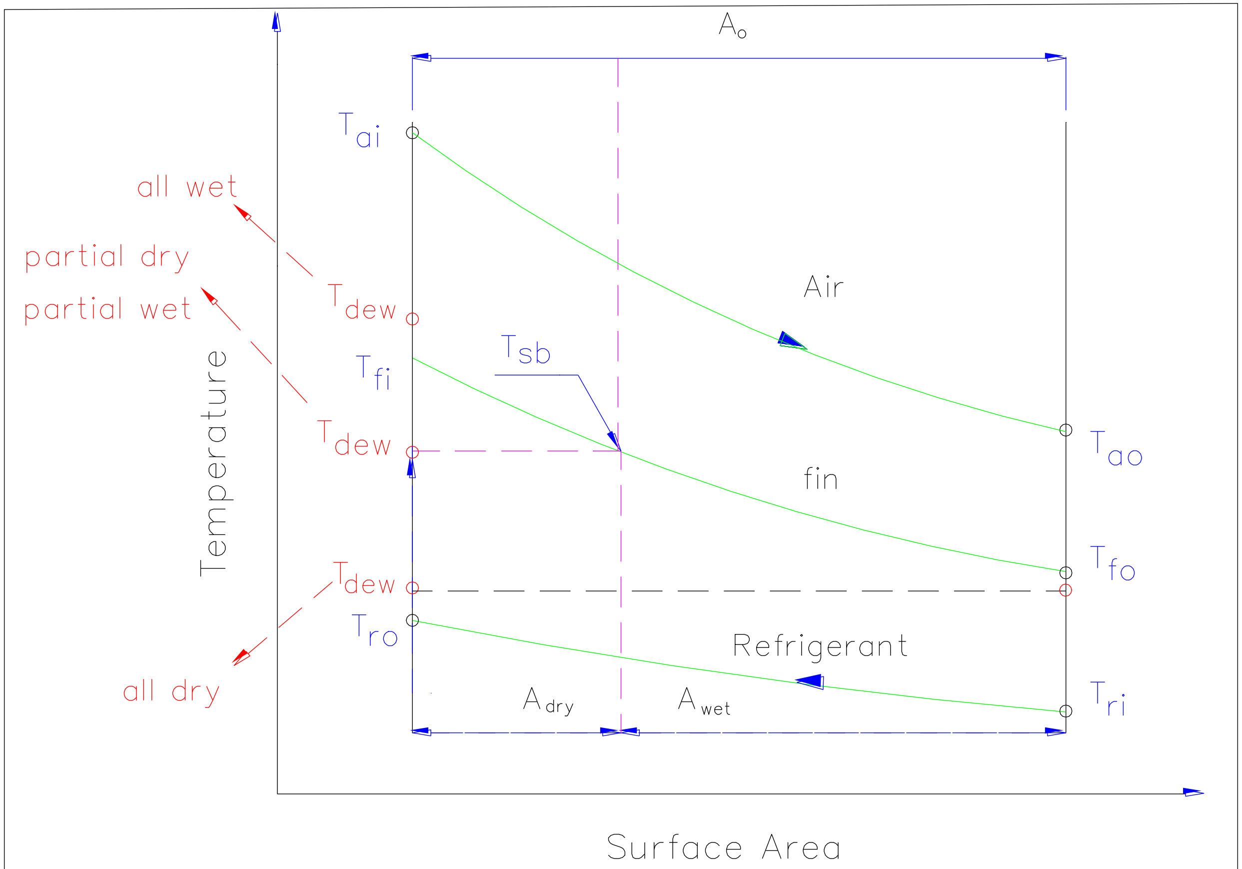


圖1熱交換器表面溫度示意圖

□ 露點溫度$T_{dew}$的定義為空氣在同一壓力及比濕條件下時，相對飽和濕空氣的乾球溫度。

# 基本介紹(Cont.)

- 當空調設備運轉時，風扇強制空氣流過蒸發器表面，除了溫度的降低外，通常會有水份凝結下來，達到冷卻和除濕的目的，此時稱盤管為濕盤管(wetted coil)。

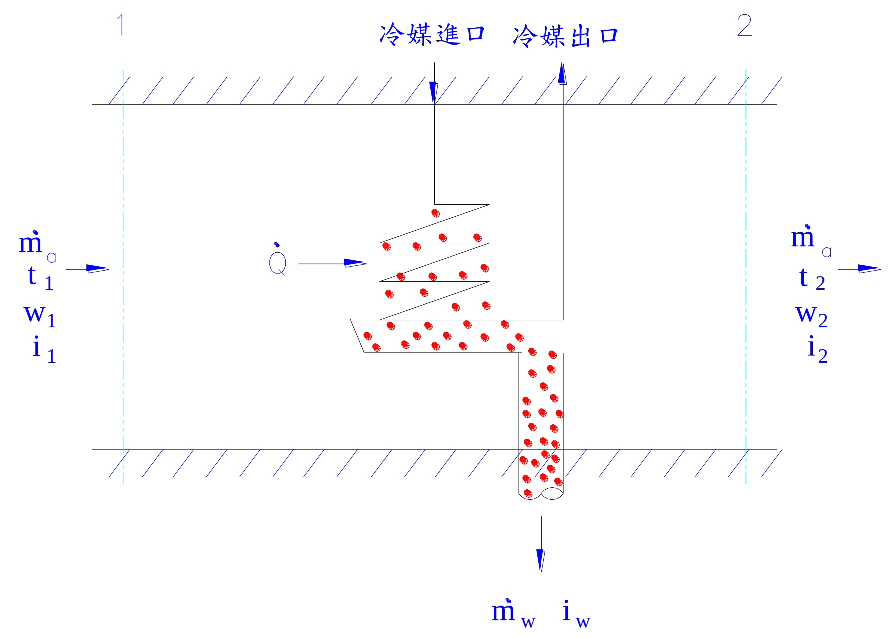


圖1 濕盤管熱力過程

Energy balance ：

$$\dot{m}_a i_1 = \dot{Q} + \dot{m}_a i_2 + \dot{m}_w i_w \qquad (1)$$

Mass balance of water：

$$\dot{m}_a \omega_1 = \dot{m}_a \omega_2 + \dot{m}_w \qquad (2)$$

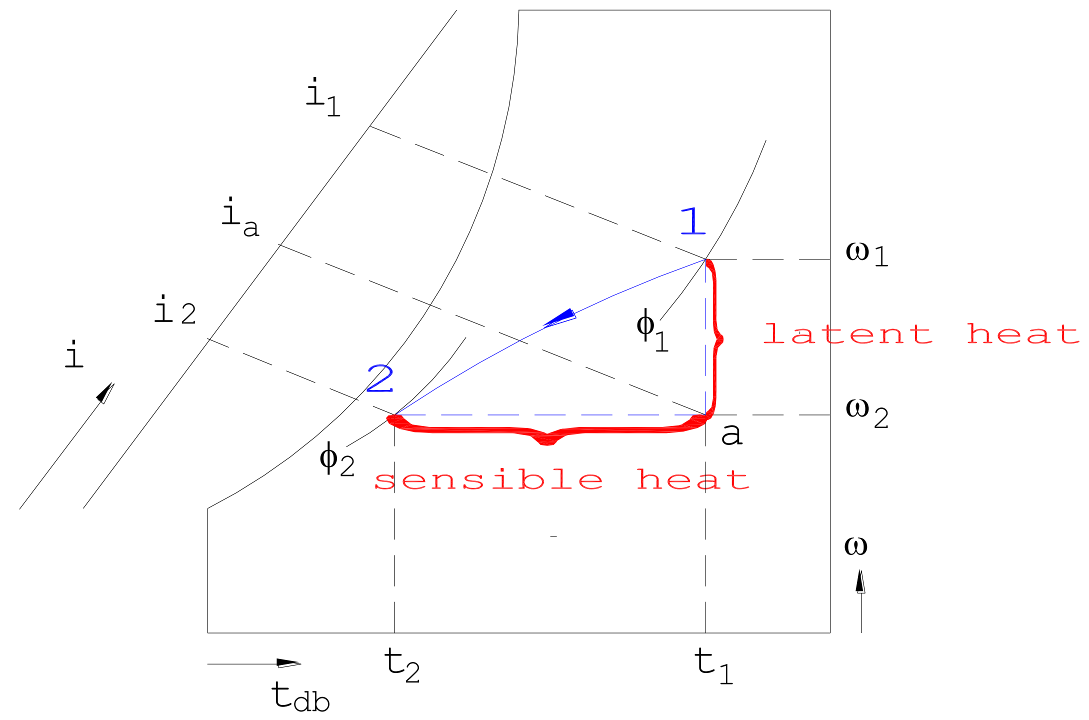
(1) & (2)式整理得：

$$\dot{Q} = \dot{m}_a (i_1 - i_2) - \cancel{\dot{m}_a (\omega_1 - \omega_2) i_w} \qquad (3)$$

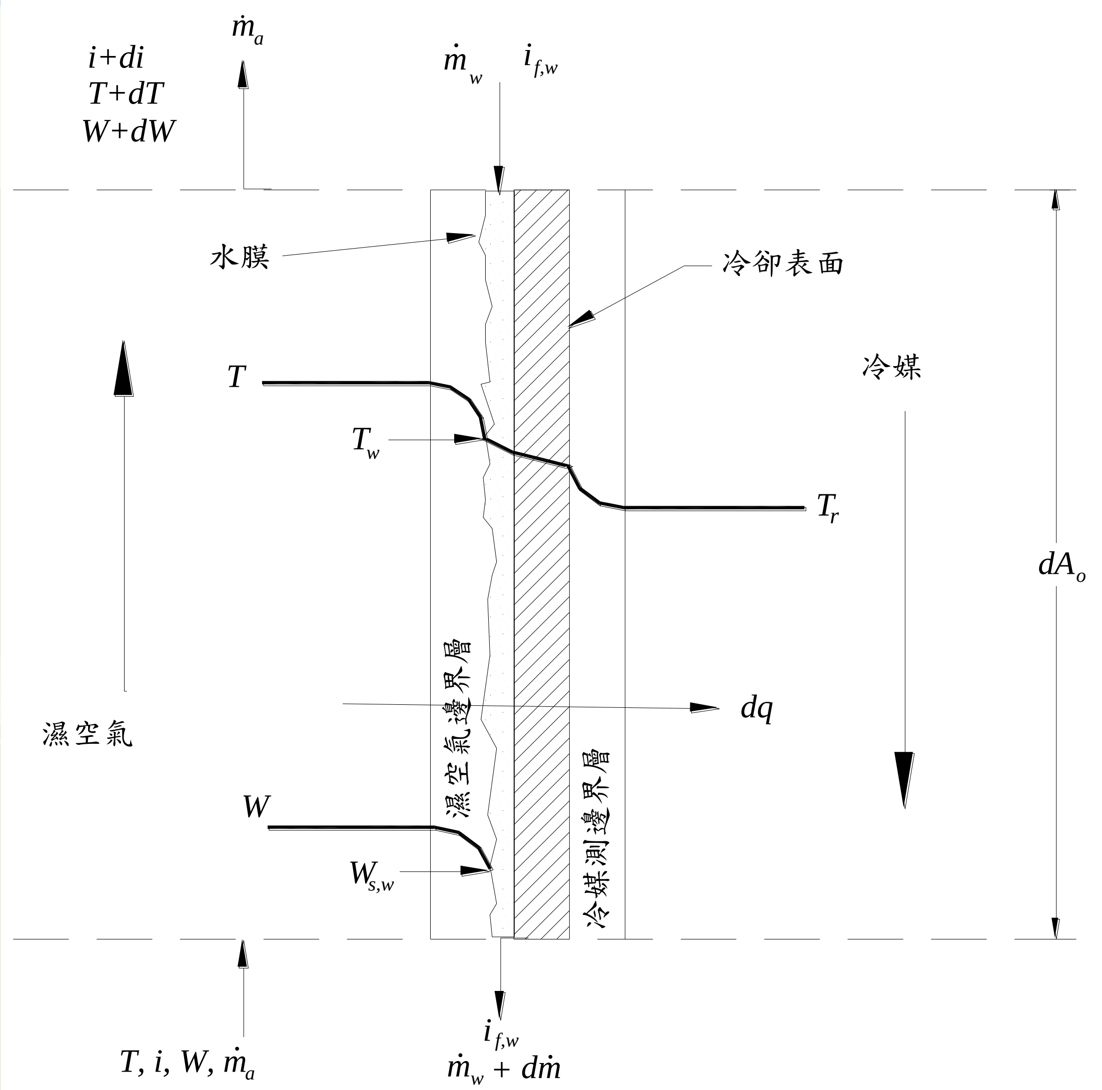
$$\Rightarrow \dot{Q} \cong \dot{m}_a (i_1 - i_2)$$

# 基本介紹(Cont.)

□濕盤管熱傳現象包括**顯熱(sensible heat)**和**潛熱(latent heat)**兩種變化，以熱傳的觀點而言，**顯熱變化係由溫度差所造成；潛熱變化則起因於相的變化(例如凝結)**。而代表溫差與相變化的驅動能力，可以用**焓勢(enthalpy potential)**來表示。



Cooling and Dehumidifying process


$\dot{m}_a$
$i+di$
$T+dT$
$W+dW$
$\dot{m}_w$
$i_{f,w}$
水膜
冷卻表面
冷媒
$T$
$T_w$
$T_r$
$dA_o$
濕空氣邊界層
$dq$
濕空氣
$W$
$W_{s,w}$
冷媒測邊界層
$T, i, W, \dot{m}_a$
$i_{f,w}$
$\dot{m}_w + d\dot{m}$

濕空氣熱傳的過程

首先我們考慮如圖6-4所示的濕空氣熱質傳過程，在這個物理模式中，我們假設濕空氣冷凝水會均勻產生一液膜覆蓋在熱交換器的表面上，圖中的$W$為比濕，$i$ 為濕空氣的焓值，$\dot{m}_a$為乾空氣的質量流率；若我們考慮一甚小的熱傳面積$dA_o$上的能量平衡，可得：

$$-\dot{m}_a di = dQ - \dot{m}_a dW \times i_{f,w} \tag{6-23}$$

其中，$i_{f,w}$為冷凝水的焓值。

由熱傳平衡的方程式可知：總熱傳量(total heat transfer) = 顯熱熱傳量(sensible heat transfer) + 潛熱熱傳量(latent heat transfer)，顯熱熱傳來自於乾球溫度差，即$dQ_s = h_{c,o}dA_o(T - T_w)$；而潛熱的驅動力來自於氣液態間的質量傳遞，質量傳遞是由濃度差所造成，在濕空氣中常用的濃度為比濕，因此$dQ_l = h_{D,o}dA_o(W - W_{s,w})(i_{g,t} - i_{f,w})$，所以：

$$dQ = h_{c,o}dA_o\left(T - T_w\right) + h_{D,o}dA_o\left(W - W_{s,w}\right)\left(i_{g,t} - i_{f,w}\right) \tag{6-24}$$

其中$h_{c,o}$代表濕空氣的顯熱熱傳係數，而$h_{D,o}$為質傳係數，故

$$-\dot{m}_a dW = h_{D,o} dA_o (W - W_{s,w}) \qquad (6\text{-}25)$$

如果我們定義一個新的參數， $Le$ (Lewis number)[1]，即

$$Le = \frac{h_{c,o}}{h_{D,o} c_{p,a}} \qquad (6\text{-}26)$$

所以，式6-24可以改寫如下：

$$dQ = \frac{h_{c,o} dA_o}{c_{p,a}} \left( c_{p,a}(T - T_w) + \frac{(W - W_{s,w})(i_{g,t} - i_{f,w})}{Le} \right) \qquad (6\text{-}27)$$

又在標準狀況下，濕空氣的焓值可以表示如下：

$$i = c_{p,a} T + W(2501 + 1.805T) \quad (\text{單位為kJ/kg}) \qquad (6\text{-}28)$$

在水膜溫度的飽和濕空氣焓值可表示如下：

$$i_W = c_{p,a} T_w + W_{s,w}(2501 + 1.805 T_w) \tag{6-29}$$

將式6-28減去式6-29可得

$$i - i_w = c_{p,a}(T - T_w) + 2501(W - W_{s,w}) \tag{6-30}$$

將式6-24中的溫差部份($T$-$T_w$)換成式6-30的焓差，可得：

$$\begin{aligned} dQ &= \frac{h_{c,o} dA_o}{c_{p,a}} \left( (i - i_w) - 2501(W - W_{s,w}) + \frac{(W - W_{s,w})(i_{g,t} - i_{f,w})}{Le} \right) \\ &= \frac{h_{c,o} dA_o}{c_{p,a}} \left( (i - i_w) + \frac{(W - W_{s,w})(i_{g,t} - i_{f,w} - 2501 \times Le)}{Le} \right) \end{aligned} \tag{6-31}$$

又由能量與質量的平衡方程式(式 6-23與式6-25)可得：

$$dQ = -\dot{m}_a di + \dot{m}_a dW \times i_{f,w} \tag{6-32}$$

$$-\dot{m}_a dW = h_{D,o} dA_o (W - W_{s,w}) \tag{6-33}$$

因此將式6-33代入式6-32，與式6-31合併後消去$dQ$，可寫成：

$$dQ = h_{D,o} dA_o (W - W_{s,w})\left(\frac{di}{dW} - i_{f,w}\right)$$

$$= \frac{h_{c,o} dA_o}{c_{p,a}} \left( (i - i_w) + \frac{\left(W - W_{s,w}\right)\left(i_{g,t} - i_{f,w} - 2501 \times Le\right)}{Le} \right) \tag{6-34}$$

將式6-34稍做處理後，我們可以得到：

$$\frac{di}{dW} = Le \frac{i - i_w}{W - W_{s,w}} + \left(i_{g,t} - 2501 \times Le\right) \tag{6-35}$$

式6-35稱之為空氣線圖上除濕過程的空氣調和線(process line or conditioning line)。接下來，我們進一步來討論式6-34，即：

$$dQ = \frac{h_{c,o} dA_o}{c_{p,a}} \left( (i - i_w) + \frac{\left(W - W_{s,w}\right)\left(i_{g,t} - i_{f,w} - 2501 \times Le\right)}{Le} \right) \tag{6-34}$$

假設一個典型的空調條件如下：

(a) 水膜溫度 $T_w = 10°\text{C}$

(b) 空氣的乾球溫度 $T = 20°\text{C}$

(c) 相對溼度 $\phi = 50\%$

在這個條件下，我們可以從空氣線圖中查出(圖6-2)

$W \approx 0.0074$ kg/kg dry air

$W_{s,w} \approx 0.0078$ kg/kg dry air

$i \approx 39.4$ kJ/kg dry air

$i_w \approx 29.4$ kJ/kg dry air

$i_{g,t} \approx 2454$ kJ/kg

$i_{f,w} \approx 42$ kJ/kg

$i - i_w \approx 10$ kJ/kg dry air

在一般應用中，*Le* (Lewis number) 大約在0.9 左右；在此，我們假設這個值很靠近1，即 $Le \approx 1.0$。

因此式6-34等號右邊的第二項可大致估算如下：

$$\frac{(W-W_{s,w})(i_{g,t}-i_{f,w}-2501\times Le)}{Le}$$

$$\approx(0.0074-0.0078)(2454-42-2501\times 1)\approx 0.04\,\mathrm{kJ/kg}$$

而式6-34等號右邊的第一項為$i-i_w$ = 10，顯然第一項要比第二項大很多，即：

$$\frac{\dfrac{(W-W_{s,w})(i_{g,t}-i_{f,w}-2501\times Le)}{Le}}{i-i_w}=\frac{0.04}{10}\approx 0$$

因此我們可以將式6-34等號右邊的第二項忽略，故式6-34可簡化成如下：

$$dQ=\frac{h_{c,o}dA_o}{c_{p,a}}\left(i-i_w\right) \tag{6-36}$$

式6-36的結果告訴讀者幾個重要訊息：(1)焓差是濕盤管的驅動力；(2)焓差驅動勢為熱傳與質傳整合而產生的濕盤管驅動力；(3)請特別留意，$h_{c,o}$為顯熱熱傳係數，但是這個熱傳係數是「濕潤表面下」的顯熱熱傳係數

熱交換過程不牽涉到質傳，則熱傳量可寫成如下：

$$Q = U \cdot A \cdot \Delta T_m = U \cdot A \cdot F \cdot LMTD \qquad (6\text{-}37)$$

而在6-2節中告訴讀者濕盤管總熱傳的驅動勢為焓差，本章節即採用焓差分析法(Threlkeld, 1970)，因此熱傳量可寫出類似式6-37的式子如下：

$$Q = U_{o,w} \cdot A \cdot \Delta i_m = U_{o,w} \cdot A \cdot F \cdot LMHD \qquad (6\text{-}38)$$

又：

$$Q = \dot{m}_a \left( i_{a,i} - i_{a,o} \right) \qquad (6\text{-}39)$$

其中

$A$：空氣側熱傳面積

$F$：修正因子 (correction factor)，修正係數的算法與乾盤管的算法完全相同，唯一的差異在於乾盤管使用四個端點溫度來算$P$與$R$，而濕盤管使用四個端點的空氣焓值來計算

$U_{o,w}$ ：濕盤管之總熱傳係數

$Q$：濕盤管的熱傳量

$\dot{m}_a$ ：乾空氣的質量流率

$LMHD$：對數平均焓差(log mean enthalpy difference)，表示如下：

$LMHD$：對數平均焓差(log mean enthalpy difference)，表示如下：

$$LMHD = \frac{(i_{a,i} - i_{r,o}) - (i_{a,o} - i_{r,i})}{\ln\left(\frac{i_{a,i} - i_{r,o}}{i_{a,o} - i_{r,i}}\right)} \qquad (6\text{-}40)$$

其中：

$i_{a,i}$, $i_{a,o}$：為空氣的進口和出口焓值

$i_{r,i}$：相對於冷媒進口溫度下的飽和空氣焓值 (saturated air enthalpy evaluated at the inlet refrigerant temperature)

$i_{r,o}$：相對於冷媒出口溫度下的飽和空氣焓值(saturated air enthalpy evaluated at the outlet refrigerant temperature)

這裡要提醒讀者，濕盤管的總熱傳係數$U_{ow}$與乾盤管的$U$是完全不同的(請注意它們的單位完全不同！)

$U_{ow}$應該如何算？

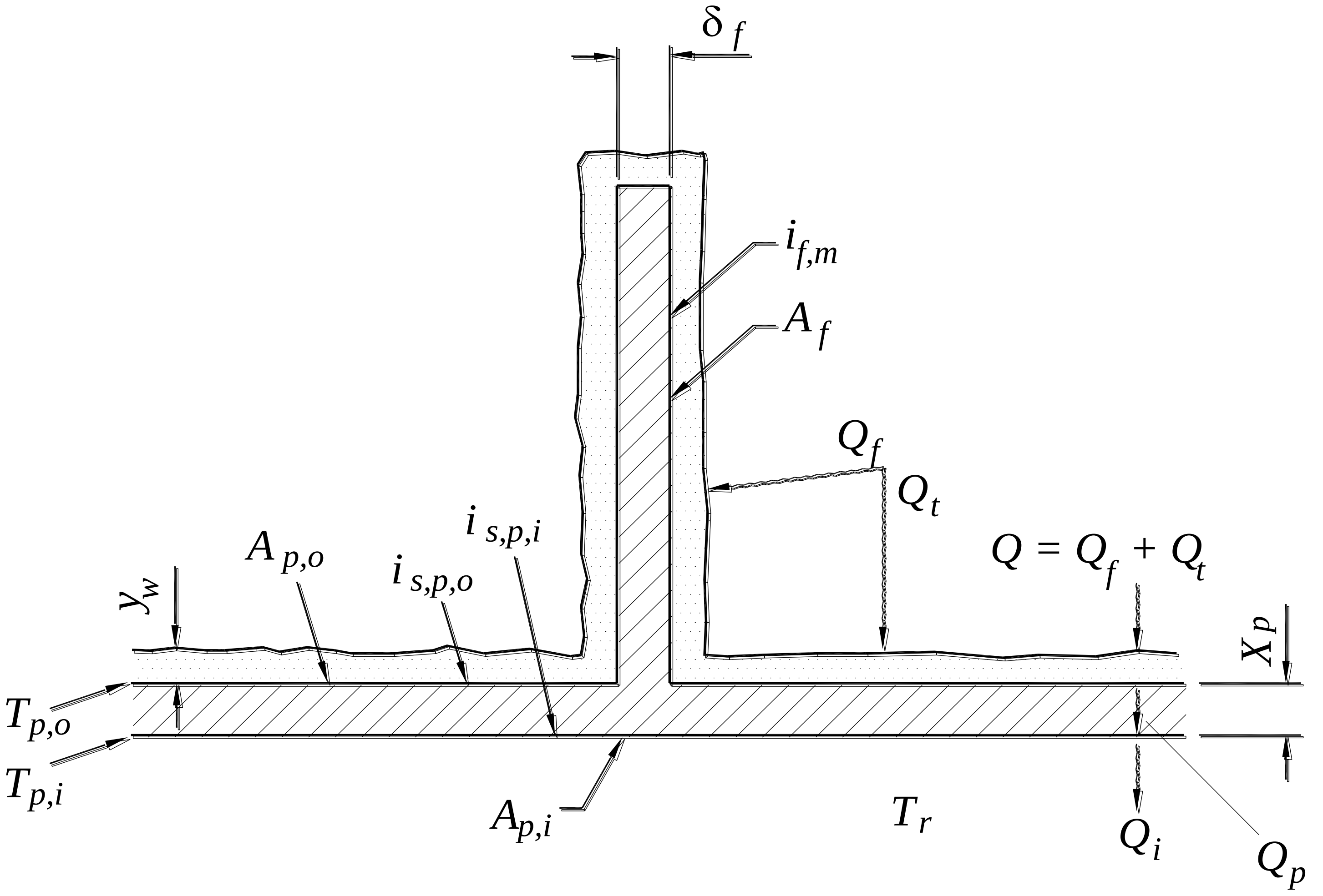


圖6-5　濕式鰭片熱傳遞途徑示意圖

# 基本介紹(Cont.)

- 濕盤管，其熱傳遞為顯熱加潛熱，**驅動勢為平均焓差**

$$\dot{Q} = (U_{ow}A_o)F(\Delta i_m)$$

若是逆向流，則 $\Delta i_m = LMHD$

$$\Delta i_m = \frac{(i_{ai} - i_{ro}) - (i_{ao} - i_{ri})}{\ln\left(\dfrac{i_{ai} - i_{ro}}{i_{ao} - i_{ri}}\right)}$$

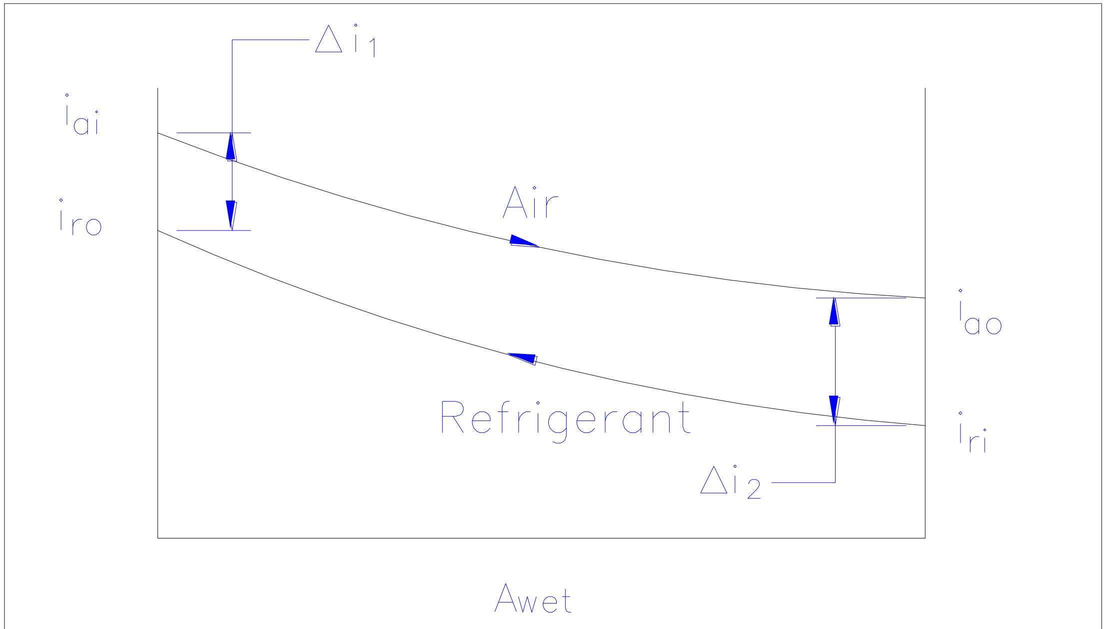


圖4 濕盤管之空氣側和管側之間enthalpy之變化(LMHD)

# 蒸發器熱傳熱傳路徑示意圖

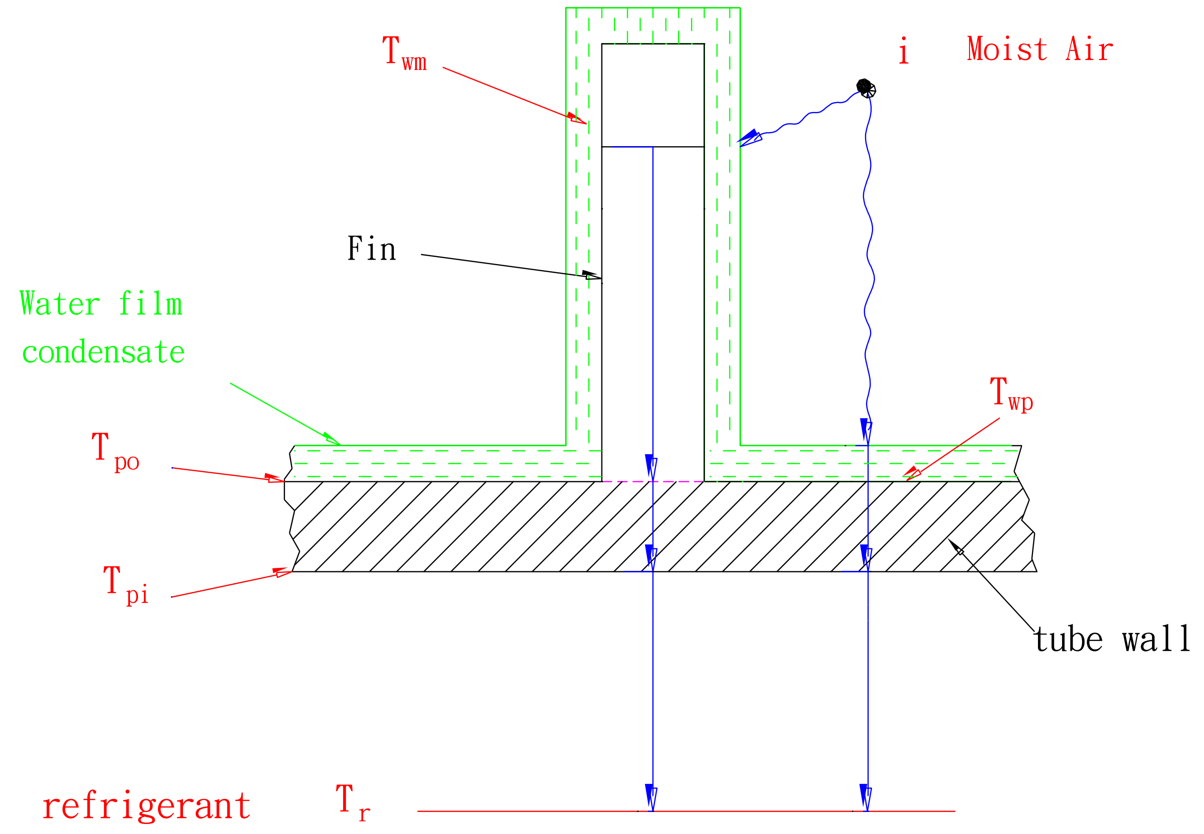


圖2冷卻除濕過程中空氣至管側的熱傳路徑示意圖

# 濕盤管熱傳分析

☞參考書籍：王啟川博士， 熱交換設計 第六章， 五南圖書公司

❑濕盤管熱傳分析模式採取 Threlkeld (1970)對濕盤管的分析方法，熱傳量計算如下：

$$\dot{Q}=\dot{m}_a(i_{ai}-i_{ao})\Rightarrow\dot{Q}=U_{ow}A_oF\Delta i_m \qquad (5)$$

$$\Delta i_m=\frac{(i_{ai}-i_{ro})-(i_{ao}-i_{ri})}{\ln\left(\dfrac{i_{ai}-i_{ro}}{i_{ao}-i_{ri}}\right)} \qquad (6)$$

$U_{ow}$為濕盤管之總熱傳係數(overall heat transfer coefficient)

$i_{ai}$ ，$i_{ao}$：為空氣的進口和出口焓值

$i_{ri}$ ：相對於冷媒進口溫度下的飽和空氣焓值(saturated air enthalpy evaluated at the inlet refrigerant temperature)

$i_{ro}$ ：相對於冷媒出口溫度下的飽和空氣焓值(saturated air enthalpy evaluated at the outlet refrigerant temperature)

# 濕盤管之總熱傳係數$U_{ow}$

$$U_{ow} = \frac{1}{\dfrac{b_{r'} A_o}{h_i A_{pi}} + \dfrac{b_{p'} X_p A_o}{k_p A_{pm}} + \dfrac{1}{h_{ow}\left(\dfrac{A_{po}}{b_{wp'} A_o} + \dfrac{A_f}{A_o}\dfrac{1}{b_{wm'}}\eta_{wet,f}\right)}} \tag{10}$$

$$h_{ow} \cong \left(\frac{C_{pa}}{b_{wm'} h_{co}}\right)^{-1} \tag{11}$$

$h_{ow}$ ：為濕式熱傳係數

$h_{c,o}$ ：為濕盤管空氣側顯熱傳係數

$$\eta_{wet,f} = \frac{2r_c}{M_w(r_{eq}^2 - r_c^2)} \times \left[\frac{K_1(M_w r_c)I_1(M_w r_{eq}) - K_1(M_w r_{eq})I_1(M_w r_c)}{K_1(M_w r_{eq})I_0(M_w r_c) + K_0(M_w r_c)I_1(M_w r_{eq})}\right] \tag{12}$$

# 濕盤管鰭片效率 $\eta_{wet,f}$

- 矩形鰭片的等效半徑：

$$r_{eq} = \sqrt{\frac{p_t \times p_l}{\pi}}$$

$$M_w = \sqrt{\frac{2\,h_{ow}}{k_f \delta}}$$

$$= \sqrt{\frac{2\,h_{co}}{k_f \delta}} \times \sqrt{\frac{b_{wm'}}{Cpa}}$$

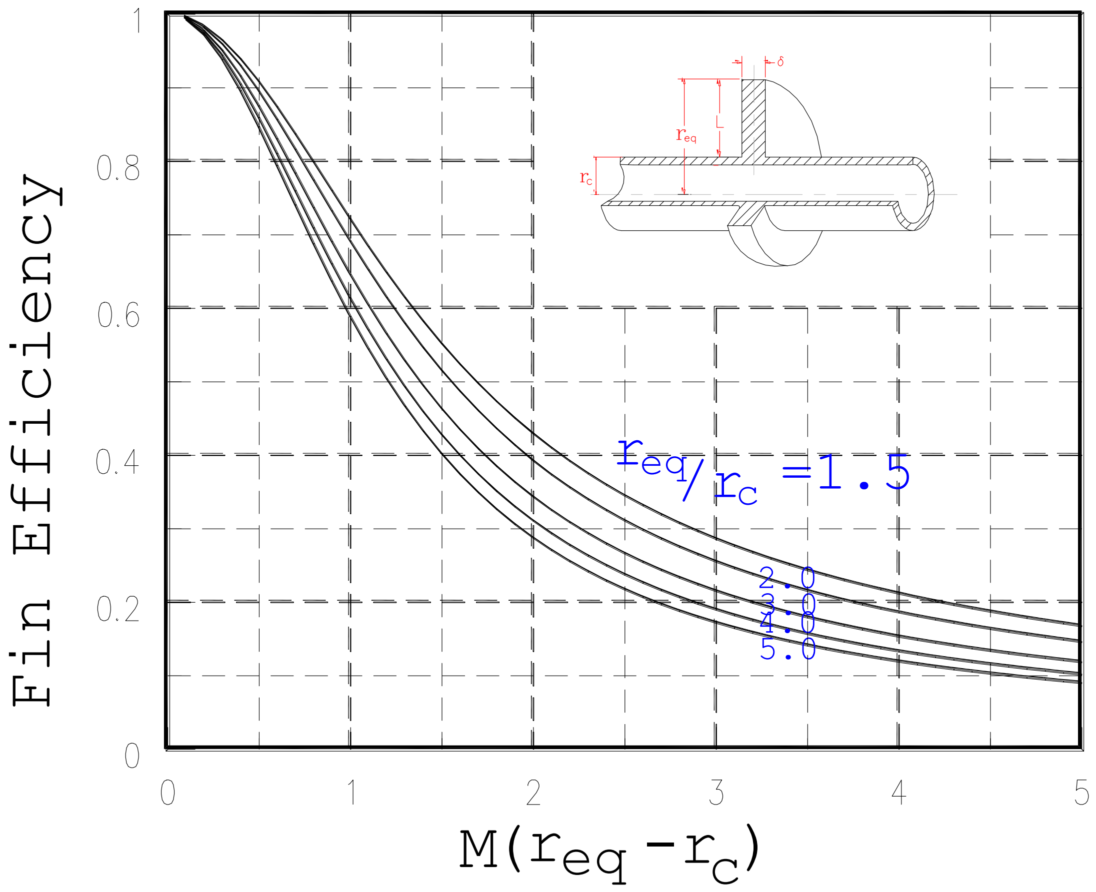


圖5 圓形鰭片的鰭片效率

# 濕盤管熱傳分析(Cont.)

□假設一小範圍的溫度下，其空氣線圖上的焓值飽和曲線可視為一直線，因此，濕空氣的飽和焓值可表示成：

$$i_s = a + b \cdot t_s \Rightarrow b = \frac{\Delta i_s}{\Delta t_s} \qquad (13)$$

$i_s$：濕空氣的飽和焓值
$t_s$：飽和空氣溫度
$b$：飽和曲線斜率

□由於濕盤管熱傳驅動勢為焓差，$b = \Delta i_s / \Delta t_s$ 的定義，可以將常用的溫差轉換成斜率$b$ 與焓間的關係，如此一來，就可將各個不同的阻抗通通換成焓差驅動勢，才能將各部分不同的阻抗加起來。

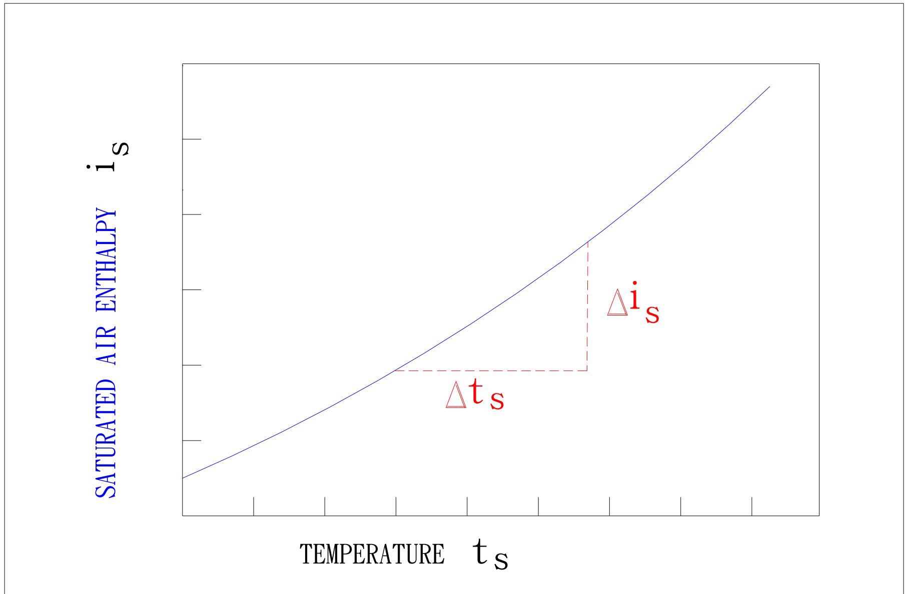


圖1飽和空氣焓值與溫度之變化關係

# 濕盤管熱傳分析(Cont.)

- □ $b_{r'}, b_{p'}, b_{wp'}, b_{wm'}$ 之值：
  - ● $b_{r'} = (i_{spi} - i_{rm})/(T_{pi} - T_{rm})$　　$b_{p'} = (i_{spo} - i_{spi})/(T_{po} - T_{pi})$
  - ● $b_{wp'}$ 相對於鰭片基部水膜溫度的飽和空氣焓值的斜率，即 $\frac{di}{dT_{po}}$（假設鰭片基部水膜溫度$T_{wp}$約等於外管壁溫度$T_{po}$）
  - ● $b_{wm'}$ 相對於鰭片基部水膜溫度的飽和空氣焓值的斜率，即 $\frac{di}{dT_{wm}}$
    - − 以trial-and--error方法及疊代的方式，先假 一個鰭片上的水膜平均溫度$T_{wm}$ ，然後疊代出最後答案 。
- □決定空氣側的出口狀態、除了出口焓值後，尚須求出另外一項的值，例如乾球溫度或者比濕度(humidity ratio)，決定空氣出口狀態。
  - ●Threlkeld (1970) 提出的除濕狀態曲線方程式(process line equation)，如圖7所示，用以決定空氣出口的比濕度：

$$\frac{di}{d\omega} = Le \times \left( \frac{i - i_{swm}}{\omega - \omega_{swm}} \right) + \left( i_{g,t} - 2500.9Le \right) \qquad (14)$$

# 圖.空氣飽和溫度($T_s$)對空氣焓值($i_s$)及斜率($di_s/dT_s$)的關係

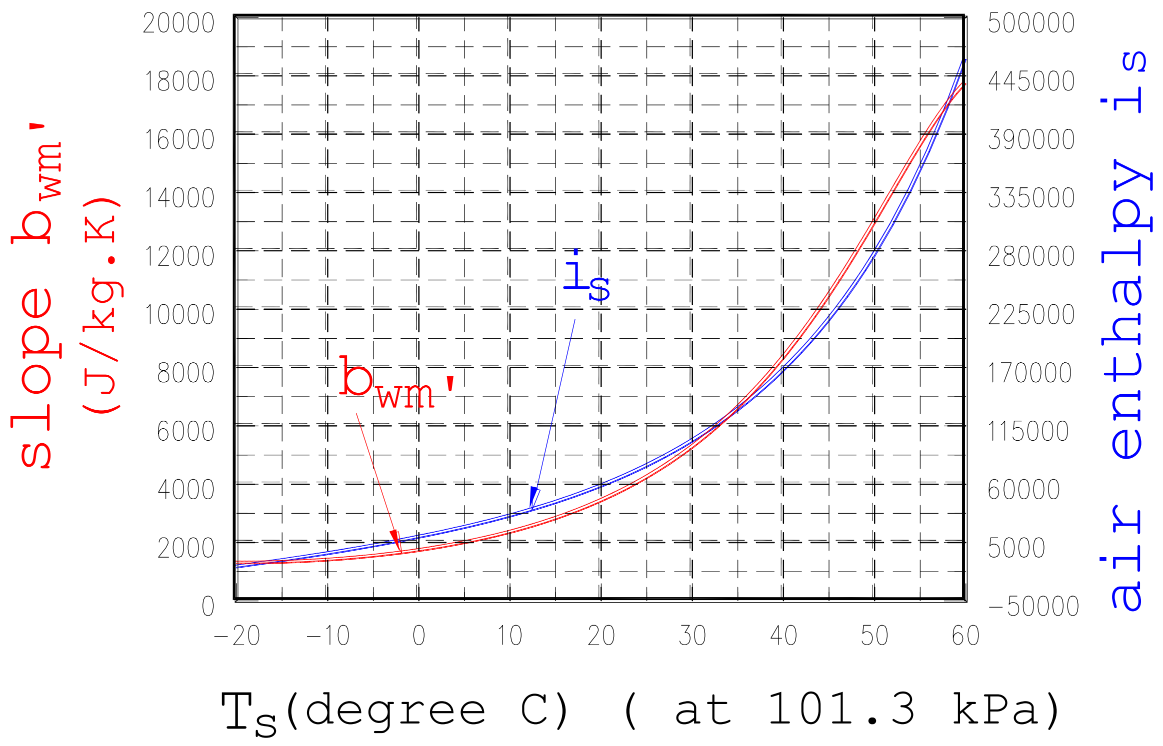
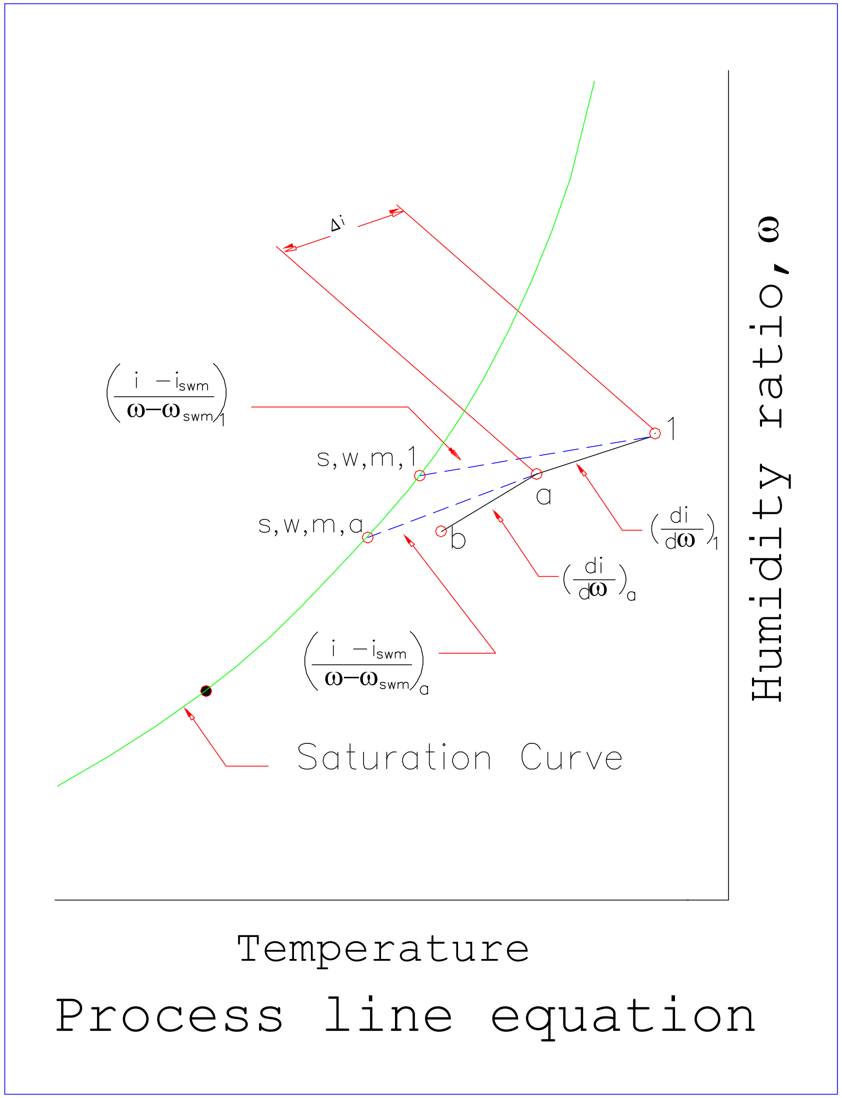
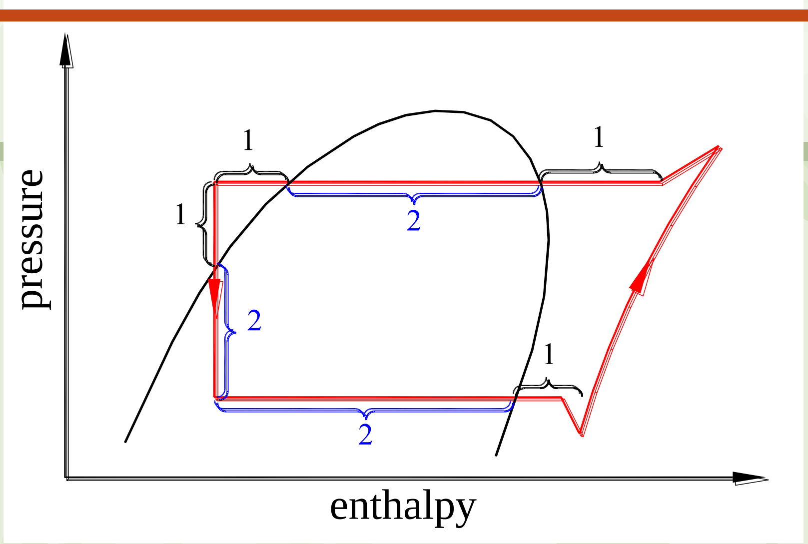


圖 6 slope of $d(i_s)/d(T_s)(=b_{wm}')$
and air saturated enthalpy

# 圖7 除濕狀態曲線方程式(process line equation)示意圖



# 濕盤管熱傳分析(Cont.)



## □冷媒側壓降

- 單相流體造成的摩擦阻力：

$$\Delta P_{f1} = \frac{2f\cdot G^{2}\cdot L_{f1}}{\rho\cdot D_{i}} \qquad (15)$$

- 兩相流區域的壓降：

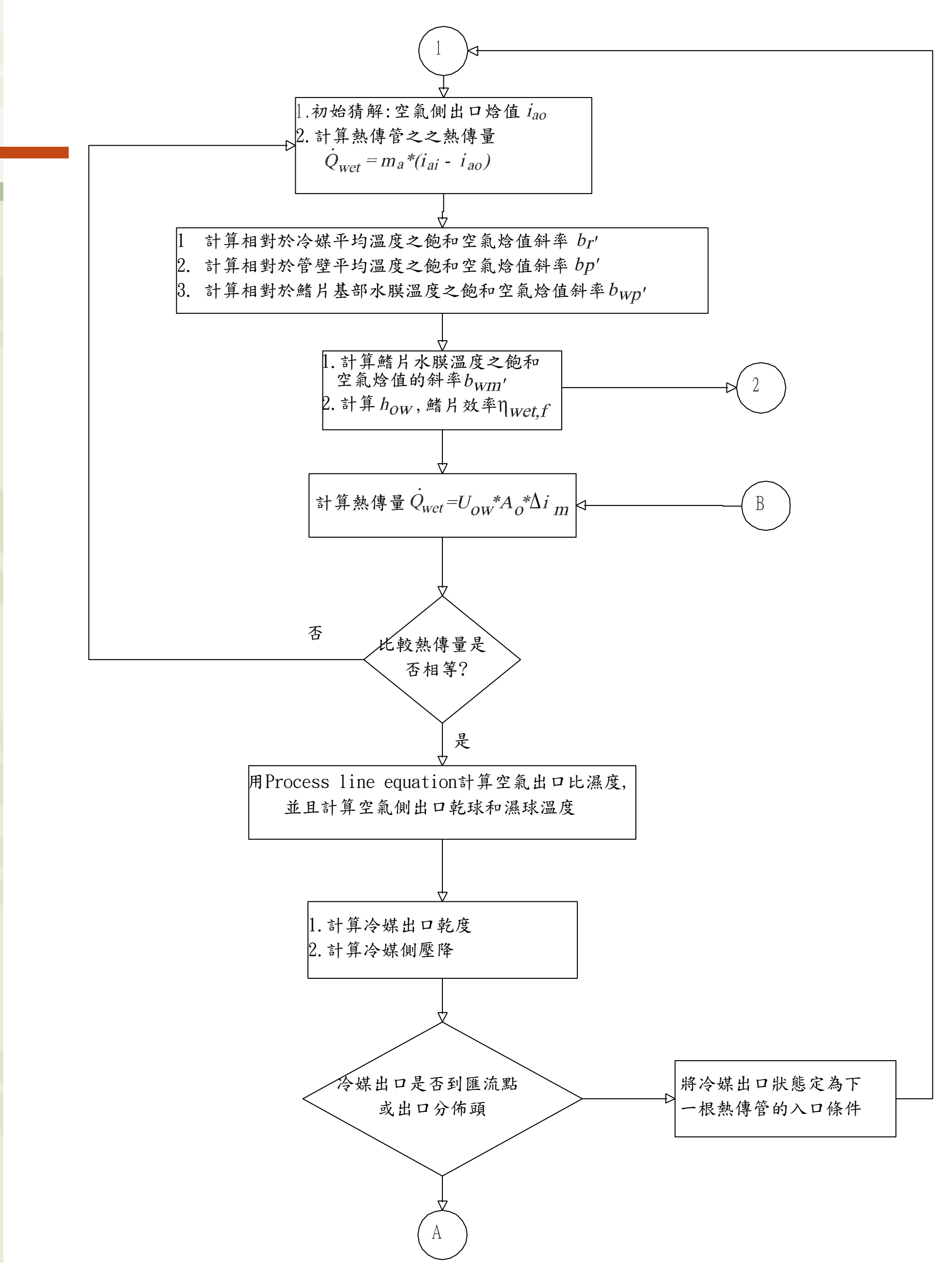
$$-\left(\frac{dp}{dz}\right) = \underbrace{-\left(\frac{dp}{dz}\right)_{fr}}_{\text{摩擦項}} + \underbrace{G^{2}\frac{d}{dz}\left[\frac{x^{2}}{\rho_{g}\alpha}+\frac{(1-x)^{2}}{\rho_{f}(1-\alpha)}\right]}_{\text{動量變化項}} + \underbrace{g\ \sin\theta\left[\alpha\rho_{g}+(1-\alpha)\rho_{f}\right]}_{\text{重力項}} \qquad (16)$$

- 冷媒側壓降可表示成：

$$\Delta P_{r} = \Delta P_{a} + \Delta P_{f} + \Delta P_{g} + \Delta P_{b} \qquad (17)$$

其中$\Delta P_a$、$\Delta P_f$、$\Delta P_g$ 、$\Delta P_b$分別代表動量變化、摩擦項、重力項及彎頭之壓力損失。

# 濕盤管熱傳量計算步驟


1
1.初始猜解:空氣側出口焓值 $i_{ao}$
2. 計算熱傳管之之熱傳量
$\dot{Q}_{wet} = m_a*(i_{ai} - i_{ao})$
1 計算相對於冷媒平均溫度之飽和空氣焓值斜率 $b_{r'}$
2. 計算相對於管壁平均溫度之飽和空氣焓值斜率 $bp'$
3. 計算相對於鰭片基部水膜溫度之飽和空氣焓值斜率 $b_{wp'}$
1. 計算鰭片水膜溫度之飽和空氣焓值的斜率 $b_{wm'}$
2. 計算 $h_{ow}$, 鰭片效率 $\eta_{wet,f}$
2
計算熱傳量 $\dot{Q}_{wet} = U_{ow}*A_o*\Delta i_m$
B
否
比較熱傳量是否相等?
是
用Process line equation計算空氣出口比濕度,並且計算空氣側出口乾球和濕球溫度
1. 計算冷媒出口乾度
2. 計算冷媒側壓降
冷媒出口是否到匯流點或出口分佈頭
將冷媒出口狀態定為下一根熱傳管的入口條件
A

蒸發器安排如冷凝器為交錯逆相流的形式，在設計上，蒸發器一共分成兩段：兩相蒸發區和單相氣體加熱區。 計算流程與先前說明的冷凝器同。

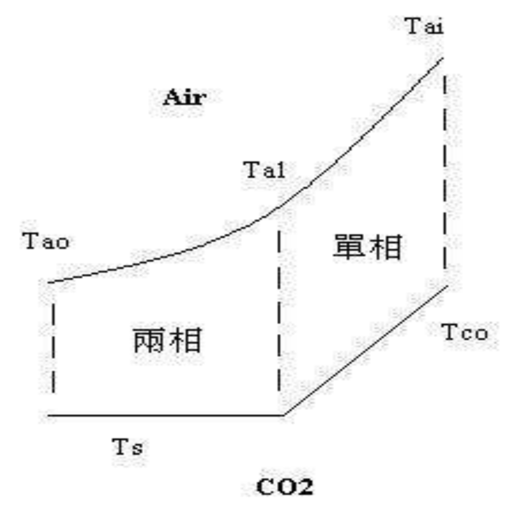
$$Q_{1c} = m_c \times i_{fg} \times (1-x) = m_a \times Cp_a \times (T_{a1} - T_{ao})$$

$$Q_{1c} = (UA)_1 \times (LMTD)_1 \times F$$

$$Q_{2c} = m_c \times Cp_c \times (T_{co} - T_s) = m_a \times Cp_a \times (T_{in} - T_{a1})$$

$$Q_{2c} = (UA)_2 \times (LMTD)_2 \times F$$

$$A_{total} = A_1 + A_2$$



冷媒側的熱傳係數計算方法，可使用任何的兩相沸騰經驗式：

單相區，冷媒側的熱傳係數則採用Gnielinski Correlation (1976) 經驗式：

# 兩相熱傳之經驗方程式-管內流動沸騰

- 沸騰模式(nucleate boiling)與強制對流蒸發模式(forced convective evaporation)。所謂蒸發，簡單的說明，乃是沒有氣泡現象的『沸騰』，蒸發發生在氣液的交界面上，而沸騰則發生於熱交換器的表面上，可想而知，當管內流速較慢時，兩相流動的主要流譜為氣泡流、波浪流或間歇流，因此主要的熱傳機制為沸騰模式，而當速度較快時，流譜為環狀流，此時的主要熱傳機制變為強制對流蒸發模式

# 合成法(superposition model)

$$q = q_{NB} + q_{CV}$$

$$q = h(T_w - T_s)$$

$$q_{NB} = h_{NB}(T_w - T_s)$$

$$q_{CV} = h_{CV}(T_w - T_s)$$

$$h = h_{NB} + h_{CV}$$

Chen (1966)認為這兩種熱傳機制會隨著流動型態的改變後，比重會有所改變，Chen認為在管內沸騰情況下，$h_{NB}$會被壓抑，而$h_{CV}$會適度的被加強，$h_{CV}$可由單相部份的熱傳係數，$h_L$，乘上一個加強係數

$$h = S \times h_{NB} + E \times h_L$$

上式中的*S*代表沸騰被壓抑的係數(suppression)，而*E*代表蒸發加強係數(enhancement)；上式為相當有名的Chen’s model

$$E = 2.35/(1/X_{tt} + 0.213)^{0.736} \tag{4-17}$$

$$S = 1/(1+2.53\times10^{-6}\times \mathrm{Re}^{1.17}) \tag{4-18}$$

其中

$$\mathrm{Re} = \mathrm{Re}_L E^{1.25} \tag{4-19}$$

有很多的研究都使用Chen的方法來歸納實驗數據以獲得$E$與$S$的方程式，例如Gungor amd Winterton (1986)歸納3700組的測試資料得到如下的結果：

$$E = 1+24000\, Bo^{1.16} + 1.23 X_{tt}^{-0.86} \tag{4-20}$$

$$S = (1+0.00000115 E^2 \mathrm{Re}_L^{1.17})^{-1} \tag{4-21}$$

其中

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.875}\left(\frac{\mu_L}{\mu_G}\right)^{0.125}\left(\frac{\rho_G}{\rho_L}\right)^{0.5} \tag{4-22}$$

$$Bo = \frac{q}{G i_{fg}} \quad \text{(boiling number)} \tag{4-23}$$

試以Chen’s correlation 計算下列條件下的兩相熱傳沸騰係數：水平擺置之平滑管，R –22，$q$ = 10 kW/m$^2$·K， $T_s$ = 5 ºC， $x$ = 0.5，$G$ = 200 kg/m$^2$·s，$d_i$ = 13 mm，$k_L$ = 94 mW/(m·K)，$\mu_L$ = 199 μPa·s，$\mu_G$ = 12 μPa·s ，$\rho_L$ = 1265 kg/m$^3$，$\rho_G$ = 25 kg/m$^3$，Pr$_L$ = 2.51，$i_{fg}$ = 200 kJ/kg·K，$P$ = 583 kPa，$P_{crit}$ = 4990 kPa (臨界壓力)，$P_r$ = 583/4990 = 0.1168 (reduced pressure)

$h = S\,h_{NB} + E\,h_L$

液體部份的雷諾數(記得要用標準SI單位！)：

$\mathrm{Re}_L = Gd_i(1 - x)/\mu_L = 200\times0.013\times(1 - 0.5)/0.000199 = 6533$

$h_L$的計算可以簡單的Dittus-Boelter方程式來計算　，見第一章表1-8，即

$Nu = 0.023\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}$

$\therefore\ h_L = 0.094/0.013\times0.023\times6533^{0.8}\times2.51^{0.4} = 251.6\ \mathrm{W/m^2\cdot K}$

$h_{NB}$ 可由Cooper方程式(式4-6)來算(Cooper的計算式中，管外沸騰取常數90而管內則與平板同，取55)：

$$h_o = 55q^{0.67}M^{-0.5}P_r^{\,m}(-\log_{10}P_r)^{-0.55}$$

$h_o = 55\times10000^{0.67}\times86.47^{-0.5}\times0.117^{0.12}\times0.932^{-0.55} = 2275\ \mathrm{W/m^2}$

上式的計算可參考例 4-2-1

接下來算$S$與$E$

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.875}\left(\frac{\mu_L}{\mu_G}\right)^{0.125}\left(\frac{\rho_G}{\rho_L}\right)^{0.5} = \left(\frac{1-0.5}{0.5}\right)^{0.875}\left(\frac{0.000199}{0.000012}\right)^{0.125}\left(\frac{1265}{25}\right)^{0.5} = 10.1$$

$E = 2.35/(1/X_{tt} + 0.213)^{0.736} = 0.696$

$\mathrm{Re} = \mathrm{Re}_L E^{1.25} = 6533\times0.696^{1.25} = 4153.5$

$S = 1/(1+2.53\times10^{-6}\times\mathrm{Re}^{1.17}) = 0.958$

$\therefore h = Sh_{NB} + Eh_L = 0.958\times2275 + 0.696\times251.6 = 2355.7\ \mathrm{W/m^2\cdot K}$

# 加強模式法 (enhanced model)

$$\Psi = \frac{h}{h_L} = f_{cn}(\text{一些特定參數})$$

# 漸進模式法(asymptotic model)

漸進模式法的觀念說明如下，由於熱傳機制為$q_{NB}$與 $q_{CV}$的加成，在合成法中，$q = q_{NB} + q_{CV}$，而在漸進模式法中認為兩者並非單純的線性加成，所以

$$q^n = q_{NB}^n + q_{CV}^n \tag{4-40}$$

$$\therefore h^n = h_{NB}^n + h_{CV}^n \tag{4-41}$$

- 由前面的步驟可得到空氣側的出口焓值。然而決定空氣側的出口狀態，除了空氣的出口焓值外，尚需求出另外一個參數，例如乾球溫度、相對濕度或比濕度，以決定空氣出口狀態。
- Threlkeld(1970)提出的除濕狀態線方程式，可用以決定空氣出口的比濕度：

$$\frac{di}{dW} = Le\times(\frac{i - i_{s,w,m}}{W - W_{s,w,m}}) + (i_{g,t} - 2501Le)$$

- 其中 $i$ 為空氣焓值，$i_w$ 為相對於水膜平均溫度的飽和空氣焓值，而 $W$ 為空氣比濕，$W_{s,w}$ 為相對於水膜平均溫度下的飽和空氣的比濕度，$i_{g,t}$ 為水蒸氣的焓值，$Le$ 為Lewis number。

式中的計算，是將空氣出口焓值和進口焓值之差，分割成n個有限片段，並且求出每一段落的空氣溫度、比濕度、水膜溫度和冷媒溫度，如此由第一段重複至最後一段，即可求出空氣側出口溫度和比濕度，簡單來說，就是解process line equation的一階起始值常微分方程式(first order ODE) 。

1. 將空氣的進口與出口之間，分割成n段

$$\Delta i = \frac{i_{a,o} - i_{a,i}}{n}$$

2. 第1段的空氣進口狀態

第一段區域中，相對於水膜平均溫度Tw,m下的飽和空氣焓值 Is,w,m

$$i_{s,w,m} = i - \eta_{wet,f}[1 - U_{o,w}A_o(\frac{b'_r}{h_i A_{p,i}} + \frac{X_p b'_p}{k_p A_{p,m}})](i - i_{r,m})$$

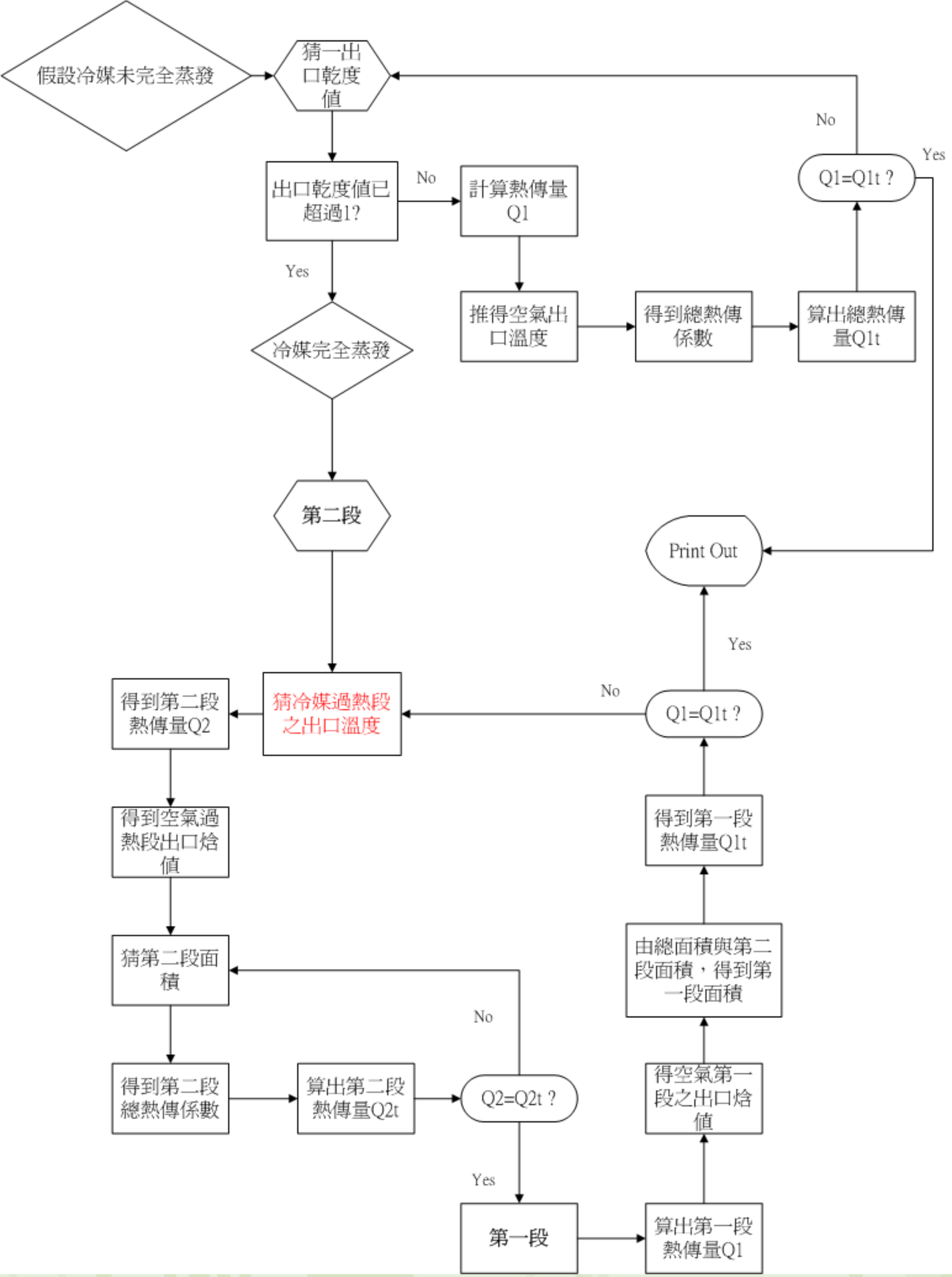
3. 計算第2段的空氣側狀態

由除濕曲線方程式求出第2段的空氣進口比濕度W2

$$W_2 = W_1 + \frac{\Delta i}{(Le \times \frac{i - i_{s,w,m}}{W - W_{s,w,m}}) + (i_{g,t} - 2501000 \times Le)}$$

$$\because i_2 = i_1 + \Delta i \qquad i_2 = C_{p,a} \times T_{a,2} + W_2 \times (2501 + 1.805 \times T_{a,2}) \quad \Rightarrow \quad T_{a,2}$$

4. 重複步驟 2→ 3 ，計算每一段空氣進出口的焓值i和比濕度W，及相對於水膜平均溫度下的飽和空氣焓值Is,w,m和飽和比濕度Ws,w,m，進而求出每一段的空氣出口溫度和比濕度及冷媒溫度，即可得知空氣最終之出口狀態



假設冷媒未完全蒸發

猜一出口乾度值

出口乾度值已超過1?

No

計算熱傳量Q1

推得空氣出口溫度

得到總熱傳係數

算出總熱傳量Q1t

Q1=Q1t ?

No

Yes

Yes

冷媒完全蒸發

第二段

猜冷媒過熱段之出口溫度

得到第二段熱傳量Q2

得到空氣過熱段出口焓值

猜第二段面積

得到第二段總熱傳係數

算出第二段熱傳量Q2t

Q2=Q2t ?

No

Yes

第一段

算出第一段熱傳量Q1

得空氣第一段之出口焓值

由總面積與第二段面積，得到第一段面積

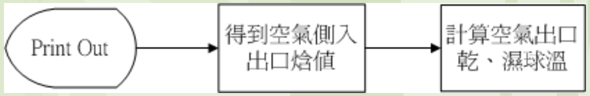
得到第一段熱傳量Q1t

Q1=Q1t ?

No

Yes

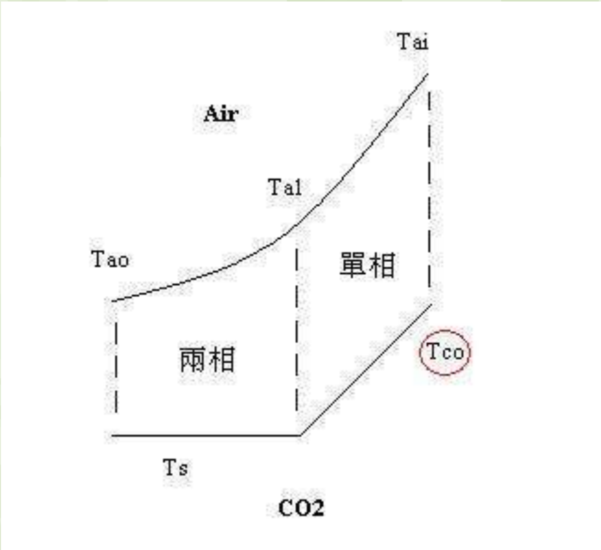
Print Out



Print Out

得到空氣側入出口焓值

計算空氣出口乾、濕球溫



# 蒸發器濕盤管熱傳分析實例演算

- 請參考 熱交換設計一書 第六章

# Thank you for not snoring

# 考題

1. 何者不在空氣空氣線圖 (1) 水蒸氣焓 (2) 濕空氣比濕 (3) 濕球溫度 (4) 露點溫度 (5) 以上皆非　。
2. 通常壓縮過程可假設為等焓過程
3. 毛細管膨脹過程可假設為等焓過程
4. 冷凝器與蒸發器內對純冷媒而言為一近乎等溫過程
5. 提升空調COP的直接作法 (1) 降高壓與低壓 (2) 昇高壓與低壓 (3) 降高壓與昇低壓 (4) 昇高壓與降低壓
6. 當高壓與低壓的變動對系統的影響，以高壓的影響比較明顯
7. 蒸發器中，空氣側的熱傳含顯熱與潛熱，通常潛熱熱傳大於顯熱的熱傳
8. 如果固定高壓，當低壓下降(未達臨界流量)，通過毛細管的流量會逐漸增大
9. 同上題，如果壓力持續下降到限流條件，則冷凍系統的流量將保持固定
10. 典型冷凝器設計必須分三區(過熱、飽和與過冷)而蒸發器分兩區(過熱與飽和)