熱交換器熱流設計

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課程大綱：

- 1. 基本流力、熱傳 (1 hr)
- 2. 熱交換器型式
- 3. UA-LMTD-F設計法 (1.5 hr)
- 4. ε-NTU 設計法 (1.5 hr)
- 5. 案例設計分析 (1 hr)
- 6. 學員實際演練 (1 hr)

課前說明

- 感謝工研院廖建順提供部分教材
- 主要教材來源：『熱交換設計』，王啟川，五南出版社，2007。
- 隨時提問
  - "There are no foolish questions and no man becomes a fool until he has stopped asking questions"
    - Charles P. Steinmetz quotes (Prussian Engineer and Inventor, 1865-1923)

基本熱流計算的目的：教導讀者正確的使用這熱流方程式觀念，一些較為深入的流力熱傳理論將不會在這個章節中敘述。

- 壓降的計算
- 基本熱傳
- 熱傳係數與總熱傳係數

壓降的計算

- 推動流體的元件 – 流體機械 (風扇、壓縮機、泵) 需要外加的動力能源
  - 風扇- 提供較小的靜壓提升 – (氣體)
  - 壓縮機 - 提供較大的靜壓提升 – (氣體)
  - 泵 - 提供液體的靜壓提升

流體機械(以泵為例)的消耗功率如下：

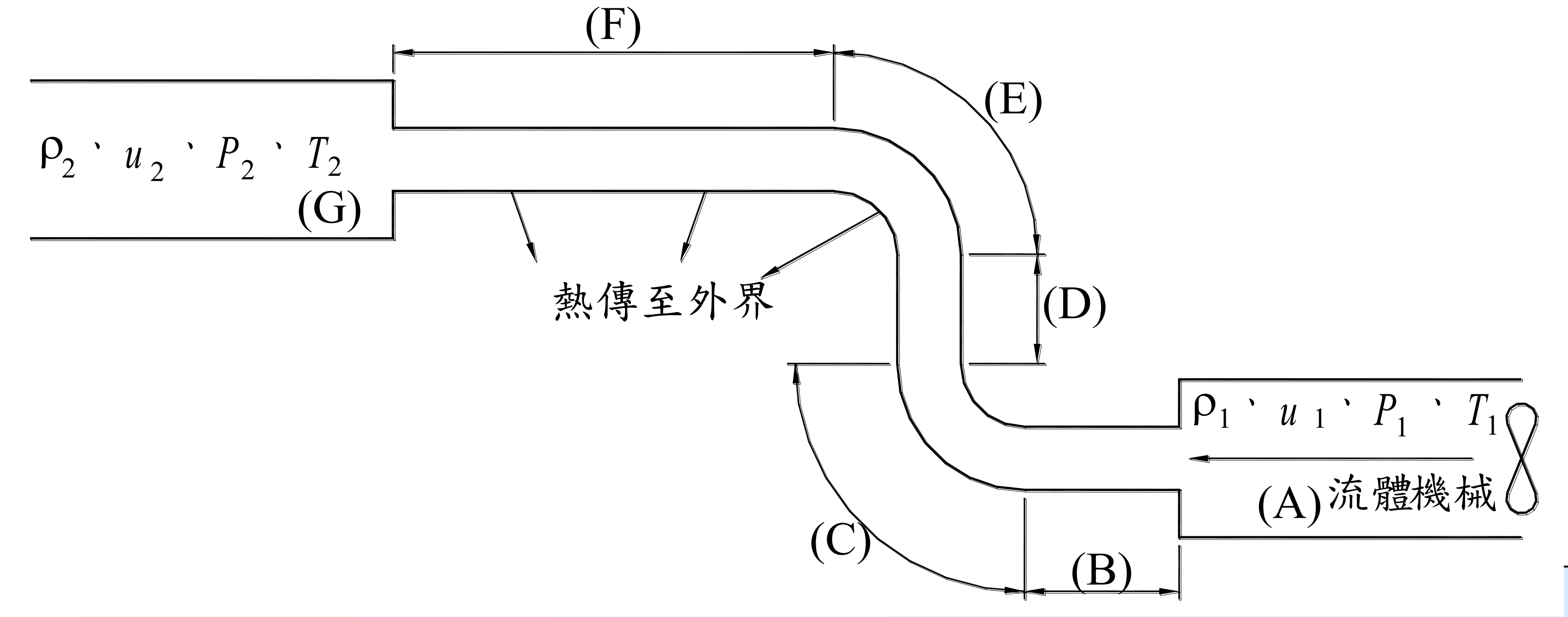
$$W_p = \frac{1}{\eta_p}\frac{\dot{m}}{\rho}\Delta P$$

因此壓降的估算資料與流體機械的選擇息息相關

壓力變化與管路高度、長度與管徑的關係示意圖

- 由圖可知需要直管、突擴、驟縮 所造成壓降的計算，以本例而言：

$$\Delta P_{\text{總壓降}} = \Sigma \Delta P = \Delta P_A + \Delta P_B + \Delta P_C + \Delta P_D + \Delta P_E + \Delta P_F + \Delta P_G$$



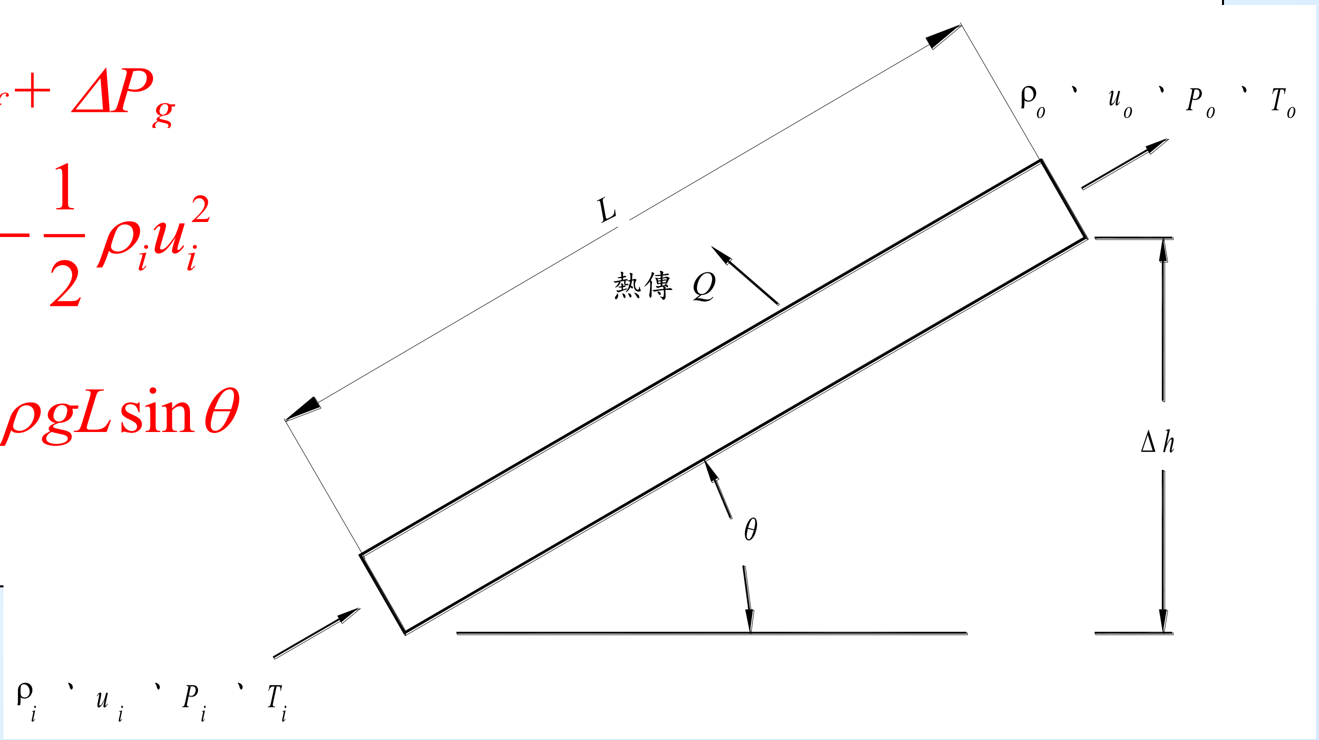
直管壓降的估算

- 其中ΔP_a為速度變化造成的壓降，ΔP_f為工作流體於管內的摩擦壓降，而ΔP_g為工作流體因高度變化所造成的壓降

$$\Delta P = \Delta P_a + \Delta P_f + \Delta P_g$$

$$\therefore \Delta P_a = \frac{1}{2}\rho_o u_o^2 - \frac{1}{2}\rho_i u_i^2$$

$$\therefore \Delta P_g = \rho g \Delta h = \rho g L \sin\theta$$



摩擦壓降的計算

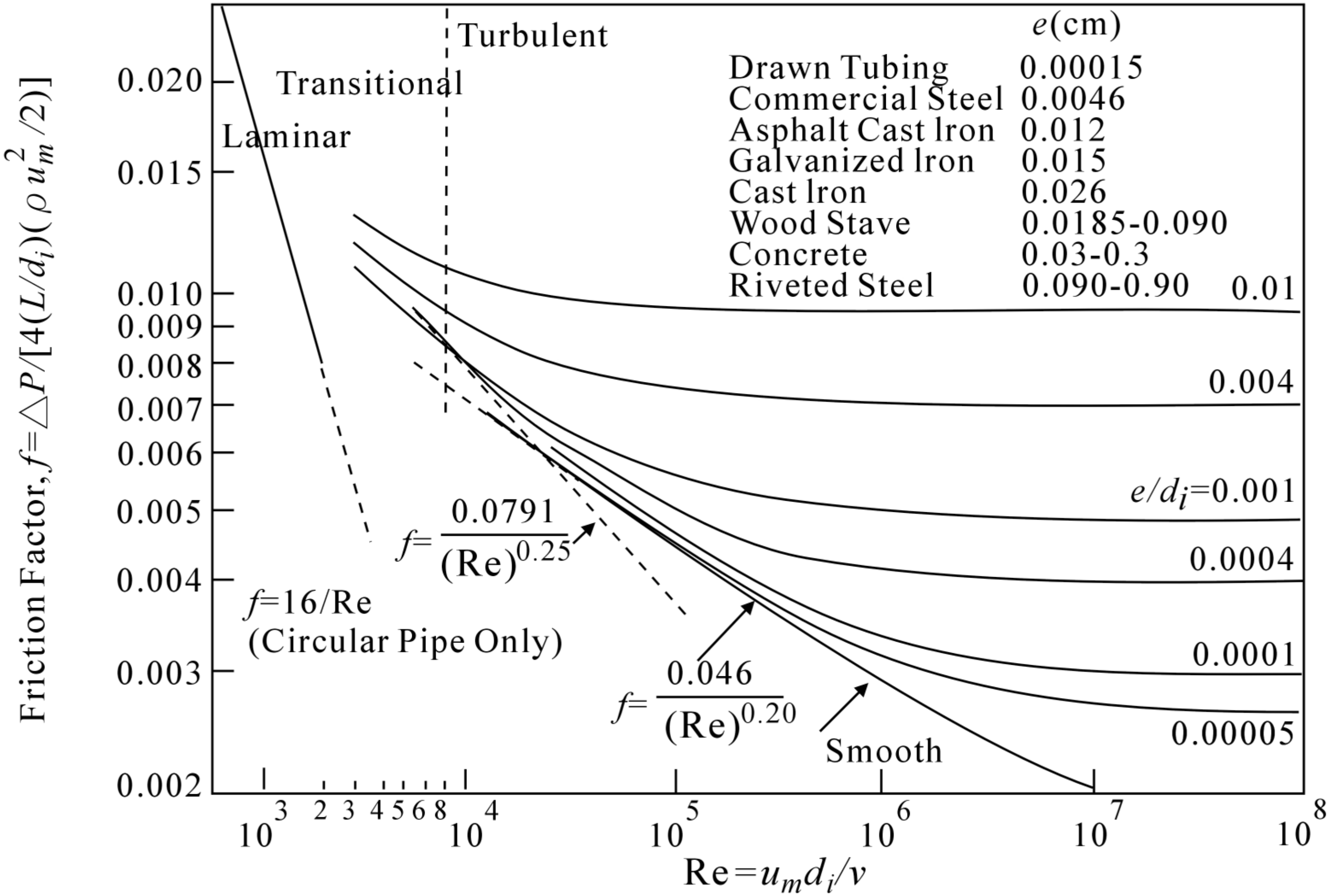
- ΔP_f為摩擦造成的壓降，在一般管線中，ΔP_f所佔的比重約略可超過85%，因此，一般快速計算管線內的壓降，可以由ΔP_f 概算得知；
  可以想見摩擦壓降ΔP_f應與流體的速度u、流體的密度ρ、流體的黏滯係數μ、管路直徑大小d_i與管路表面的粗糙度e有關

假設ΔP_f 與管路長度成正比→
$$\frac{\Delta P_f}{L} = F_{cn}(u, \rho, d_i, e, \mu)$$

經過無因次化的分析後，可以得到一個新的無因次參數 – 摩擦係數f，且摩擦係數f 與Re等參數關係如下：

$$f \equiv \frac{\Delta P}{4\left(\frac{L}{d_i}\right)\left(\frac{\rho u^2}{2}\right)} = F_{cn}\left(\frac{\rho u d_i}{\mu}, \frac{e}{d_i}\right) = F_{cn}\left(\mathrm{Re}, \frac{e}{d_i}\right)$$

單相流動管內壓損的估算-Moody Diagram



例：如下圖所示，水量30 L/min流進一長為10公尺的水平直管中，假設水入口溫度為$T = 20°\text{C}$，試估算通過管路的壓降：

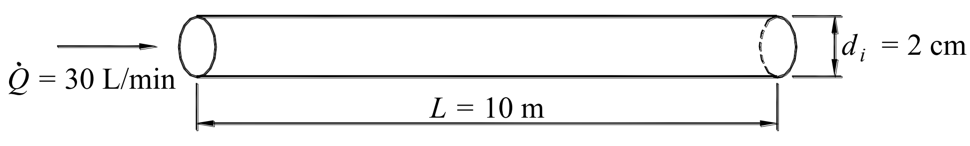
在計算過程中，建議讀者養成使用標準SI單位的習慣，如此可避免單位換算的錯誤(尤其是無因次參數，一定要用標準單位)

$d_i = 2\ \text{cm} = 0.02\ \text{m}$

$L = 10\ \text{m}$

$$\dot{Q} = 30\,\text{L/min} = {}^{30\,\text{L}}\!/_{60\,\text{s}} = 0.5\,\text{L/s} = 0.0005\,\text{m}^3/\text{s}$$

管內的截面積 $A_c = \dfrac{\pi}{4} d_i^2 = \dfrac{\pi}{4}(0.02)^2 = 0.000314\,\text{m}^2$



$\therefore \dot{Q} = A_c \cdot u$

$0.0005\ \text{m}^3/\text{s} = (0.000314\ \text{m}^2) \times u$

$\therefore\ u = 1.59\ \text{m/s}$

水在20°C的黏度、密度如下(見附錄資料)

$\mu = 1002 \times 10^{-6}\ \text{N}\cdot\text{s/m}^2$

$\rho = 998.2\,\text{kg/m}^3$

$\therefore$雷諾數

$$\text{Re} = \frac{998.2 \times 1.59 \times 0.02}{1002 \times 10^{-6}} = 31680$$

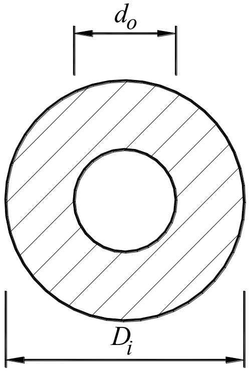
由圖1-3來估算 f 值，由於本例為平滑管且Re = 31680，故可估出 f 約為0.0058

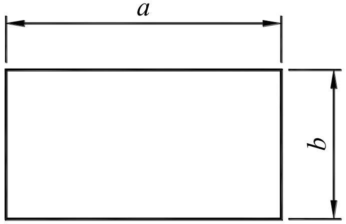
$$\therefore \Delta P = \frac{4L}{d_i} \cdot f \cdot \frac{1}{2}\rho u^2 = \frac{4 \times 10}{0.02} \times 0.0058 \times \frac{1}{2} \times 998.2 \times (1.59)^2 = 14636.6\ \text{Pa}$$

如果不是圓管…

- 可使用水力直徑(類似等效直徑)，水力直徑(hydraulic diameter)的定義為：

$$D_h = \frac{4A_c}{P} = \frac{4(\text{淨截面面積})}{\text{截面周長}}$$

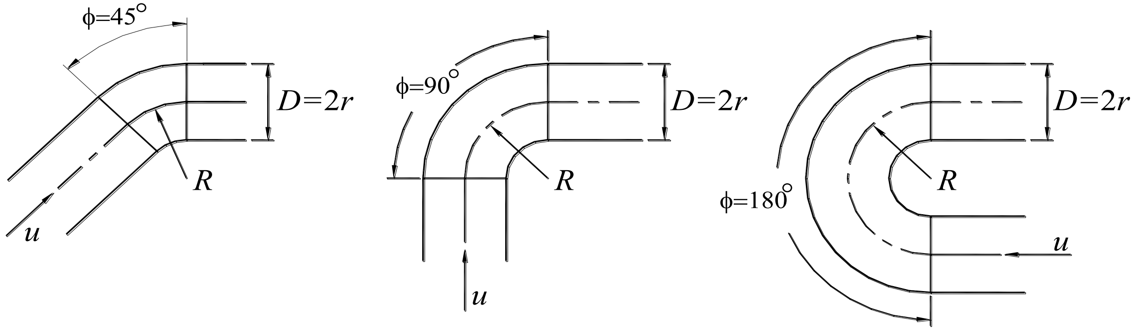




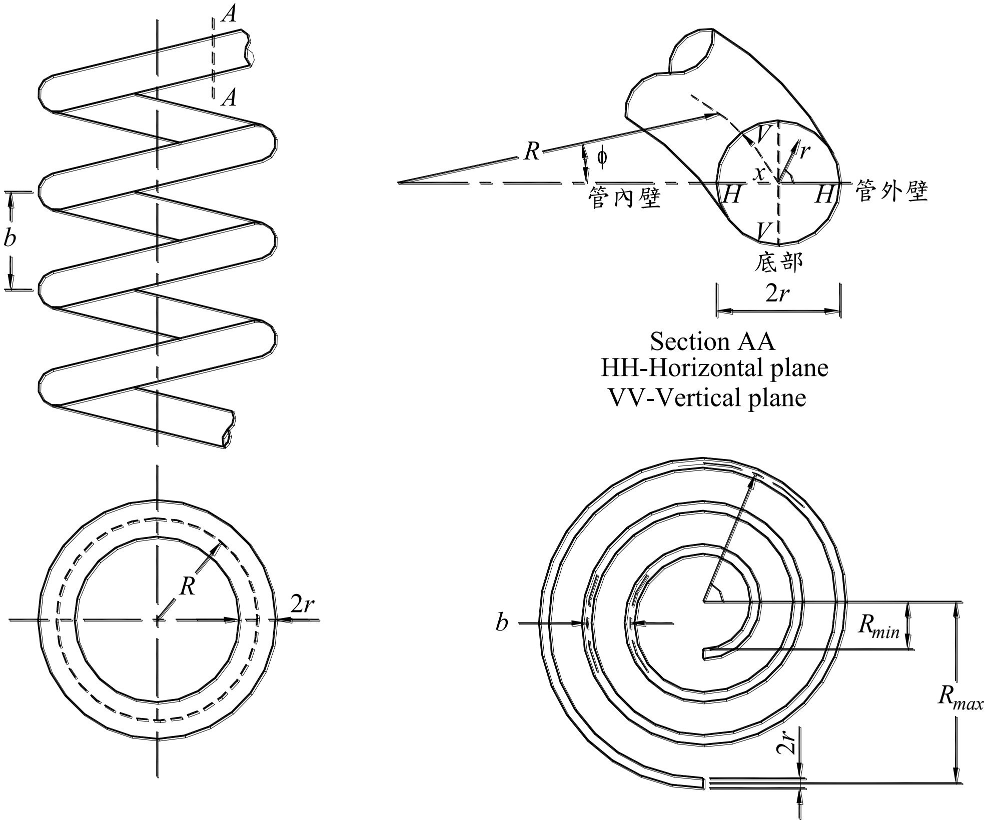
$$D_{h,\text{圓環}} = \frac{4\times(\frac{\pi}{4}D_i^2 - \frac{\pi}{4}d_o^2)}{\pi D_i + \pi d_o} = D_i - d_o$$

$$D_{h,\text{方管}} = \frac{4\times(a\times b)}{2a+2b} = \frac{2ab}{a+b}$$

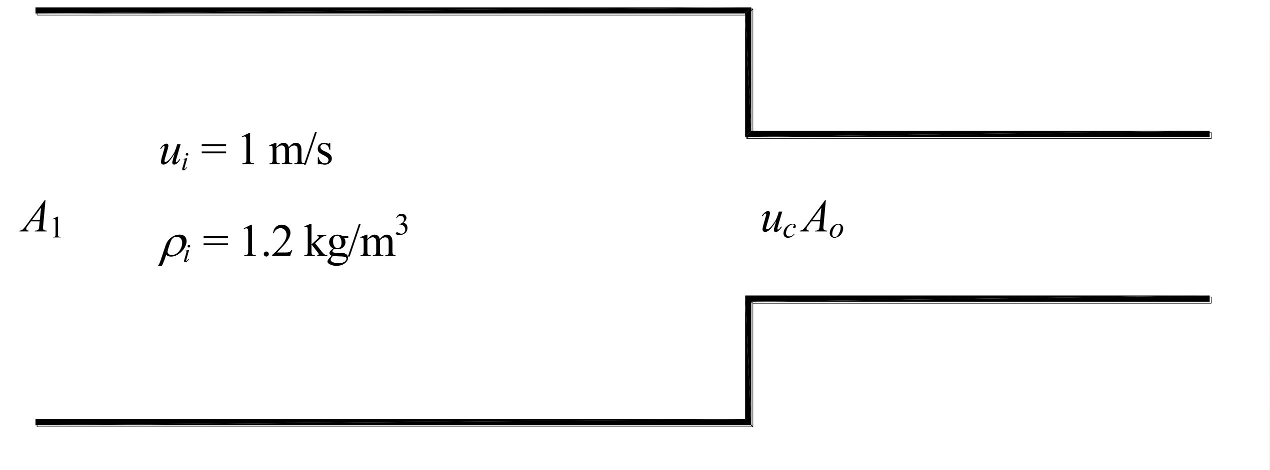
螺旋管、渦管與彎管



(a) 螺旋管 (b) 渦管



例1-1-1：如下圖所示，$A_1/A_0=4$，空氣之進入驟縮段前的風速為1 m/s， 空氣的密度為1.2 kg/m^3，通過驟縮段後產生的壓降為何？



1-1-1 解：

驟縮後的工作流體速度會增加，由於流體為不可壓縮，所以：

$$\frac{A_1}{A_0} = \frac{u_C}{u_i}$$

$A_1/A_0 = 4$

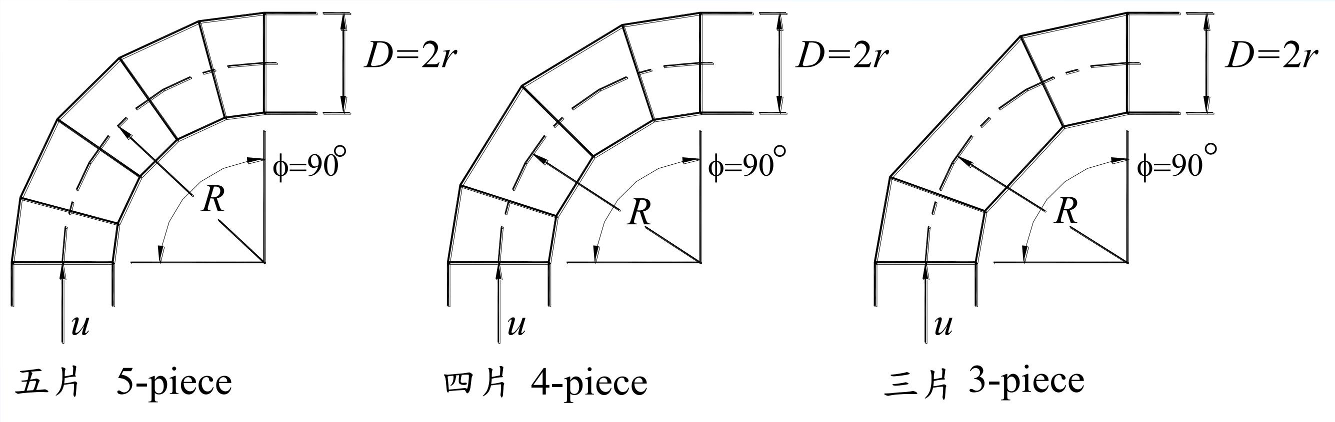
所以$u_C = 4$ m/s

C_o由表1-1可知為0.41

故 $\Delta P_i = 0.41 \times \frac{1.2 \times 4^2}{2} = 3.9$ Pa

多片組合彎管 (常用於風管)

表 1-4　90° 多片組合型式之轉彎之全壓損失係數的 C_o (適用圓管)



五片　5-piece　四片　4-piece　三片　3-piece

圖 1-7　多片組合型式之彎管

	R/D				
彎頭組成片數	0.5	0.75	1.0	1.5	2.0
5	--	0.46	0.33	0.24	0.19
4	--	0.50	0.37	0.27	0.24
3	0.90	0.54	0.42	0.34	0.33

擴張管的壓降

表 1-3　擴張段的全壓損失係數的 C_o (適用圓管)

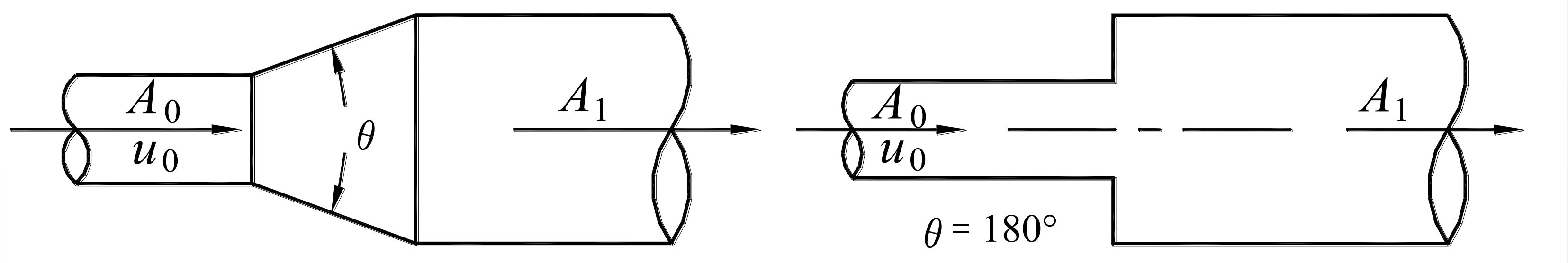


圖 1-6　擴張段示意圖

	角度 θ，度									
A_1/A_0	8	12	16	20	30	45	60	90	120	180
2	0.11	0.11	0.14	0.19	0.32	0.33	0.33	0.32	0.31	0.30
4	0.15	0.17	0.23	0.30	0.46	0.61	0.68	0.64	0.63	0.62
6	0.17	0.20	0.27	0.33	0.48	0.66	0.77	0.74	0.73	0.72
10	0.19	0.23	0.29	0.38	0.59	0.76	0.80	0.83	0.84	0.83
16	0.19	0.22	0.31	0.38	0.60	0.84	0.88	0.88	0.88	0.88

資料參考來源：ASHRAE Handbook，Fundamentals，1997，32章

常用單相管內壓損計算方程式

表 1-2　紊流流動下等溫摩擦係數方程式

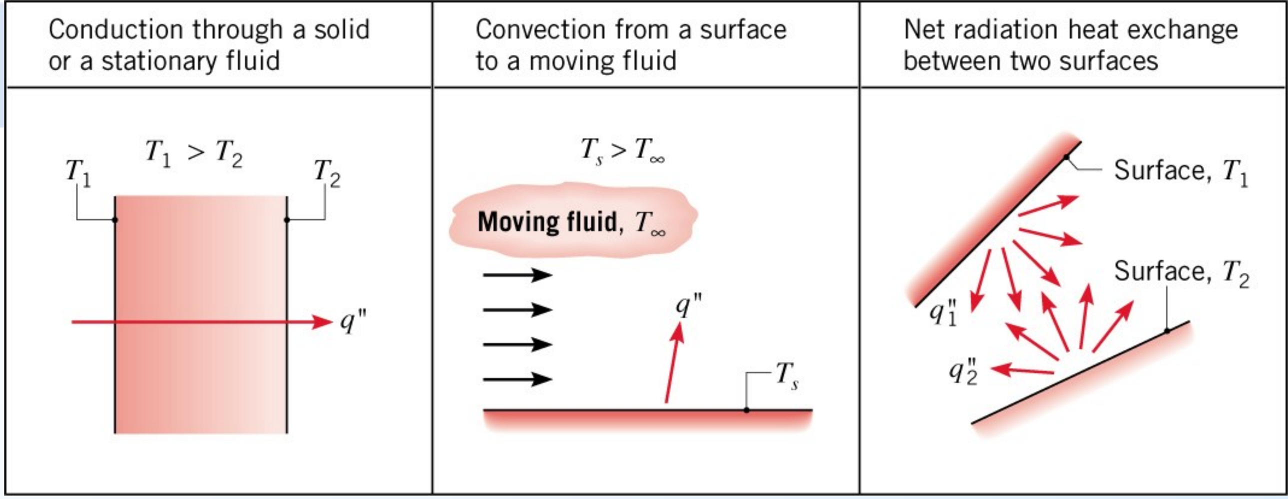
資料來源 (參考 Kakaç and Liu 1998一書)	方程式[a]	適用範圍
Blasius	$f=0.0791\mathrm{Re}^{-0.25}$	$3\times10^{3}<\mathrm{Re}<10^{5}$
Drew, Koo, and McAdams	$f=0.00140+0.125\mathrm{Re}^{-0.32}$	$3\times10^{3}<\mathrm{Re}<5\times10^{6}$
Karman-Nikuradse	$\frac{1}{\sqrt{f}}=1.737\ln(\mathrm{Re}\sqrt{f})-0.4$ 或 $\frac{1}{\sqrt{f}}=4\log_{10}(\mathrm{Re}\sqrt{f})-0.4$ 可近似為：$f=0.46\,\mathrm{Re}^{-0.2}$	$3\times10^{3}<\mathrm{Re}<3\times10^{6}$
Filonenko	$f=(3.64\log_{10}\mathrm{Re}-3.28)^{-2}$	$3\times10^{4}<\mathrm{Re}<10^{6}$

[a] 計算時以流體的混和溫度為基準　(bulk temperature)

基本熱傳學簡介

- 能量可以不同型態貯存，也可以藉由熱傳遞和做功方式來進行彼此的交換。
- －當存在有溫度梯度(temperature gradient)時，能量以熱能的方式來傳遞；當有作用力加在物體上，且使物體有沿著作用力方向位移時，則能量以功的方式進行傳遞。
- 當系統和其外界周圍存在有溫度差異時，就會有發生能量的交換，此能量即就是熱量(heat)或稱為熱能(thermal energy)。熱傳遞模式為熱傳導(heat conduction)、熱對流(heat convection)和熱輻射(thermal radiation)
- －熱傳導為固體或靜止流體分子的熱擴散行為，熱對流則以流體分子的熱擴散和運動行為來進行熱量的傳遞；至於熱輻射則不需要藉助任何介質做媒介，它是以電磁波方式進行熱量交換。
- ※熱傳量單位為焦耳(J)或仟焦耳(kJ)，每單位質量所具有的能量為kJ/kg
- *1 Joule = 1 N ·m*
- 單位時間所做的功，稱為功率(power)，單位為瓦特(Watts)。
- *1 W=Joule/sec*

熱傳模式



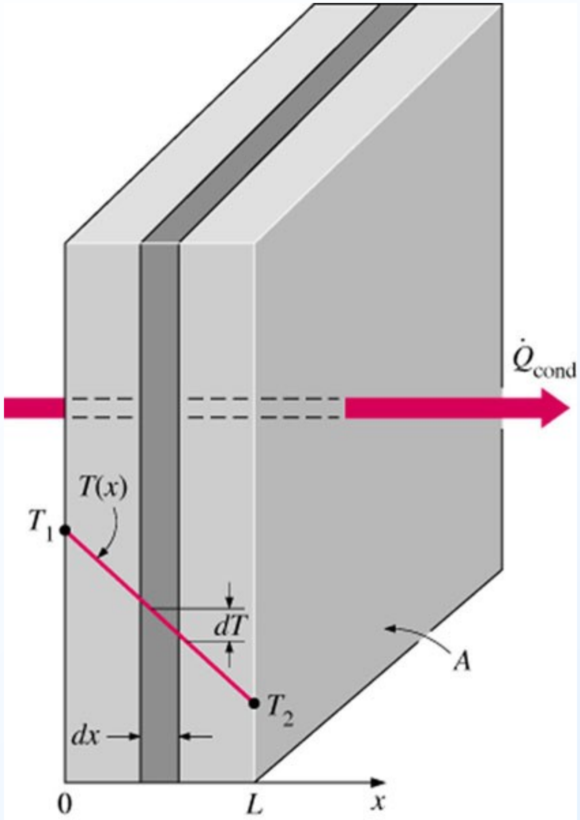
Conduction: Heat transfer in a solid or a stationary fluid (gas or liquid) due to the random motion of its constituent atoms, molecules and/or electrons.

Convection: Heat transfer due to the combined influence of bulk and random motion for fluid flow over a surface.

Radiation: Energy that is emitted by matter due to changes in the electron configurations of its atoms or molecules and is transported as electromagnetic waves (or photons).

- Conduction and convection require the presence of temperature variations in a material medium.
- Although radiation originates from matter, its transport does not require a material medium and occurs most efficiently in a vacuum.

熱傳導(heat conduction)

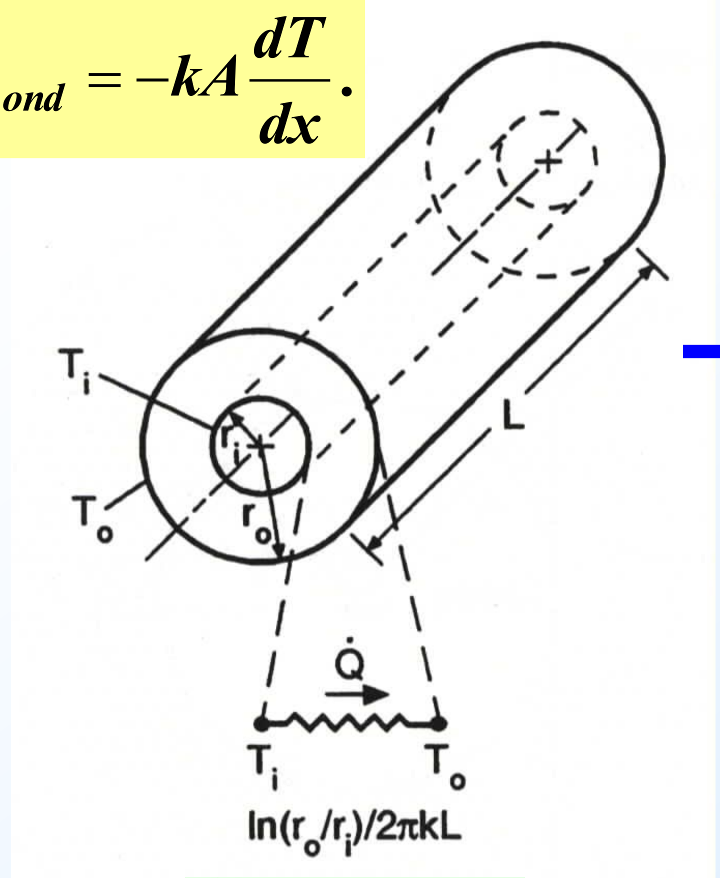


$$\dot{Q}_{\text{cond,wall}} = \frac{kA}{L}(T_1 - T_2)$$

k = thermal conductivity (W/m.ºC)

熱阻抗觀念(Thermal Resistance)

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}.$$



$$\dot{Q}_{cond} = -k(2\pi rL)\frac{dT}{dr}.$$

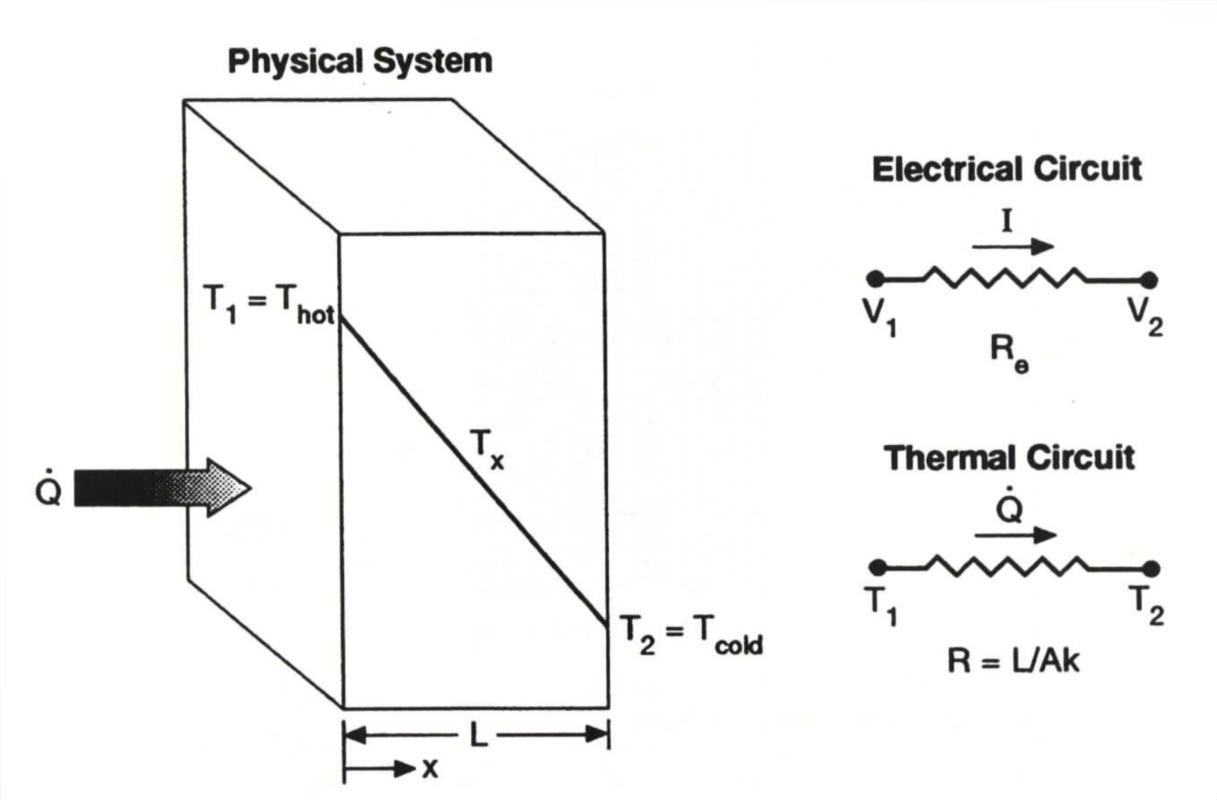
$$\dot{Q}_{cond}\int_{r_i}^{r_o}\frac{dr}{r} = -2\pi kL\int_{T_i}^{T_o} dT$$

$$\Rightarrow \dot{Q}_{cond}\, ln\left(\frac{r_o}{r_i}\right) = 2\pi kL(T_i - T_o)$$

$$\Rightarrow \dot{Q}_{cond} = \frac{T_i - T_o}{\frac{ln\left(\frac{r_o}{r_i}\right)}{2\pi kL}}$$

$$R = \frac{ln\left(\frac{r_o}{r_i}\right)}{2\pi kL}$$

熱阻抗觀念(Thermal Resistance)



管壁內熱傳導溫度變化示意圖

$$I = \frac{V}{R}$$

$$\dot{Q} = \frac{\Delta T}{\dfrac{L}{kA}}$$

Fourier's Law ⇨ $\dot{Q}_{cond} = -kA \int_{T_1,0}^{T_2,L} \frac{dT}{dx} = kA \frac{T_1 - T_2}{L}.$

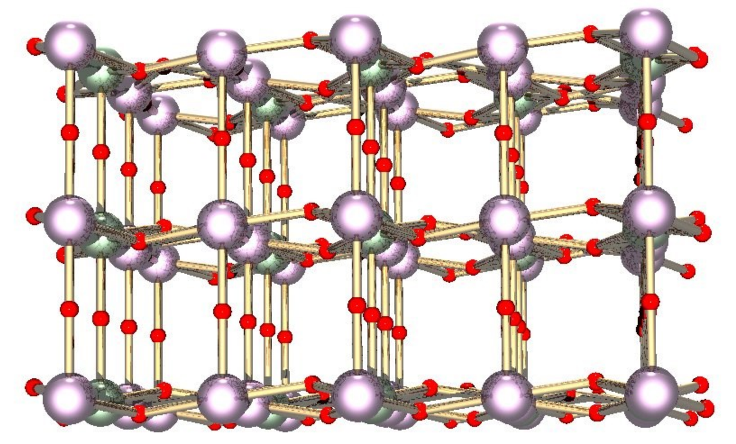
$$R_{thermal} = \frac{L}{kA}$$

常用金屬熱傳導係數值(at 0℃)

材質	熱傳導係數 k,(W/m.K)
銀，Silver (pure)	410
銅，Copper (pure)	385
鋁，Aluminum (pure)	202
鎳，Nickel (pure)	93
鐵，Iron (pure)	73
碳鋼，Carbon steel, 1%C	43
鉛，Lead (pure)	35
鎳鉻合金，Chrome-nickel steel (18%Cr, 8% Ni)	16.3

More on Conduction

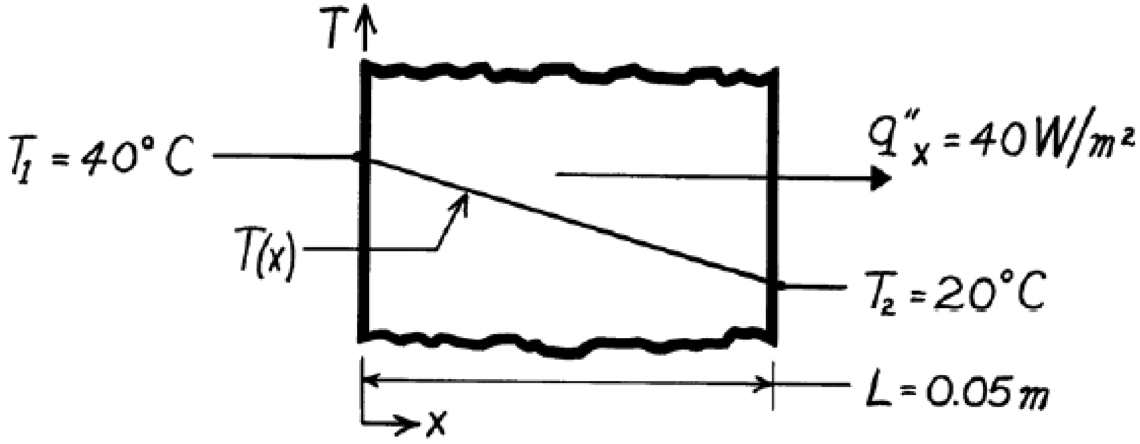
- Conduction is regarded as the energy transfer from more energetic molecules to less energetic molecules
    - Applicable to solid & fluid
- In solid, the energy transfer is in the form of lattice wave induced by atomic motion. In electrical conductor, an additional mechanism is via translational motion of free electrons.



KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^\circ C}$$

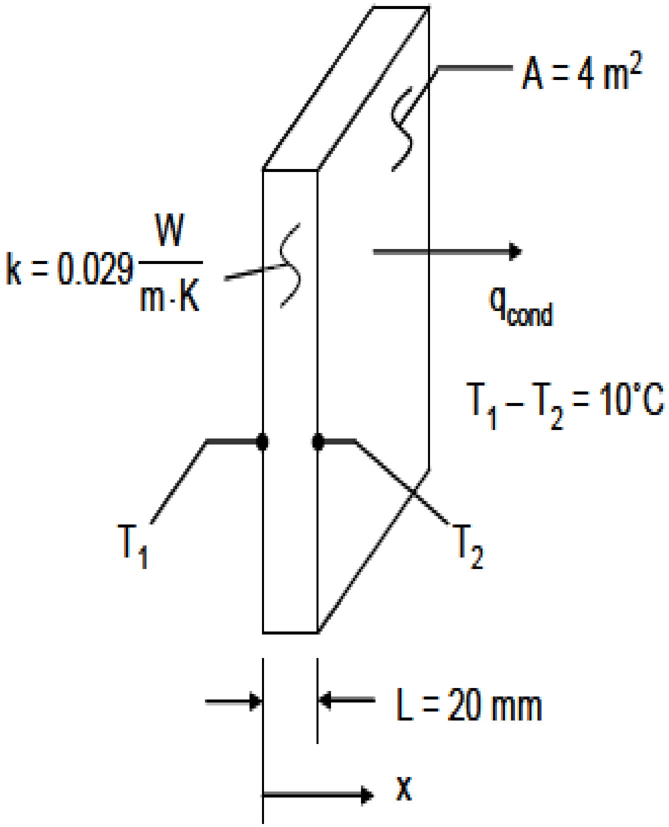
$$k = 0.10\ W/m \cdot K.$$

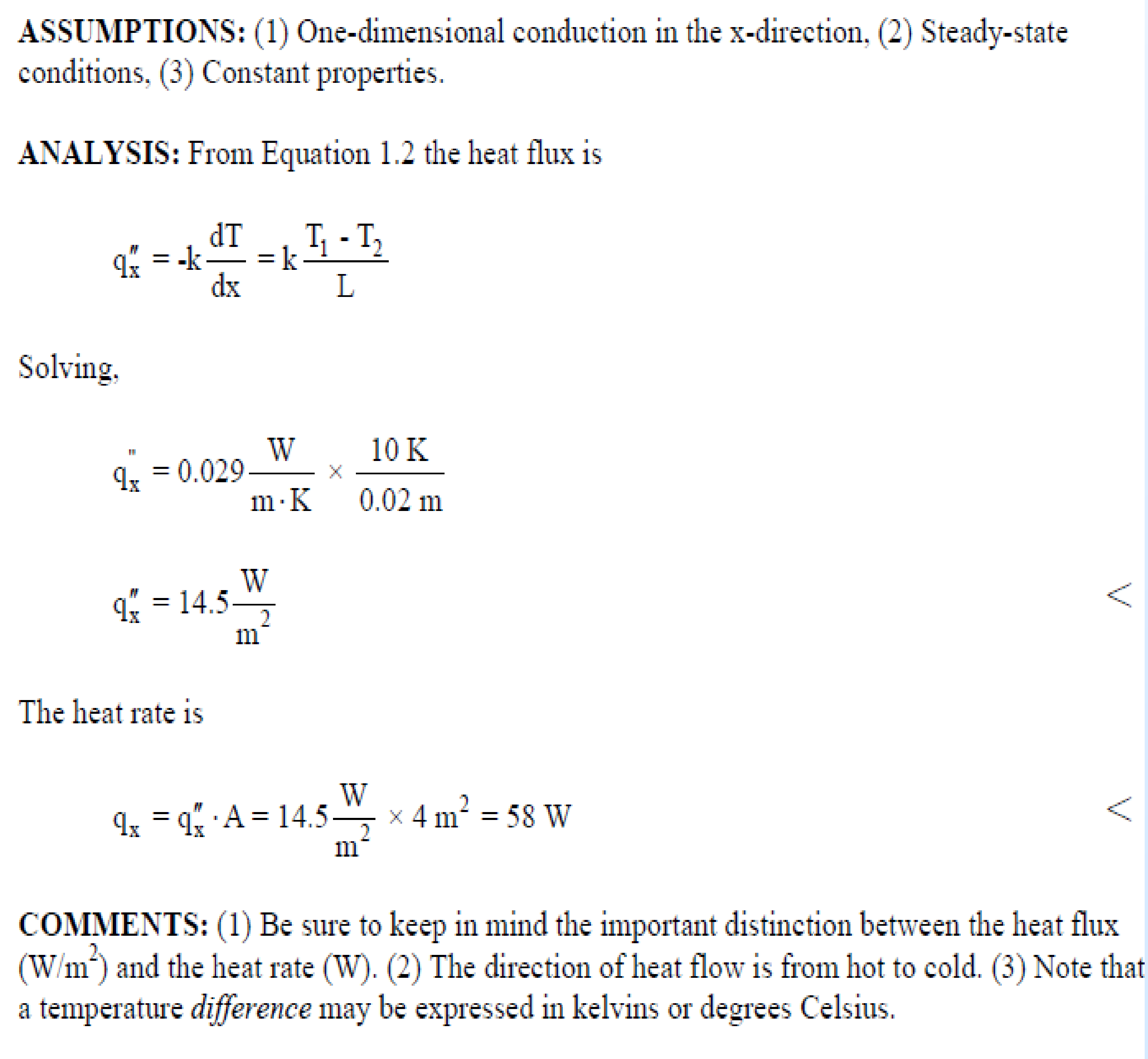
<

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation.

FIND: (a) The heat flux through a 2 m × 2 m sheet of the insulation, and (b) The heat rate through the sheet.

SCHEMATIC:





ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Equation 1.2 the heat flux is

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L}$$

Solving,

$$q''_x = 0.029\frac{\mathrm{W}}{\mathrm{m \cdot K}} \times \frac{10\ \mathrm{K}}{0.02\ \mathrm{m}}$$

$$q''_x = 14.5\frac{\mathrm{W}}{\mathrm{m}^2} \qquad <$$

The heat rate is

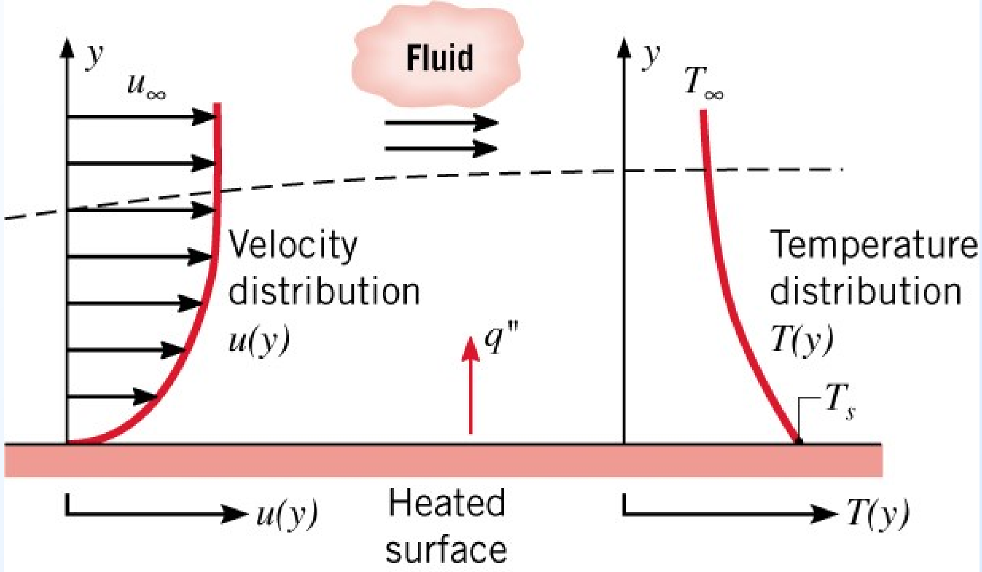
$$q_x = q''_x \cdot A = 14.5\frac{\mathrm{W}}{\mathrm{m}^2} \times 4\ \mathrm{m}^2 = 58\ \mathrm{W} \qquad <$$

COMMENTS: (1) Be sure to keep in mind the important distinction between the heat flux ($\mathrm{W/m^2}$) and the heat rate (W). (2) The direction of heat flow is from hot to cold. (3) Note that a temperature *difference* may be expressed in kelvins or degrees Celsius.

Heat Transfer Rates

Convection

Relation of convection to flow over a surface and development of velocity and thermal boundary layers:



Newton’s law of cooling:

$$q'' = h\left(T_s - T_\infty\right)$$

h: Convection heat transfer coefficient ($\mathrm{W/m^2 \cdot K}$)

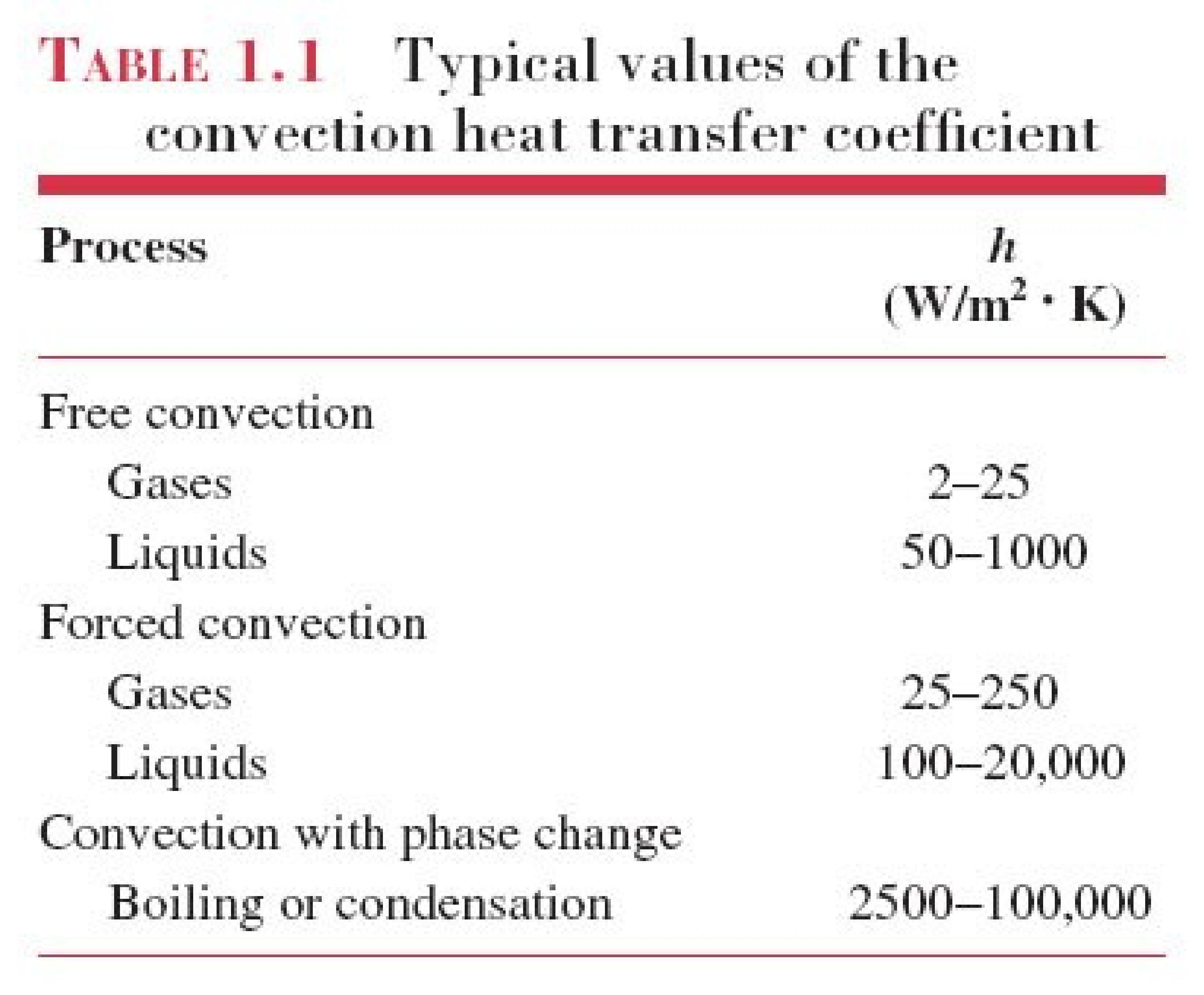


TABLE 1.1 Typical values of the convection heat transfer coefficient

Process	h $(\mathrm{W/m^2 \cdot K})$
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–100,000

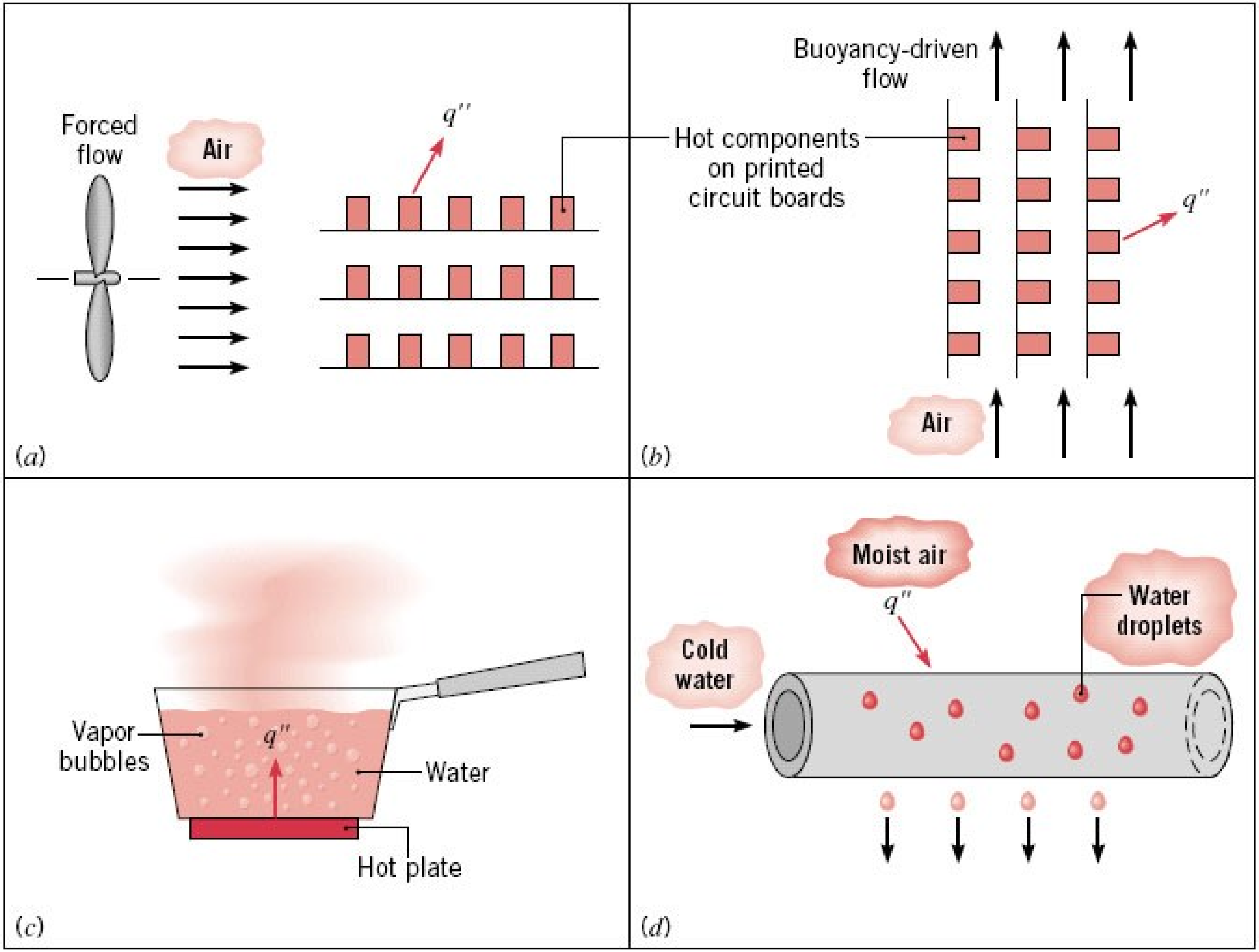


FIGURE 1.5 Convection heat transfer processes. (*a*) Forced convection. (*b*) Natural convection. (*c*) Boiling. (*d*) Condensation.

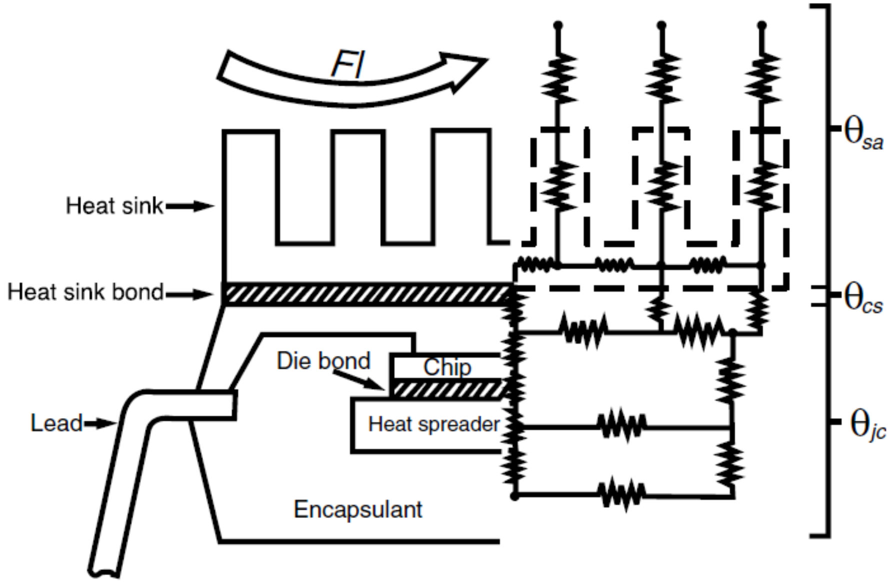


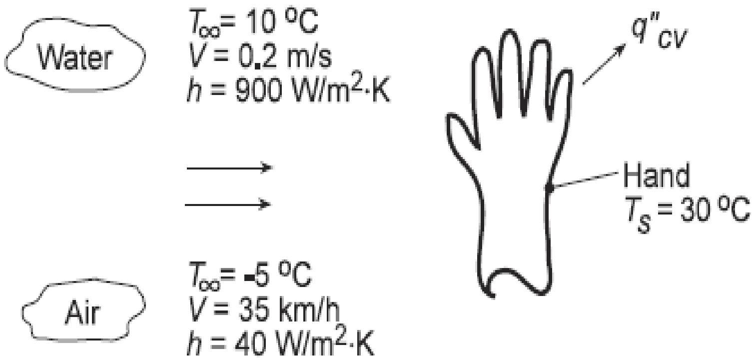
FIGURE 1.2 Primary thermal resistances in a chip/heat sink assembly. θ_{jc} is resistance from the die junction to the device case. θ_{cs} is resistance from the device case to the heat sink. θ_{sa} is resistance from the heat sink to the ambient air. (Adapted from Kraus, A. D. and Bar-Cohen, A., *Design and Analysis of Heat Sinks,* John Wiley & Sons, New York, 1995. With permission.)

FIGURE 1.11
A finned heat sink and fan assembly (left) and microprocessor (right).

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m^2 under normal room conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{\text{air}} = 40\,\text{W}/\text{m}^2\cdot\text{K}\left[30-(-5)\right]\text{K} = 1,400\,\text{W}/\text{m}^2 \qquad <$$

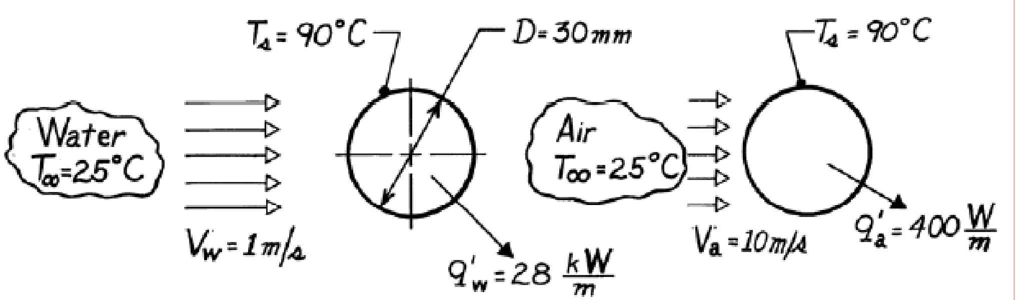
For the water stream:

$$q''_{\text{water}} = 900\,\text{W}/\text{m}^2\cdot\text{K}\left(30-10\right)\text{K} = 18,000\,\text{W}/\text{m}^2 \qquad <$$

KNOWN: Long, 30mm-diameter cylinder with embedded electrical heater; power required to maintain a specified surface temperature for water and air flows.

FIND: Convection coefficients for the water and air flow convection processes, h_w and h_a, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Flow is cross-wise over cylinder which is very long in the direction normal to flow.

ANALYSIS: The convection heat rate from the cylinder per unit length of the cylinder has the form

$$q' = h(\pi D)\ (T_s - T_\infty)$$

and solving for the heat transfer convection coefficient, find

$$h = \frac{q'}{\pi D\ (T_s - T_\infty)}.$$

Substituting numerical values for the water and air situations:

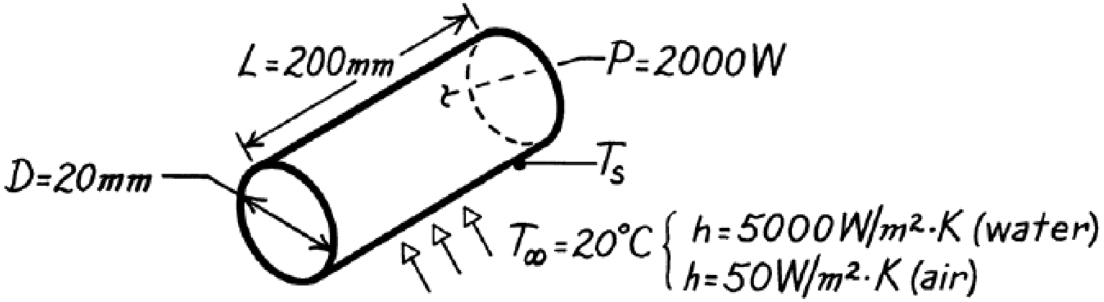
Water $$h_w = \frac{28 \times 10^3\ W/m}{\pi \times 0.030m\ (90\text{-}25)^\circ\ C} = 4{,}570\ W/m^2 \cdot K$$ <

Air $$h_a = \frac{400\ W/m}{\pi \times 0.030m\ (90\text{-}25)^\circ\ C} = 65\ W/m^2 \cdot K.$$ <

KNOWN: Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

FIND: Heater surface temperatures in water and air.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

ANALYSIS: With $P = q_{conv}$, Newton's law of cooling yields

$$P=hA\left(T_S - T_\infty\right) = h\pi DL\left(T_S - T_\infty\right)$$

$$T_S = T_\infty + \frac{P}{h\pi DL}.$$

In water,

$$T_s = 20^\circ C + \frac{2000\ W}{5000\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 31.8^\circ C = 51.8^\circ C. \qquad <$$

In air,

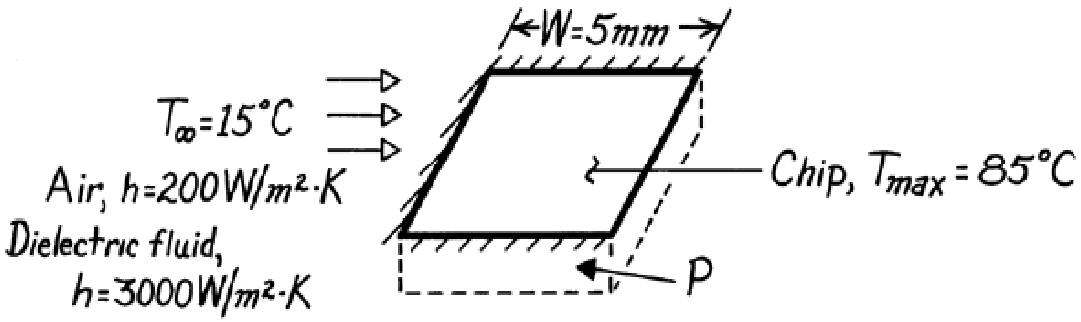
$$T_s = 20^\circ C + \frac{2000\ W}{50\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 3183^\circ C = 3203^\circ C. \qquad <$$

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_{\infty}) = h\,W^2(T - T_{\infty}).$$

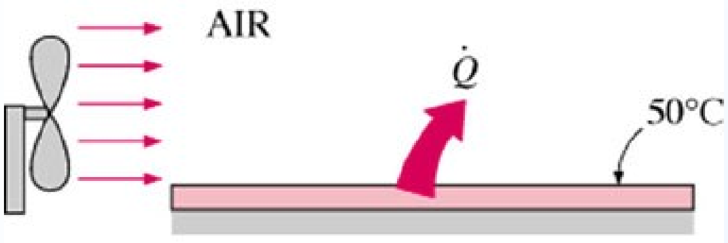
In *air*,

$$P_{max} = 200\ \mathrm{W/m^2 \cdot K}(0.005\ \mathrm{m})^2(85 - 15)\ ^{\circ}\mathrm{C} = 0.35\ \mathrm{W}. \qquad <$$

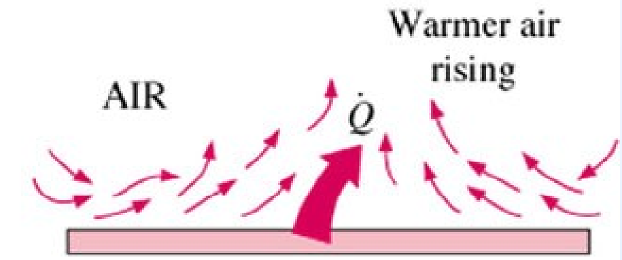
In the *dielectric liquid*

$$P_{max} = 3000\ \mathrm{W/m^2 \cdot K}(0.005\ \mathrm{m})^2(85\text{-}15)\ ^{\circ}\mathrm{C} = 5.25\ \mathrm{W}. \qquad <$$

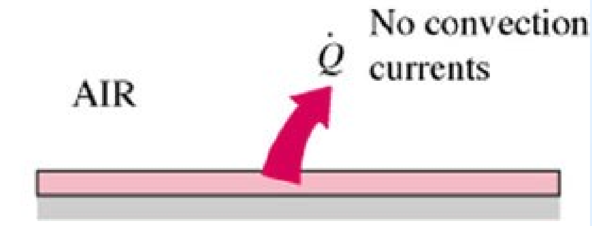
熱對流(heat convection)



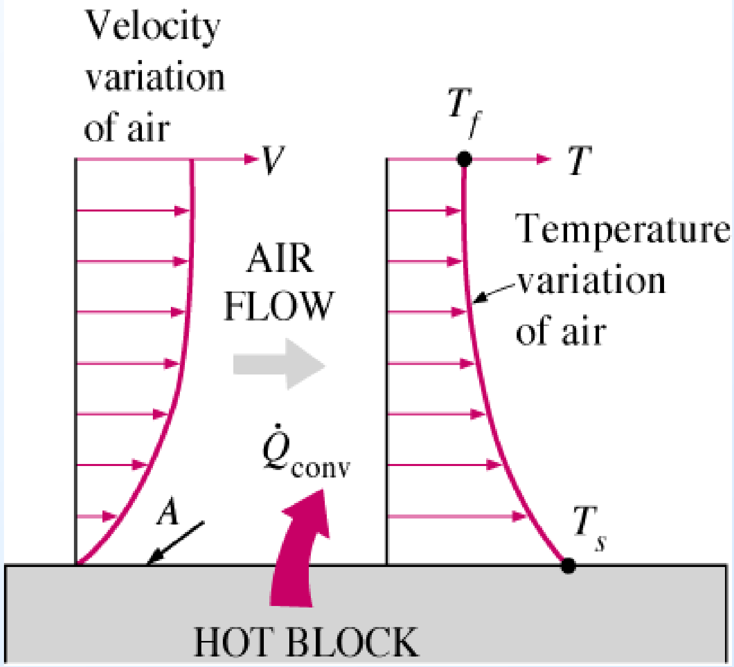
(*a*) Forced convection



(*b*) Free convection



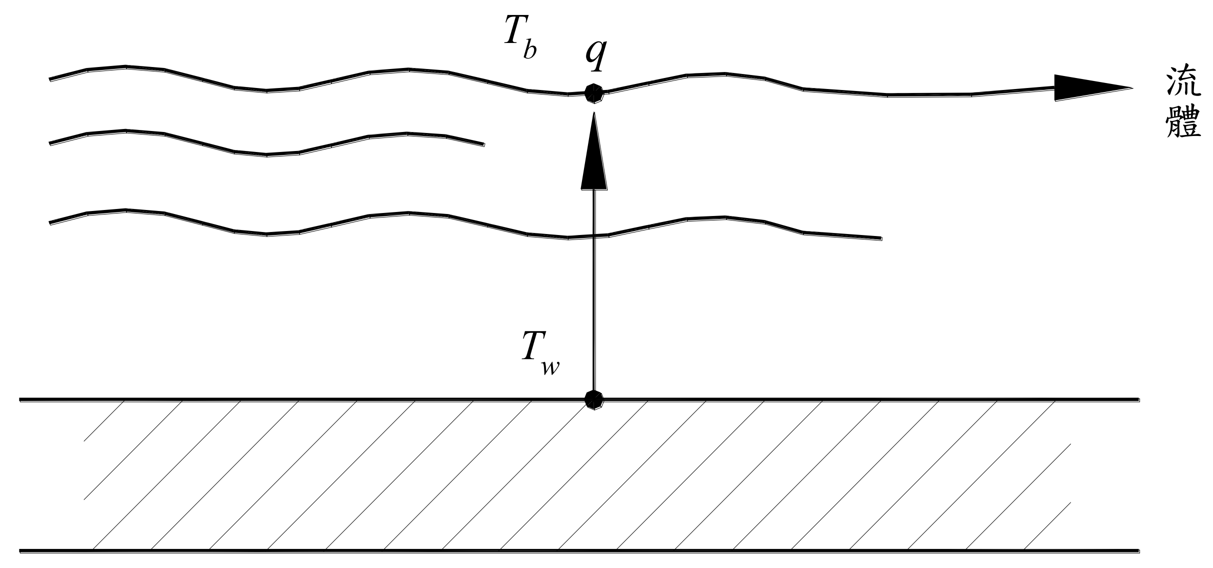
(*c*) Conduction



$$\dot{Q}_{conv} = hA(T_s - T_f)$$

h = heat transfer coefficient (W/m^2.^{o}C)

熱阻抗觀念(Thermal Resistance)



熱對流熱傳示意圖

Newton's Law of Cooling ⇨

$$\dot{Q}_{convection} = hA(T_w - T_b).$$

$$\dot{Q} = \frac{\Delta T}{\frac{1}{hA}}$$

⬇

$$R_{thermal} = \frac{1}{hA}$$

熱交換器原理介紹(Cont.)

一些常見的流體熱傳係數(heat transfer coefficient)

fluid	state	h，W/m^2.K
Gas	1 bar	80 - 125
Gas	10 bar	250 - 400
Water	Single phase	5000 - 7500
Water	Boiling < 5 bar	3000 - 10000
Steam	Condensation 1 Bar	10000 - 15000

熱交換器原理介紹(Cont.)

一些常見的流體熱傳係數(heat transfer coefficient)

fluid	*h*，$W/m^2.K$
Air (Natural Convection)	5-25
Air/ superheated steam (Forced Convection)	20-300
Oil (Forced Convection)	60-1800
Water (Forced Convection)	300-6000
Water (Boiling)	3000-60000
Steam (Condensing)	6000-120,000

常用單相管內熱傳計算方程式 (直管)

表1-7　單相常用熱傳方程式

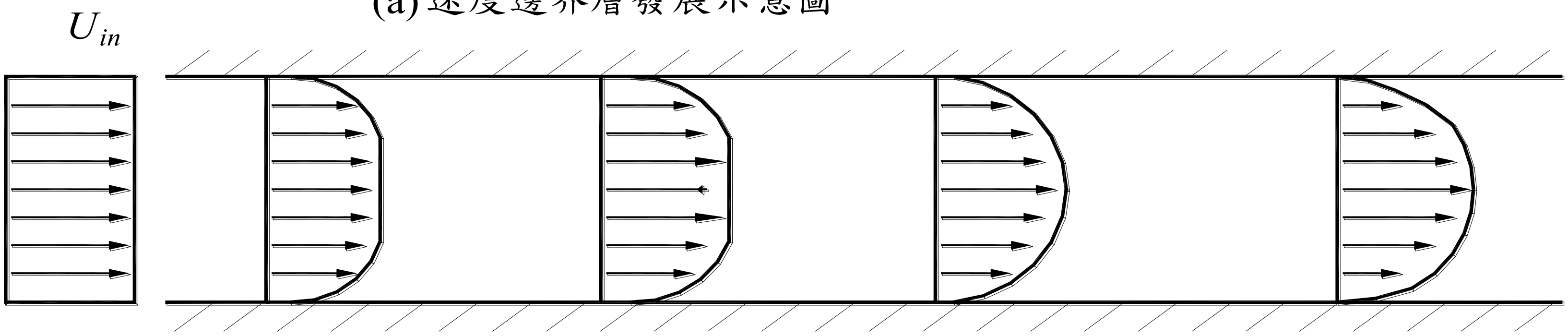
編號	方程式	適用範圍說明
1.	$Nu = 0.023\,\mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}$ (被加熱的流體) $Nu = 0.023\,\mathrm{Re}^{0.8}\mathrm{Pr}^{0.3}$ (被冷卻的流體)	$\mathrm{Re} > 10^4$，Dittus-Boelter 方程式
2.	$Nu = \dfrac{\left(f/2\right)(\mathrm{Re}-1000)\mathrm{Pr}}{1.07 + 12.7\sqrt{f/2}\left(\mathrm{Pr}^{2/3}-1\right)}$ $f = (1.58\ln\mathrm{Re} - 3.28)^{-2}$	$2300 < \mathrm{Re} < 10^5$ Gnielinski 方程式
3.	$Nu = 0.022\,\mathrm{Re}^{0.8}\mathrm{Pr}^{0.5}$	適用氣體且 $\mathrm{Re} > 5000$

熱傳與壓降方程式選用注意事項

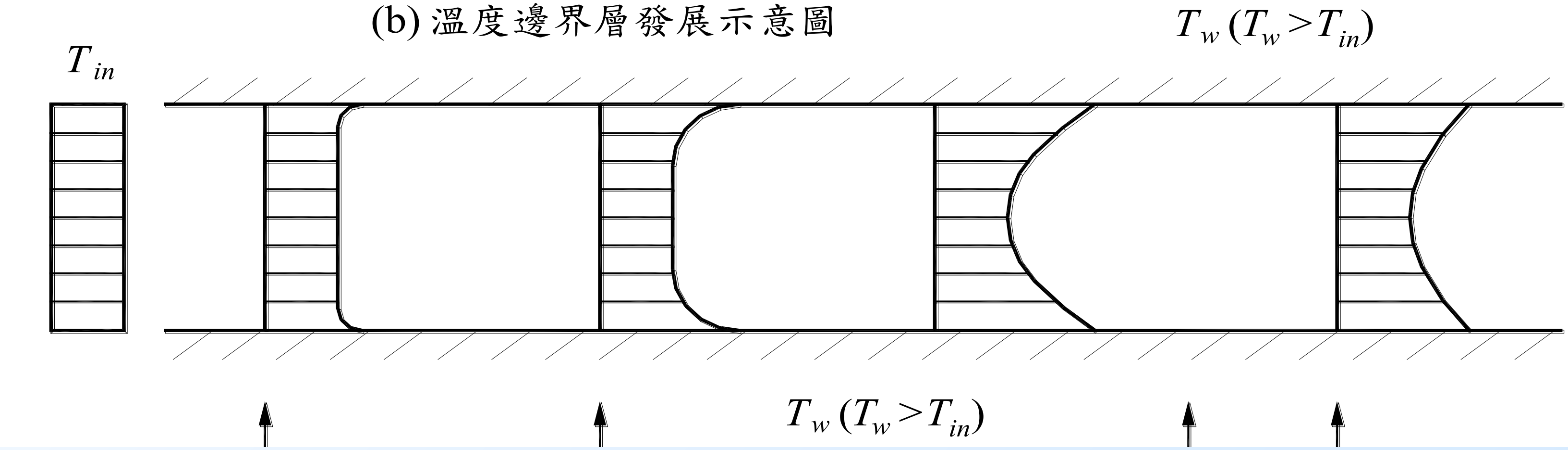
- 單位 (無因次方程式或有因次方程式)
- 特徵長度
- 邊界條件 (尤其是層流流動)
- 適用範圍
    - 溫度、壓力、工作流體
- 溫度效應的影響
- 流動條件 (入口效應)
- 幾何形狀變動

入口效應

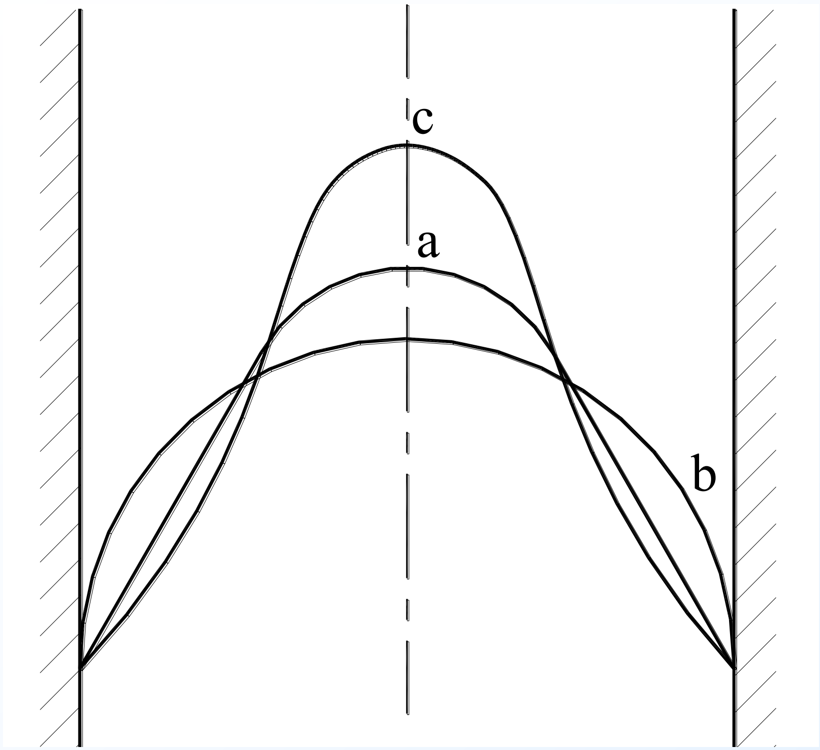
(a) 速度邊界層發展示意圖



(b) 溫度邊界層發展示意圖



溫度效應



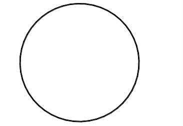
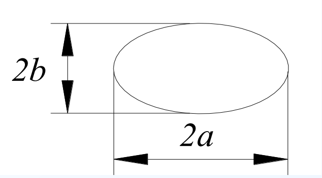
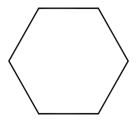
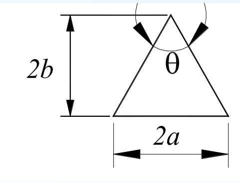
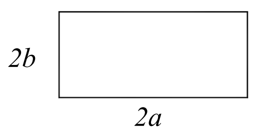
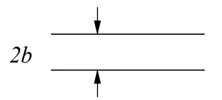
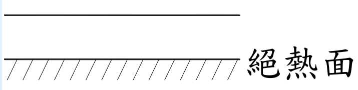
等溫條件下的流速分布為曲線a；當溫度上升時，流體黏性變小，所以靠近壁面的速度會變快(曲線b)，相反的，若流體將熱傳至外界降溫時，靠近壁面的流體速度會下降(曲線c)，但中心的速度反而會變快。流體速度分布的變化當然也會影響熱傳係數

因此若要考慮溫度效應的影響，常用的方程式乘上一個修正係數

$$(\frac{\mu_b}{\mu_w})^{0.14}$$

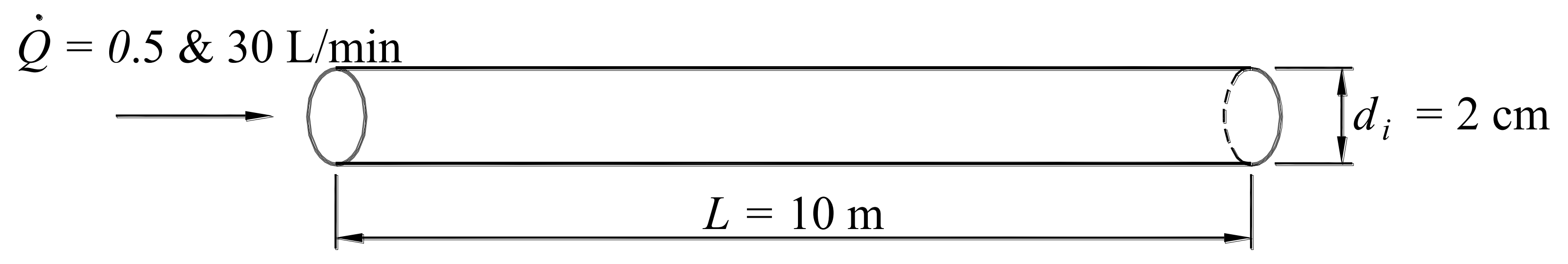
其中μ_b為流體中心溫度的黏度(viscosity)μ_w為壁面溫度下的流體黏度

幾何形狀變化的影響

Geometry ($L/D_h > 100$，完全發展)		Nu_T	Nu_{H1}	Nu_{H2}	$f \cdot \mathrm{Re}$
		3.657	4.364	4.364	16.00
	$2b/2a = 0.5$	3.742	4.558	3.802	16.83
	$2b/2a = 0.25$	3.792	4.88	2.333	18.24
	$2b/2a = 0.125$	3.725	5.085	0.9433	19.146
		3.34	4.002	3.682	16.06
	$\theta = 120°,\ \frac{2b}{2a} = 0.289$	2.0	2.68	0.62	12.744
	$\theta = 90°,\ \frac{2b}{2a} = \frac{1}{2}$	2.34	2.982	1.34	13.153
	$\theta = 60°,\ \frac{2b}{2a} = \frac{\sqrt{3}}{2}$	2.47	3.111	1.892	13.333
	$\theta = 30°,\ \frac{2b}{2a} = 1.866$	2.26	2.91	0.851	13.065
	$\frac{2b}{2a} = 1$	2.976	3.608	3.091	14.23
	$\frac{2b}{2a} = \frac{1}{2}$	3.391	4.123	3.017	15.55
	$\frac{2b}{2a} = \frac{1}{4}$	3.66	5.099	4.35	18.7
	$\frac{2b}{2a} = \frac{1}{8}$	5.597	6.490	2.904	20.59
	$\frac{2b}{2a} = 0$	7.541	8.235	8.235	24.0
	$\frac{b}{a} = 0$	4.861	5.385	--	24.0

Question 1：如下圖所示，水量0.5 L/min流進一長為10公尺的水平直管中，假設水入口溫度為$T = 20°C$，試估算通過管路的熱傳係數？

Question 2：如果水量為30 L/min，試估算通過管路的熱傳係數？



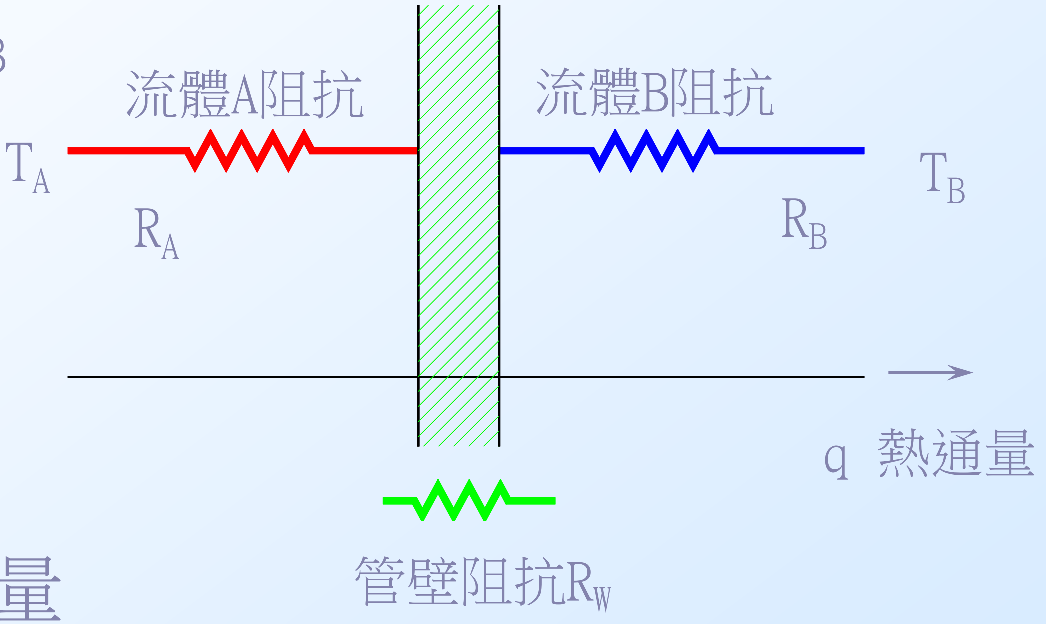
基本熱流介紹-熱交換原理

總阻抗：$R=R_A+R_W+R_B$

熱通量q

$q=(T_A-T_B)/R$

$Q=q\cdot A \Rightarrow$ 總熱傳量



因此若能確切掌握流體阻抗，即能精確設計熱交換器。

熱交換原理介紹(Cont.)

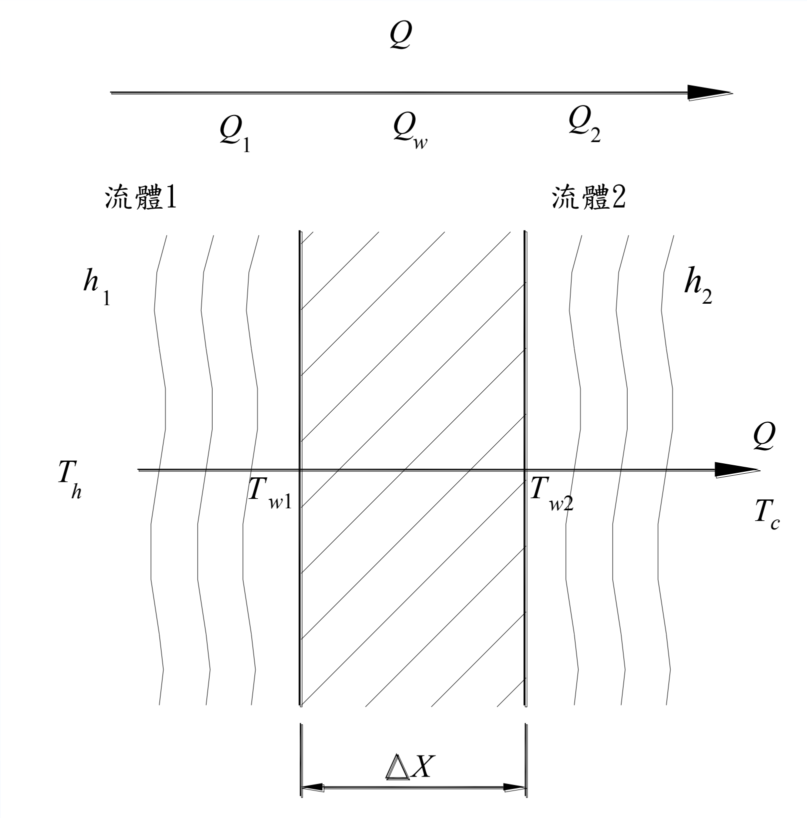
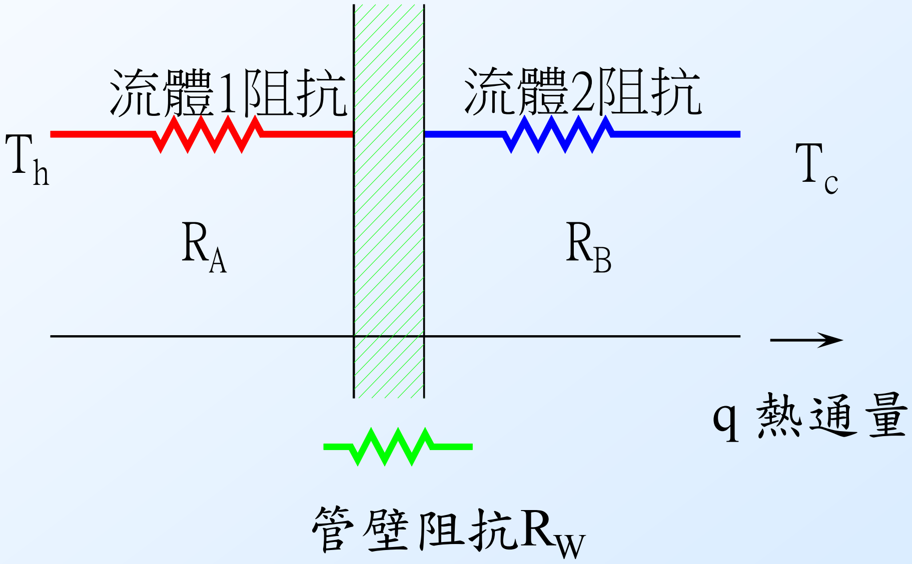


Fig.熱交換器兩側熱對流熱傳示意圖



總阻抗：$R=R_A+R_W+R_B$

熱通量 $q=(T_h-T_c)/R$

$Q=q\cdot A$ ⇒ 總熱傳量

因此若能確切掌握流體阻抗，即能精確設計熱交換器

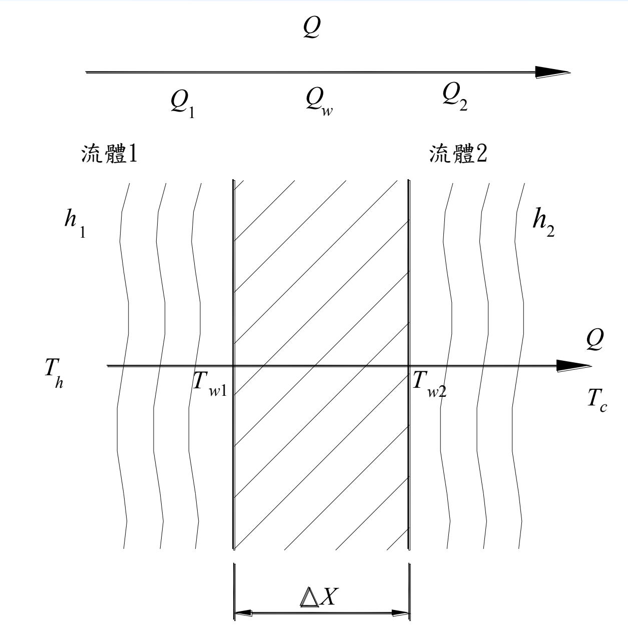
熱交換原理介紹(Cont.)

在穩定狀態(steady state)下熱傳量是平衡的，故$Q_1 = Q_w = Q_2 = Q$

$$\dot{Q} = h_1 A_{w1}\left(T_h - T_{w,1}\right) \Rightarrow \frac{\dot{Q}}{h_1 A_{w1}} = T_h - T_{w,1}$$

$$\dot{Q} = kA\frac{\left(T_{w,1} - T_{w,2}\right)}{\Delta X} \Rightarrow \frac{\dot{Q}}{\frac{kA}{\Delta X}} = T_{w,1} - T_{w,2}$$

$$\dot{Q} = h_2 A_{w2}\left(T_{w,2} - T_c\right) \Rightarrow \frac{\dot{Q}}{h_2 A_{w2}} = T_{w,2} - T_c$$



三式相加，得

$$\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)\dot{Q} = T_h - T_c$$

$$\Rightarrow \dot{Q} = \frac{1}{\underbrace{\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)}_{UA}}\left(T_h - T_c\right)$$

熱交換原理介紹(Cont.)

熱傳量Q 表示成：

$$\therefore \dot{Q} = \frac{1}{\underbrace{\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)}_{UA}} (T_h - T_c)$$

$$\Rightarrow \dot{Q} = UA\Delta T_m$$

$$\boxed{(\dot{Q} = \frac{\Delta T_m}{\frac{1}{UA}} = \frac{\Delta T_m}{R_t})}$$

其中 U一般習稱為總熱傳係數(單位： $W/m^2.K$)，A為參考面積

$$\therefore \frac{1}{\underbrace{UA}_{\text{總阻抗}}} = \frac{1}{\underbrace{h_1 A_{w1}}_{\text{熱側阻抗}}} + \frac{\Delta X}{\underbrace{kA}_{\text{管壁阻抗}}} + \frac{1}{\underbrace{h_2 A_{w2}}_{\text{冷側阻抗}}} \qquad \Rightarrow R_t = R_1 + R_w + R_2$$

$$\boxed{U_o A_o = U_i A_i = \frac{1}{R_t} = \frac{1}{\frac{1}{\underbrace{h_i A_i}_{\text{冷媒側阻抗}}} + \frac{X_p}{\underbrace{k_p A_p}_{\text{管壁阻抗}}} + \frac{1}{\underbrace{h_o A_o}_{\text{空氣側阻抗}}}}}$$

管壁組抗

$$\begin{cases} R_w = \frac{X_p}{k_p A_p} & \text{For plane wall} \\ R_w = \frac{\ln(r_o / r_i)}{2\pi k_p L} & \text{For tube wall} \end{cases}$$

熱交換器原理介紹(Cont.)

For fin-and-tube HX ➔

$$U_oA_o = U_iA_i = \frac{1}{\underbrace{\frac{1}{h_iA_i}}_{\text{冷媒側阻抗}} + \underbrace{\frac{X_p}{k_pA_p}}_{\text{管壁阻抗}} + \underbrace{\frac{1}{h_oA_o\eta_s}}_{\text{空氣側阻抗}}}$$

其中 U_o稱為總熱傳係數，A_o和A_i為管外和管內側熱傳面積，A_p為管壁平均面積，k_p為管壁熱傳導係數，η_s稱為鰭片表面有效度(fin surface effectiveness, 後續說明)，h_o為空氣側之熱傳係數，h_i為管內流體熱傳係數。

- 總熱傳係數U_o的計算，須先求得空氣側熱傳係數h_o 、 管內流體熱傳係數h_i 、鰭片表面有效度η_s和熱傳面積A_o和及A_i。

熱交換器原理介紹(Cont.)

一些常見流體的總熱傳係數(overall heat transfer coefficient)

FLUID COMBINATION	$U(W/m^2.K)$
Water to water	850-1700
Water to oil	110-350
Steam condenser (water in tubes)	1000-6000
Ammonia condenser (water in tubes)	800-1400
Alcohol condenser (water in tubes)	250-700
Finned – tube heat exchanger (water in tubes, air in cross flow)	25-50

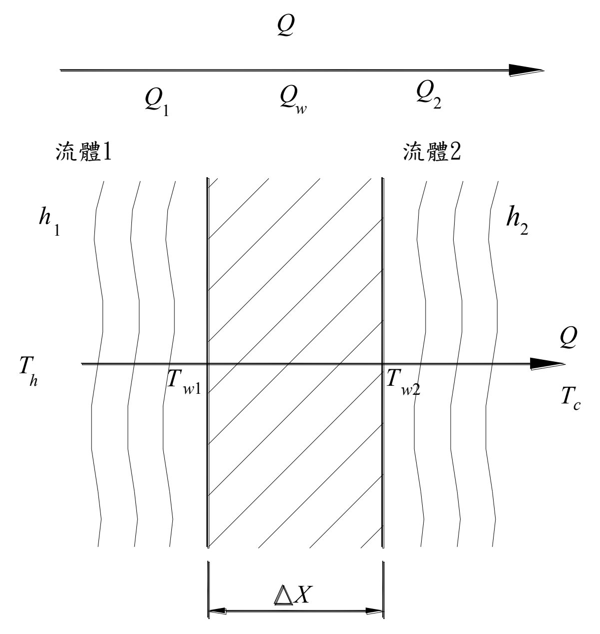
熱交換器原理介紹(Cont.)

在穩定狀態(steady state)下熱傳量是平衡的，故$Q_1 = Q_w = Q_2 = Q$

$$\dot{Q} = h_1 A_{w1}\left(T_h - T_{w,1}\right) \Rightarrow \frac{\dot{Q}}{h_1 A_{w1}} = T_h - T_{w,1}$$

$$\dot{Q} = kA\frac{\left(T_{w,1} - T_{w,2}\right)}{\Delta X} \Rightarrow \frac{\dot{Q}}{\frac{kA}{\Delta X}} = T_{w,1} - T_{w,2}$$

$$\dot{Q} = h_2 A_{w2}\left(T_{w,2} - T_c\right) \Rightarrow \frac{\dot{Q}}{h_2 A_{w2}} = T_{w,2} - T_c$$



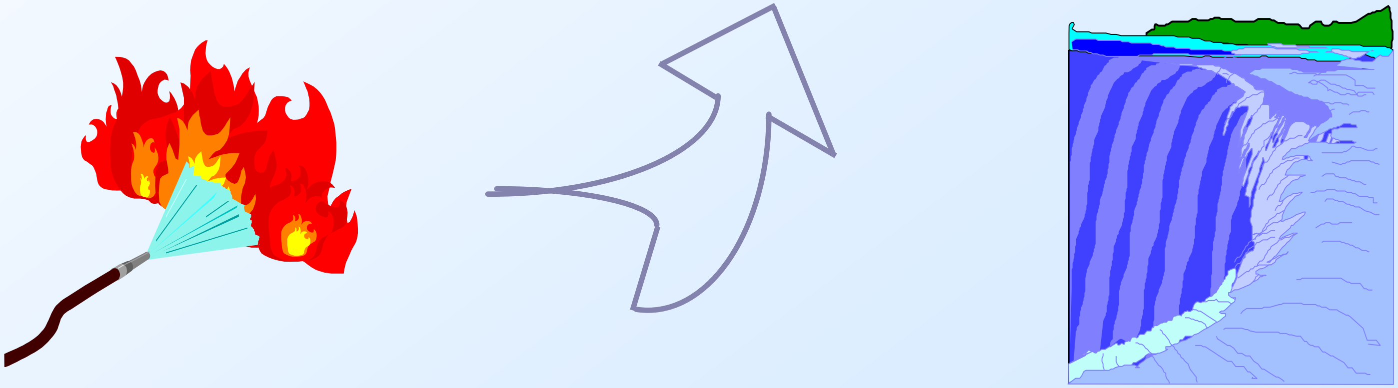
三式相加，得

$$\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)\dot{Q} = T_h - T_c$$

$$\Rightarrow \dot{Q} = \frac{1}{\underbrace{\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)}_{UA}}\left(T_h - T_c\right)$$

熱交換器的功用

- 不同流體間的換熱
- 易於控制與使用
- 避免流體於熱交換過程的「汙染」

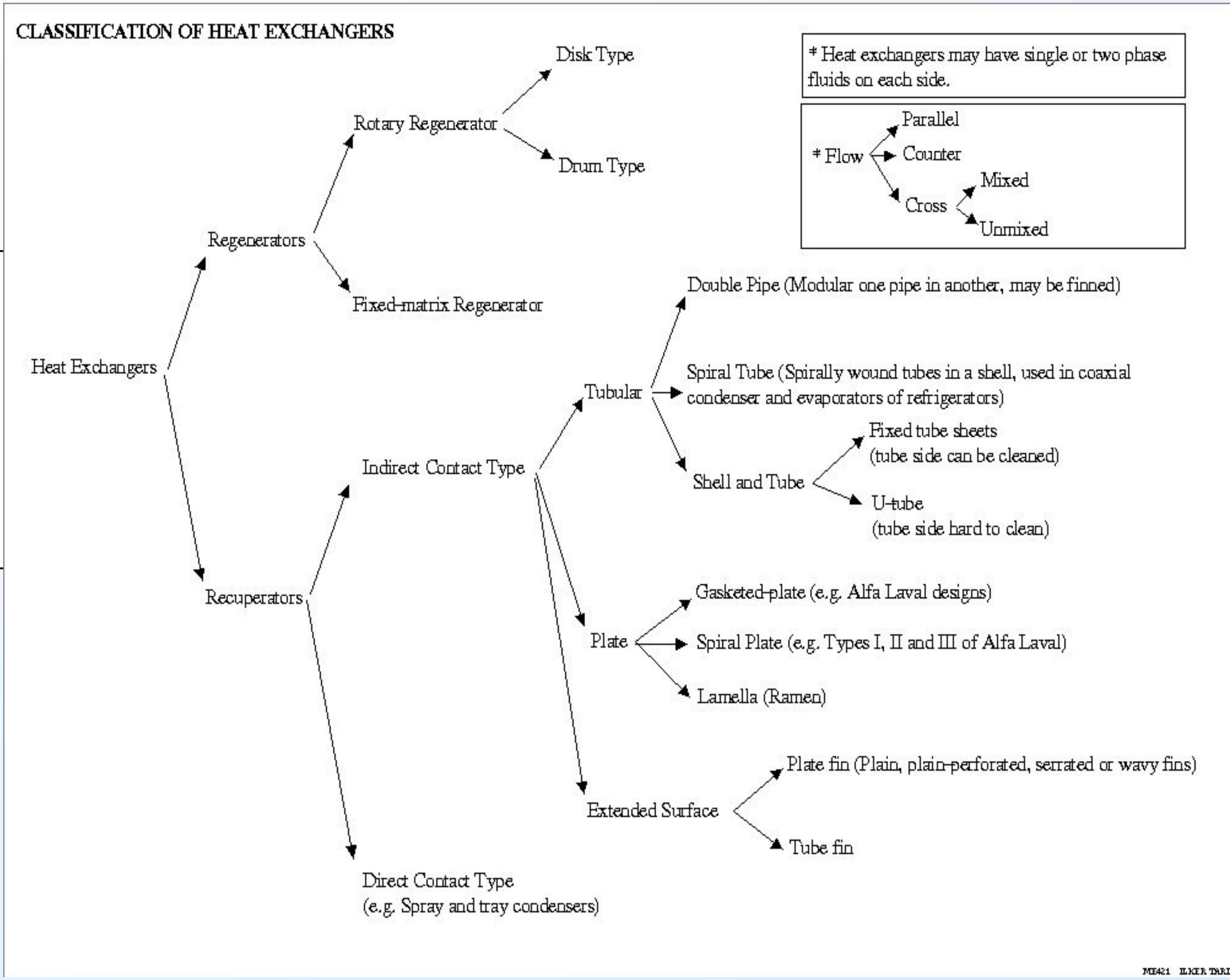


熱交換器的型式

回覆式

Recuperator

→ 最普遍型式



CLASSIFICATION OF HEAT EXCHANGERS

- Heat Exchangers
  - Regenerators
    - Rotary Regenerator
      - Disk Type
      - Drum Type
    - Fixed-matrix Regenerator
  - Recuperators
    - Indirect Contact Type
      - Tubular
        - Double Pipe (Modular one pipe in another, may be finned)
        - Spiral Tube (Spirally wound tubes in a shell, used in coaxial condenser and evaporators of refrigerators)
        - Shell and Tube
          - Fixed tube sheets (tube side can be cleaned)
          - U-tube (tube side hard to clean)
      - Plate
        - Gasketed-plate (e.g. Alfa Laval designs)
        - Spiral Plate (e.g. Types I, II and III of Alfa Laval)
        - Lamella (Ramen)
      - Extended Surface
        - Plate fin (Plain, plain-perforated, serrated or wavy fins)
        - Tube fin
    - Direct Contact Type (e.g. Spray and tray condensers)

* Heat exchangers may have single or two phase fluids on each side.

- * Flow
  - Parallel
  - Counter
  - Cross
    - Mixed
    - Unmixed

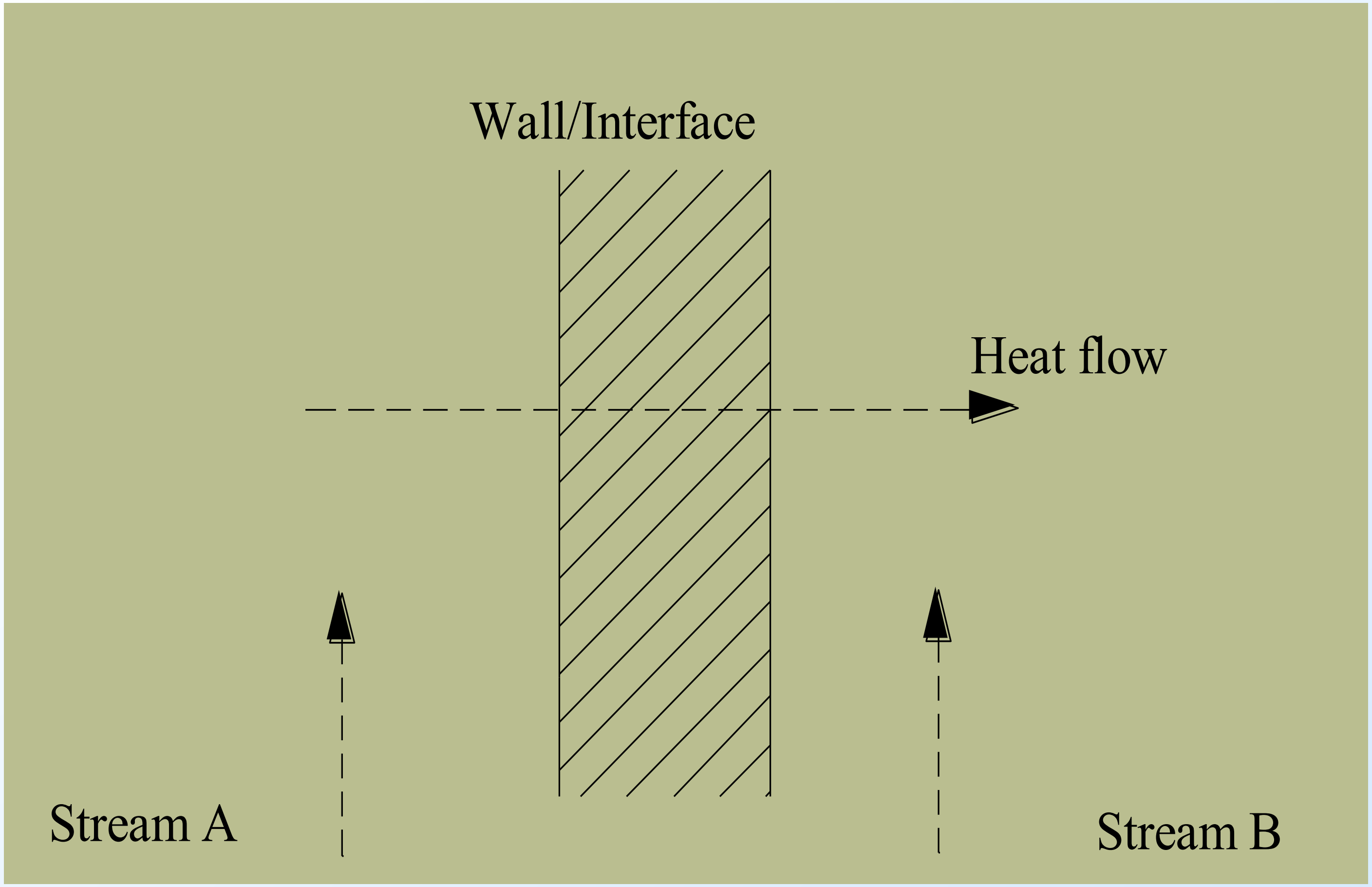
ME421 ILKER TARI

再生式

Regenerator

直接接觸式

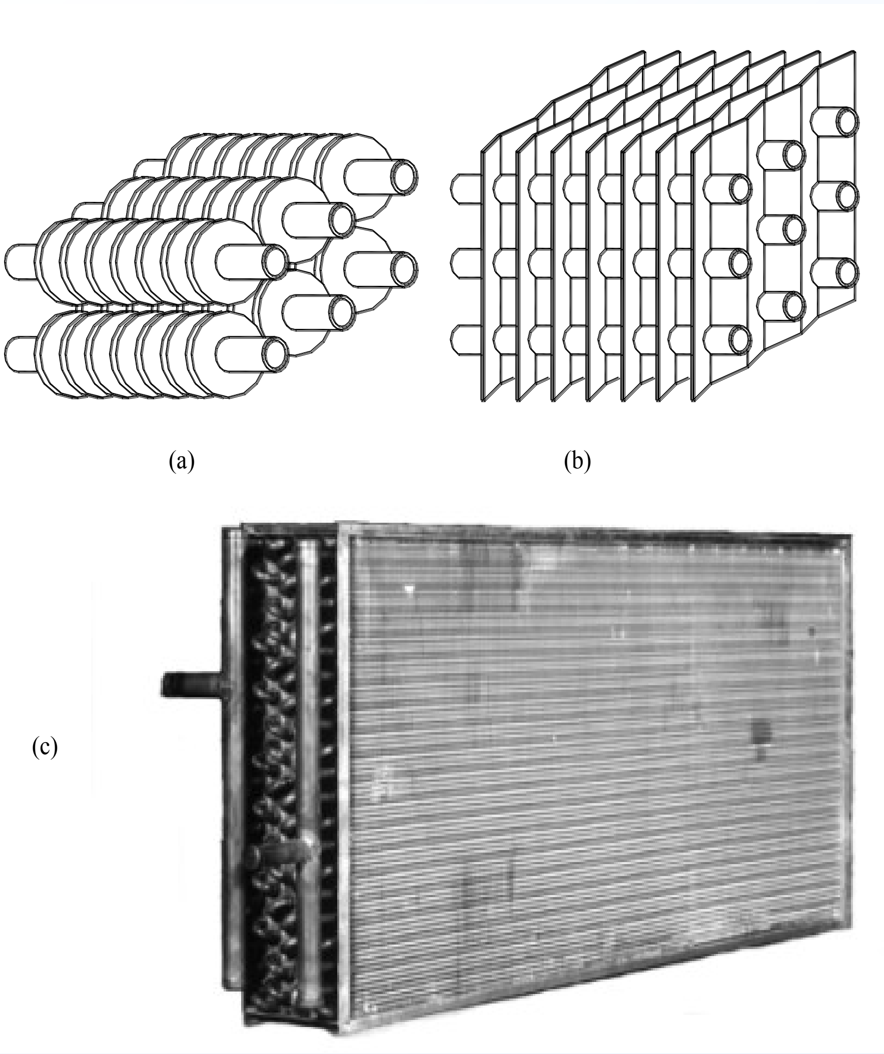
Direct Contact Heat Exchanger

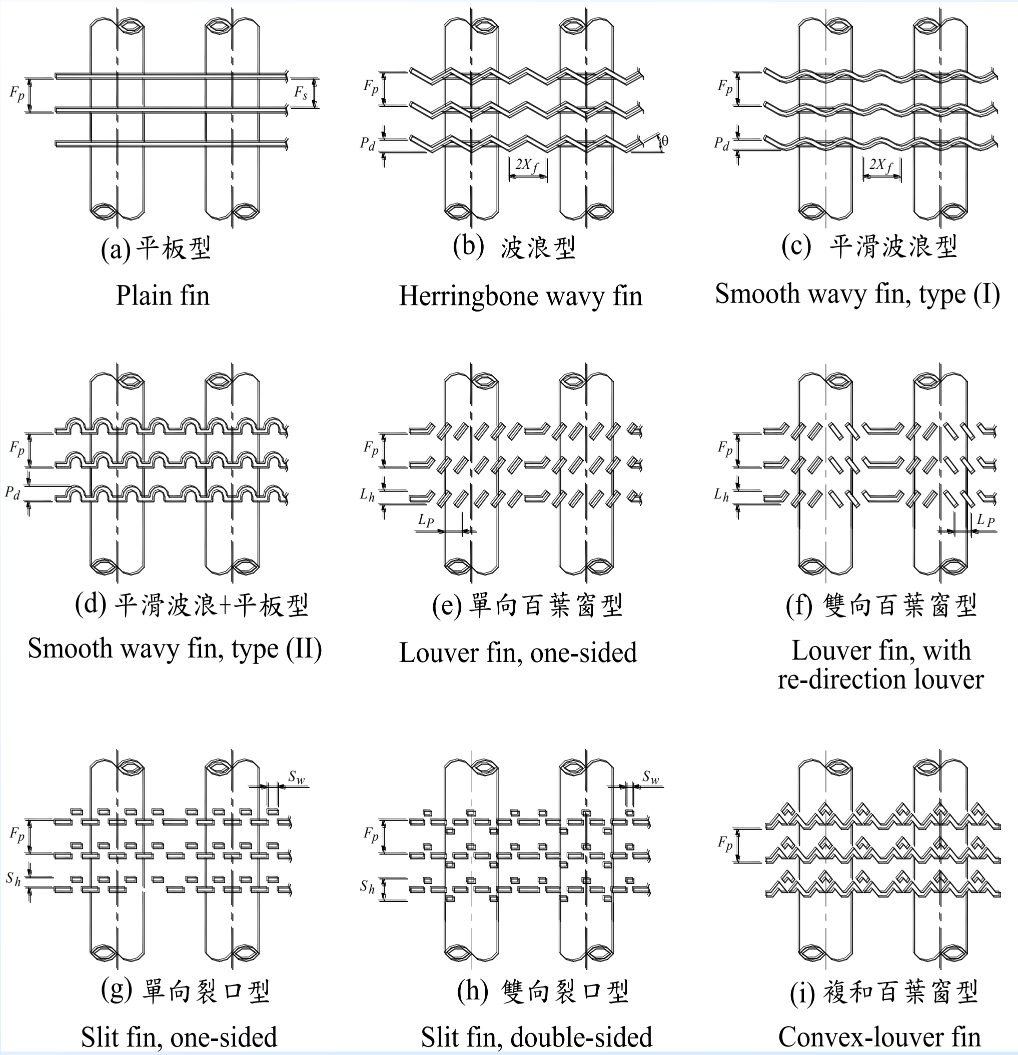


熱交換工作原理

典型 Recuperator

- 鰭管式 (Fin-and-tube Heat Exchanger)
- 殼管式 (Shell and Tube Heat Exchanger)
- 板式 (Plate Heat Exchanger)
- 螺旋式 (Spiral Heat Exchanger)
- 套管式 (Tube in Tube Heat Exchanger)
- Cross flow Heat Exchanger





(a) 平板型 Plain fin

(b) 波浪型 Herringbone wavy fin

(c) 平滑波浪型 Smooth wavy fin, type (I)

(d) 平滑波浪+平板型 Smooth wavy fin, type (II)

(e) 單向百葉窗型 Louver fin, one-sided

(f) 雙向百葉窗型 Louver fin, with re-direction louver

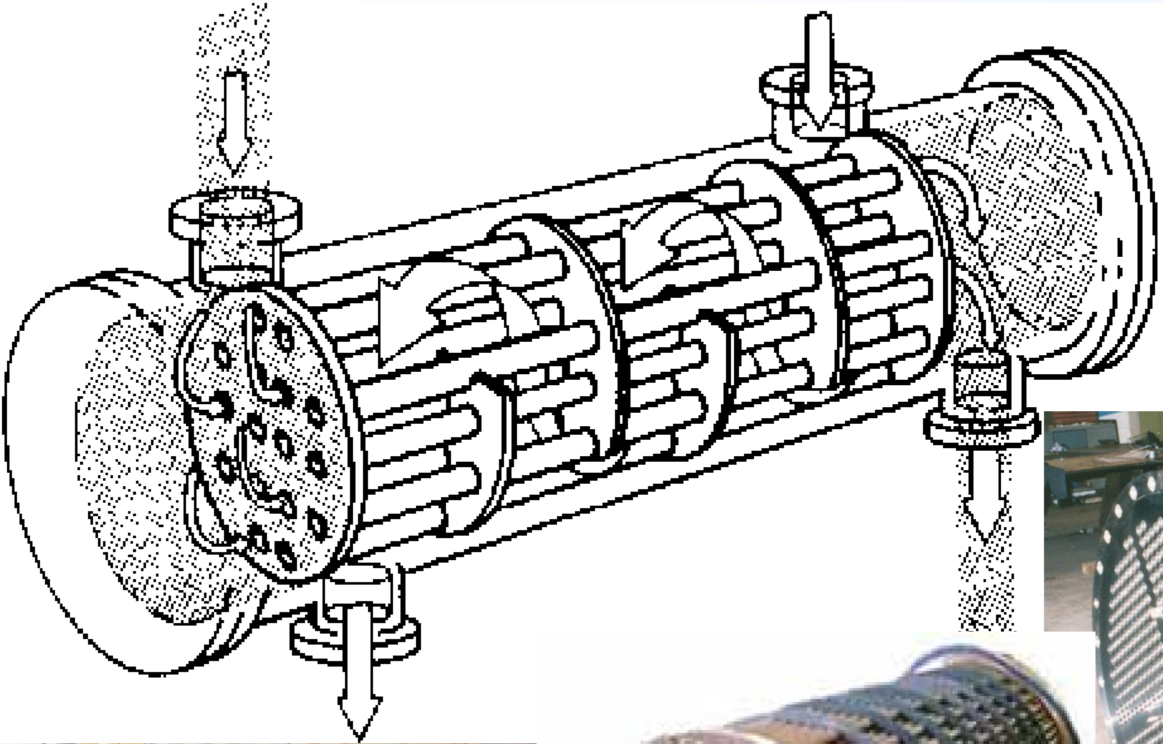
(g) 單向裂口型 Slit fin, one-sided

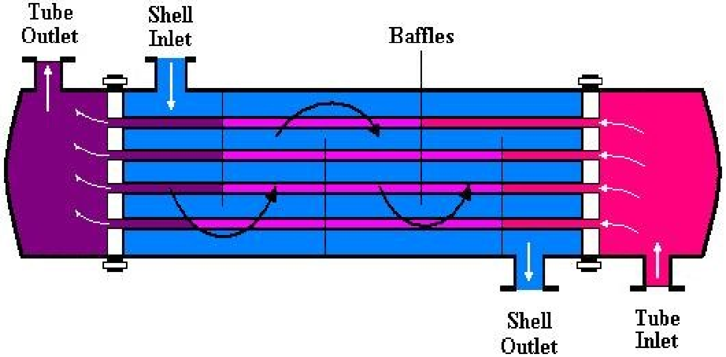
(h) 雙向裂口型 Slit fin, double-sided

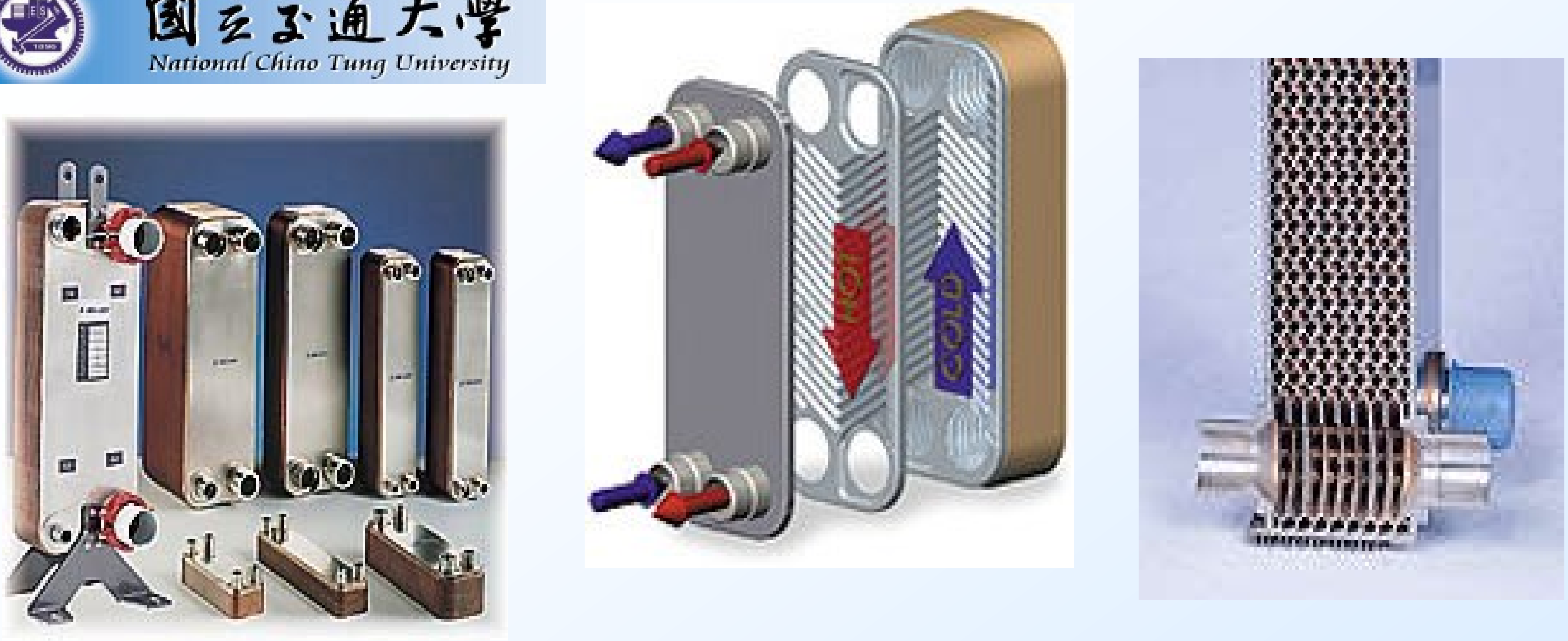
(i) 複和百葉窗型 Convex-louver fin

Typical fin-and-tube heat exchanger (a) individual circular fin; (b) continuous wavy fin; (c) conventional air-cooled heat exchanger

殼管式 Shell and Tube Heat Exchanger



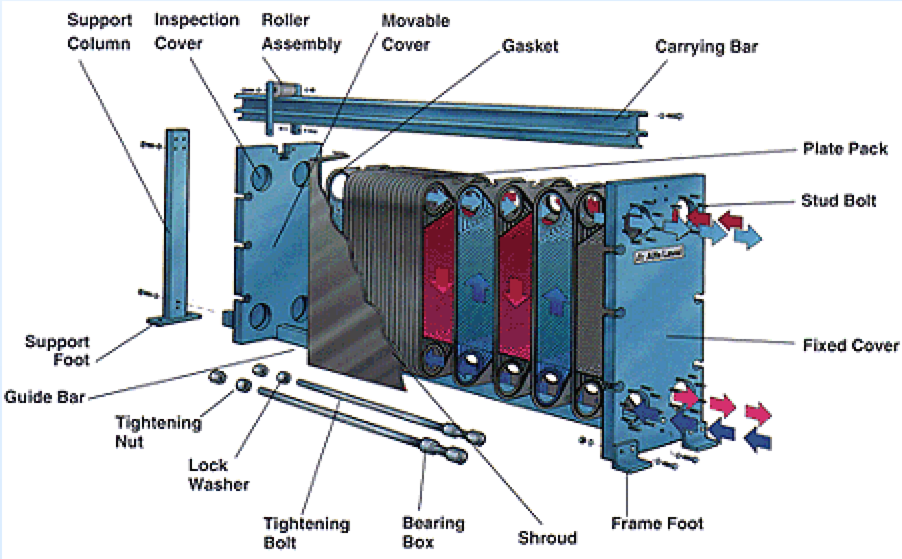




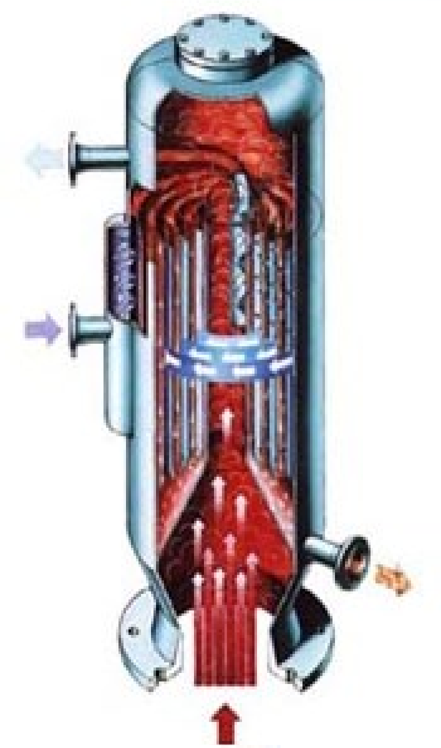
(a) (b) (c)

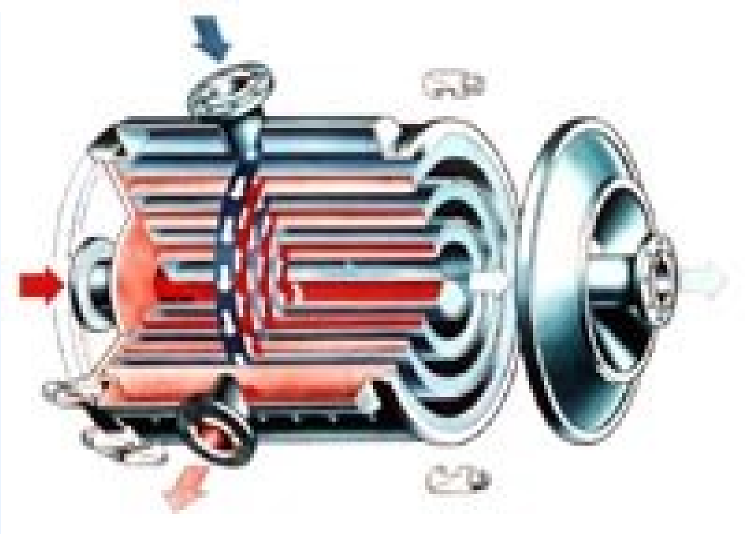
板式熱交換器 (a) brazed type; (b) working principle; (c) cross section

可拆卸式板式熱交換器

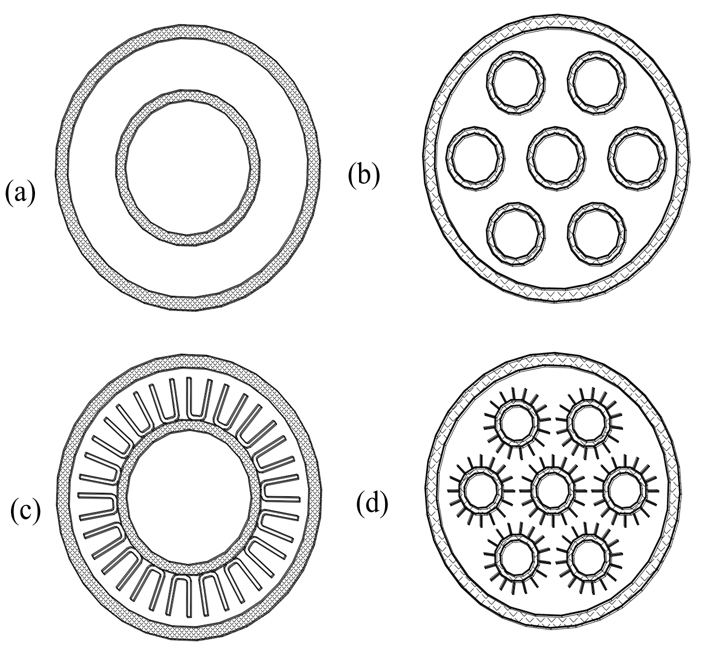


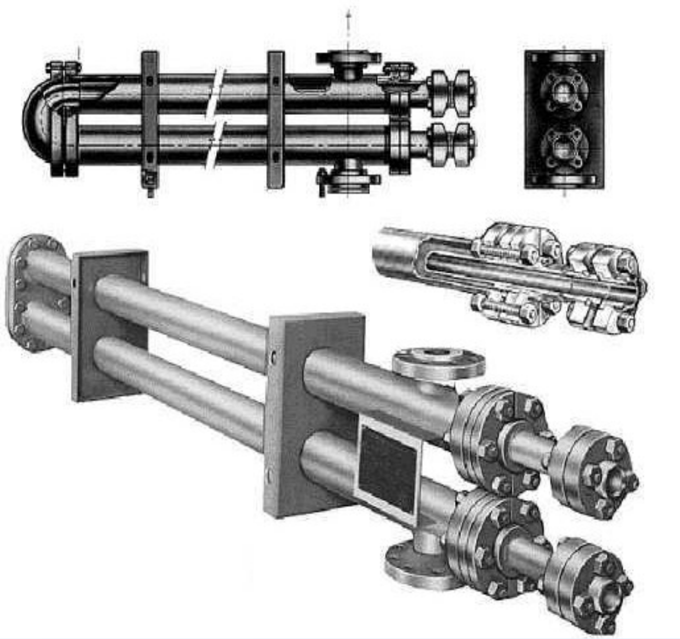
Spiral Heat Exchanger

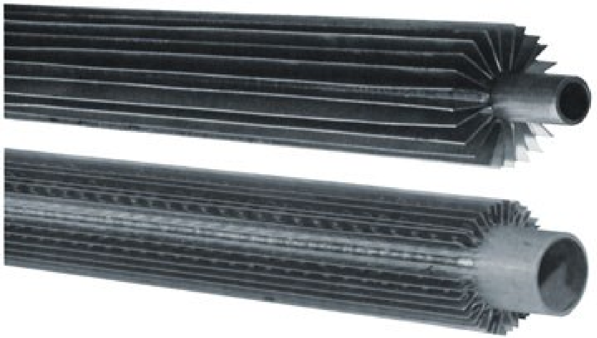


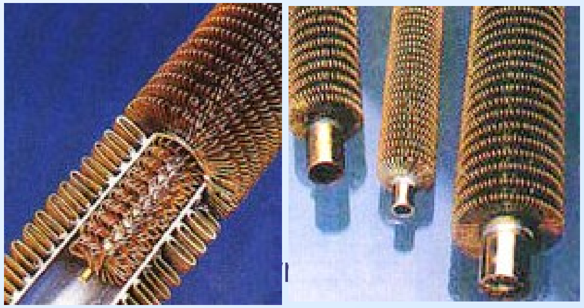


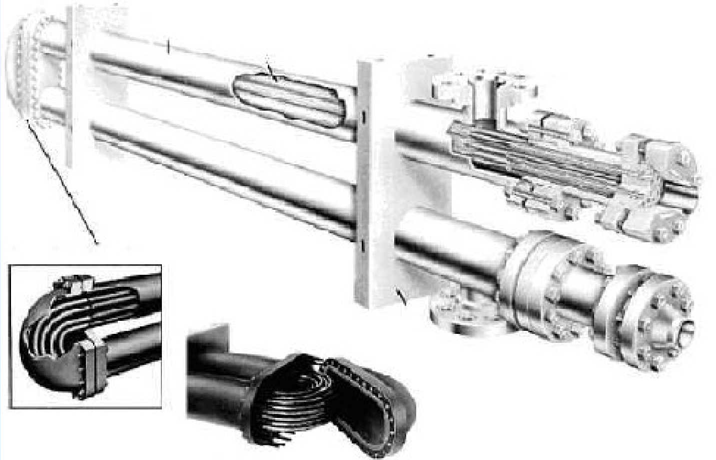
套管式 (Tube-in-Tube Heat Exchanger)



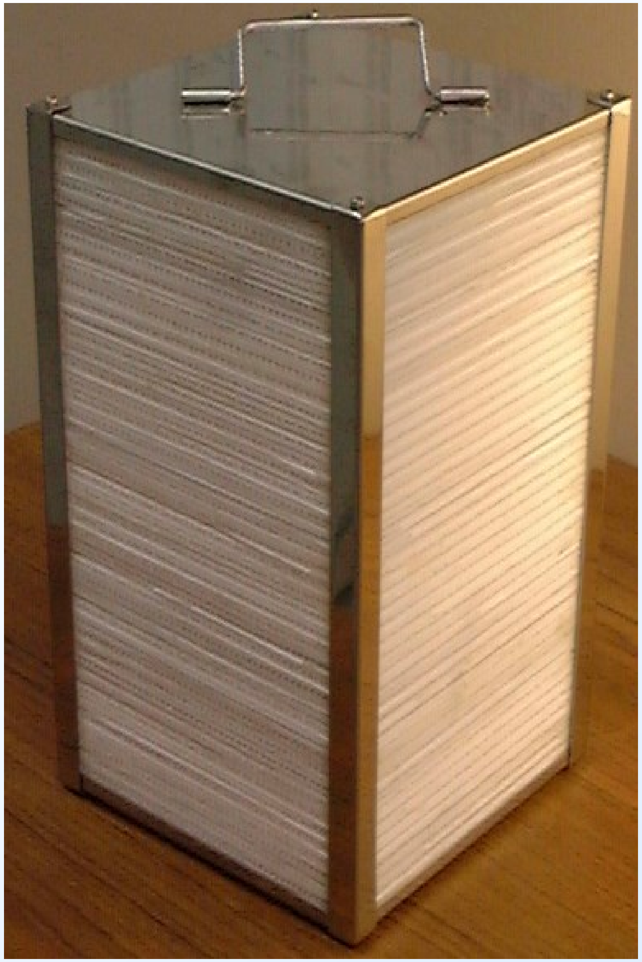


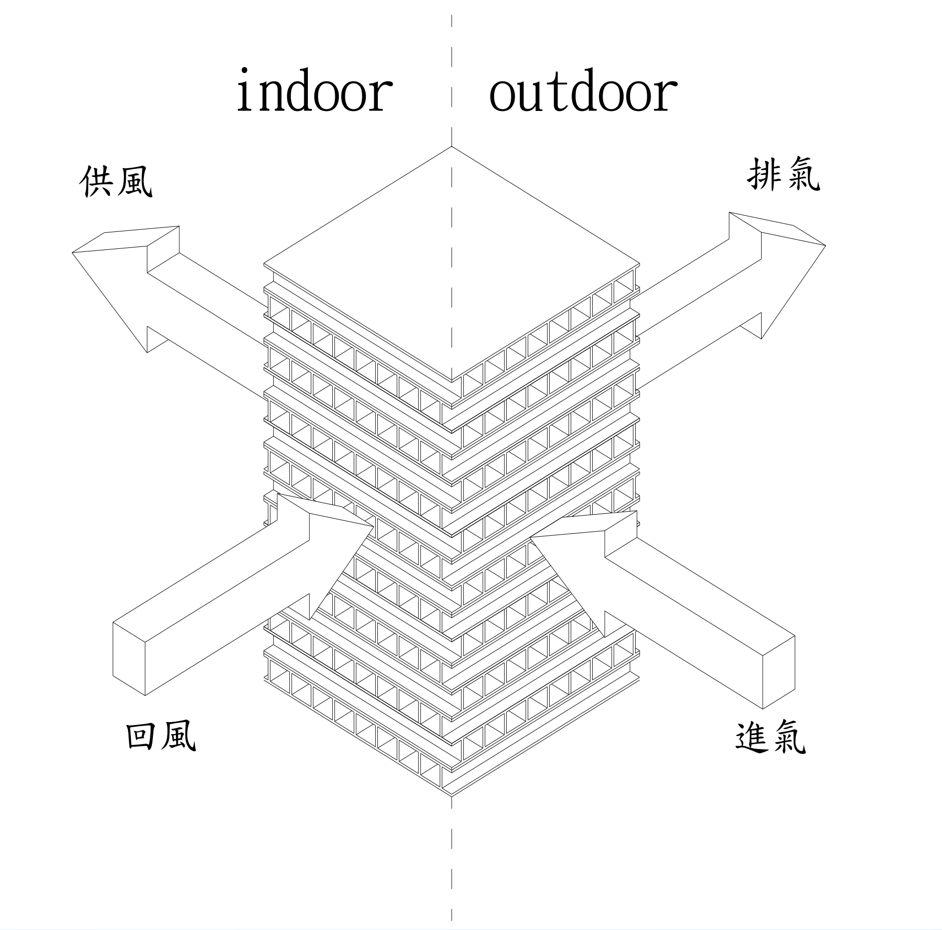


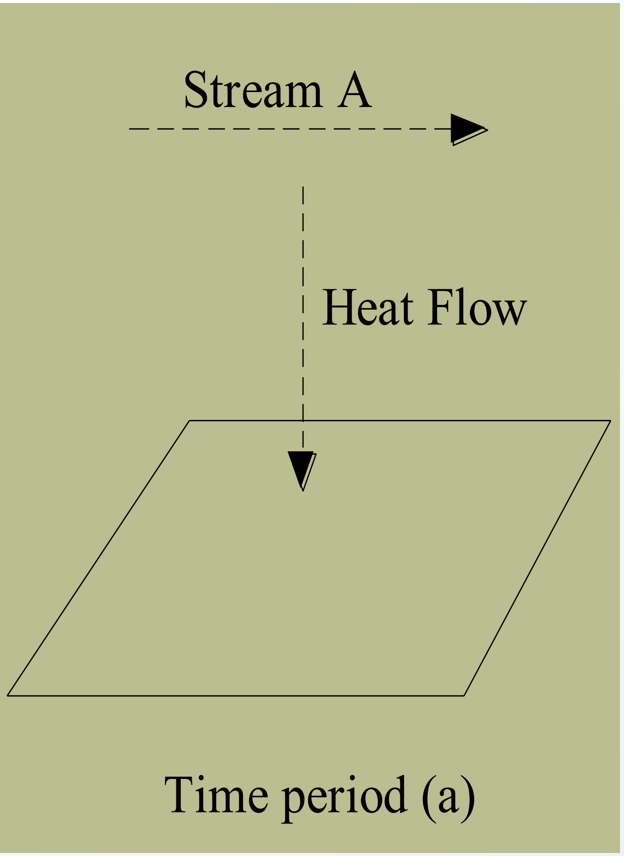


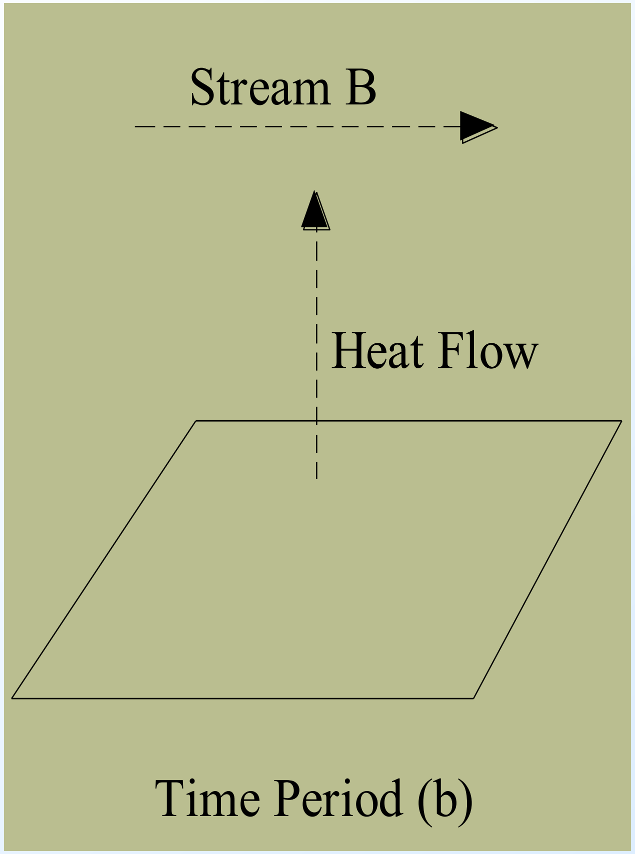


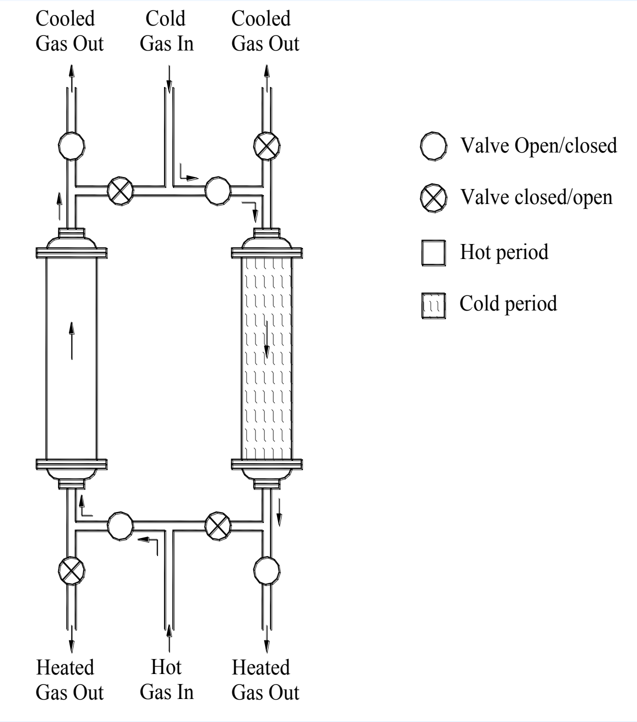
氣對氣交流式熱交換器
Air-to-air Cross Flow Heat Exchanger



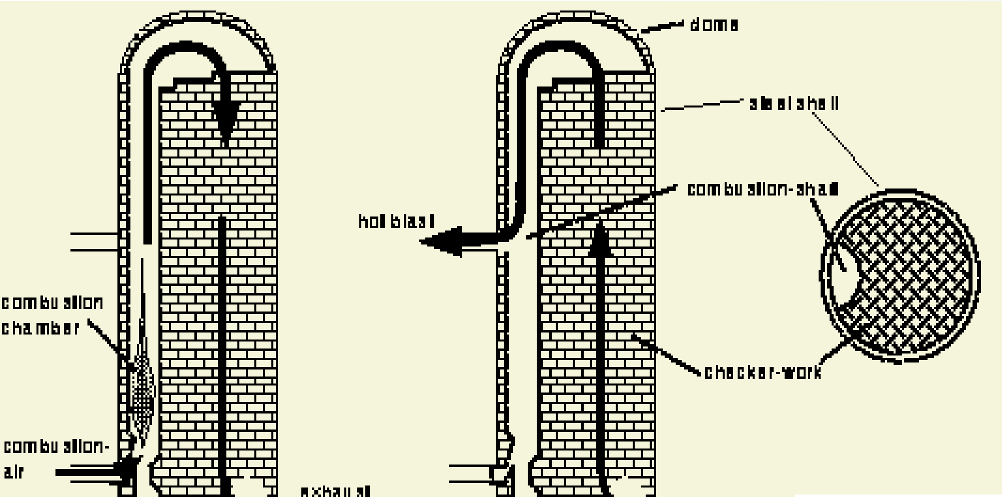






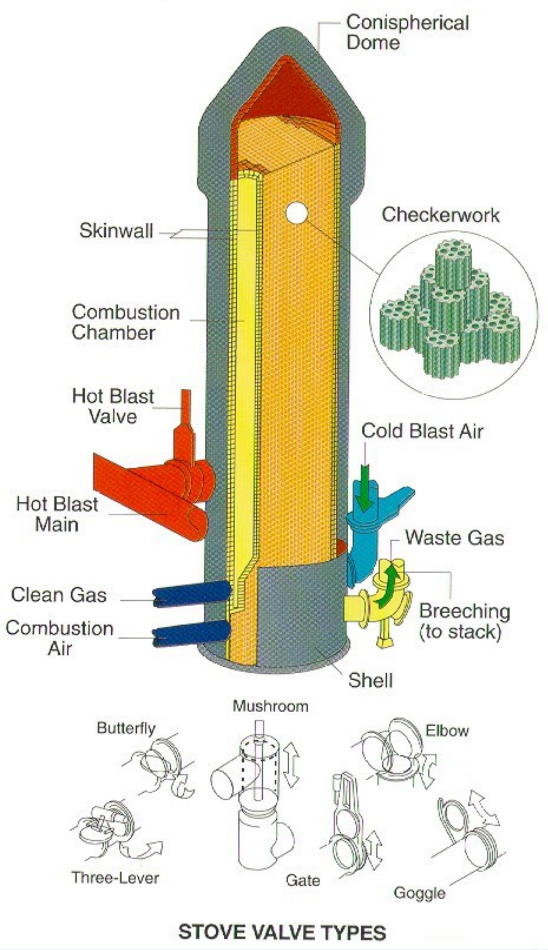


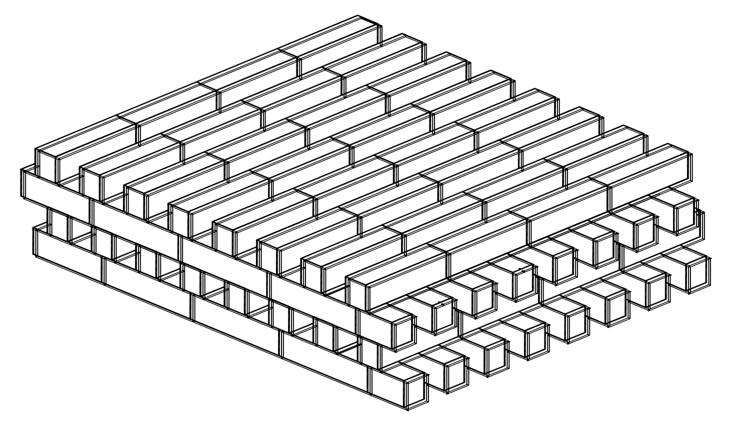
再生式熱交換器工作原理

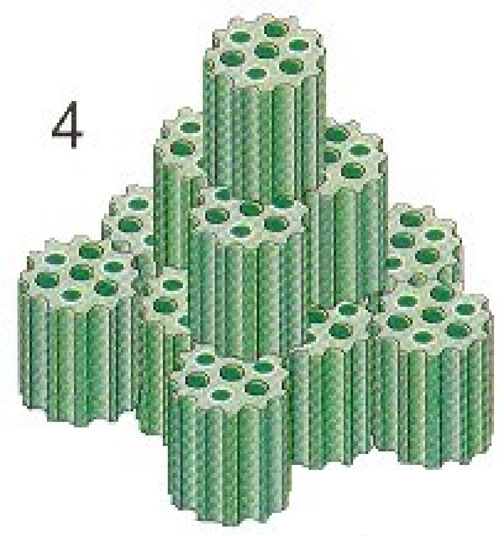


a) on-gas phase

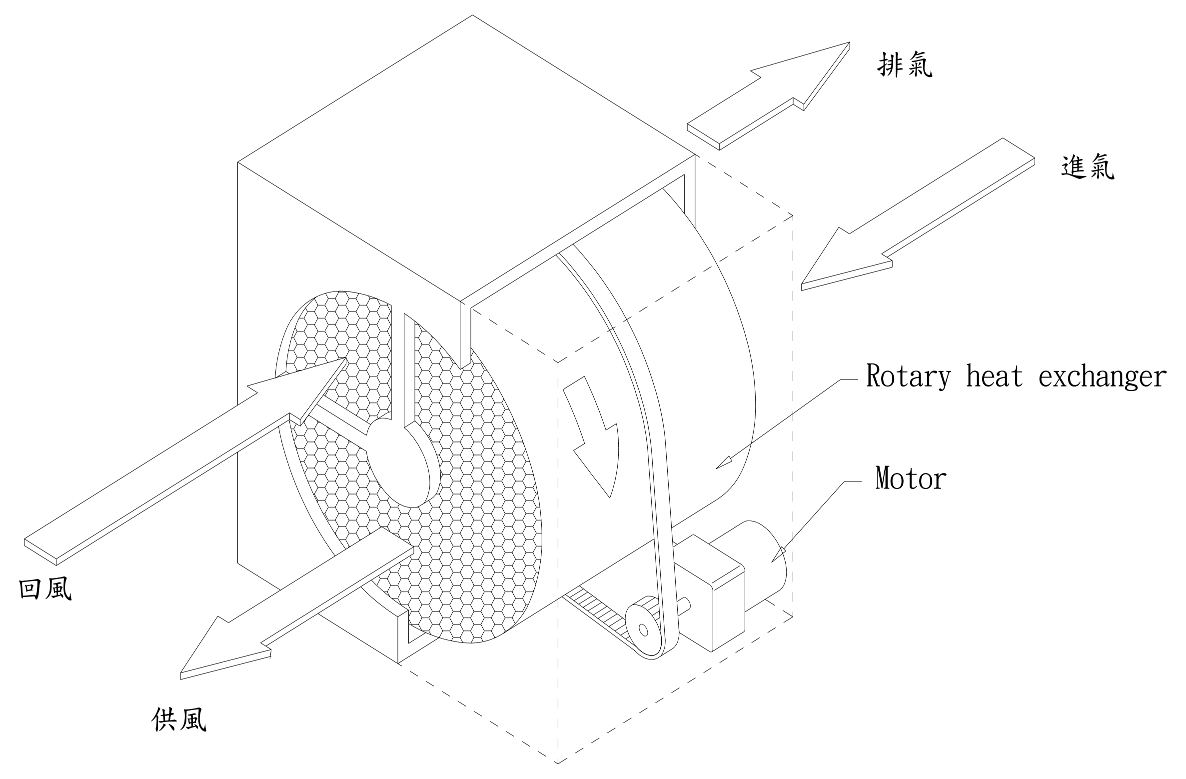
b) on-blast phase





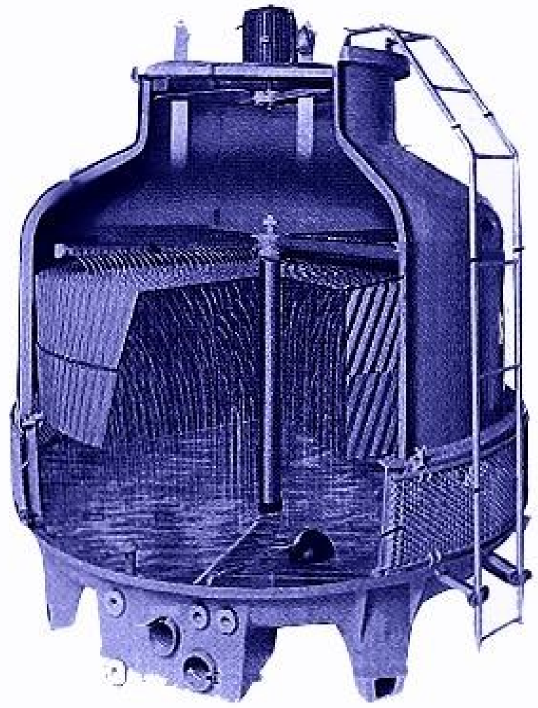


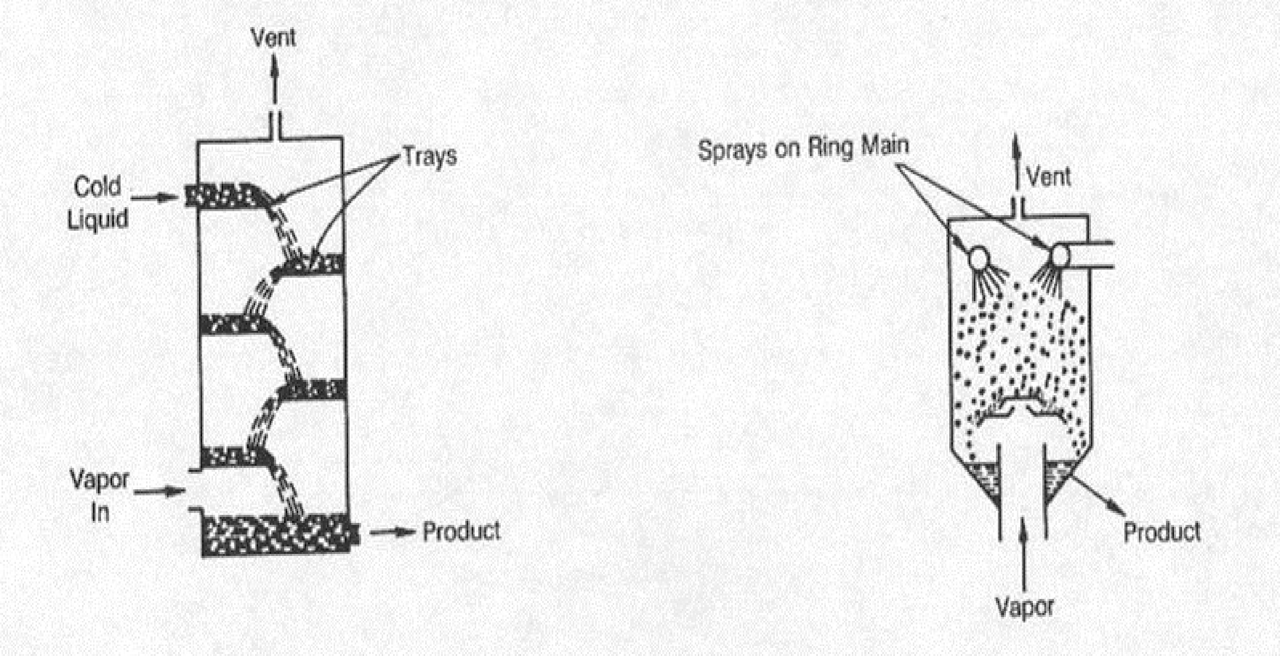
Hot blast stove Fixed Bed type regenerator



轉輪式再生式熱交換器 (a) Working Principle & (b) Rotary wheel

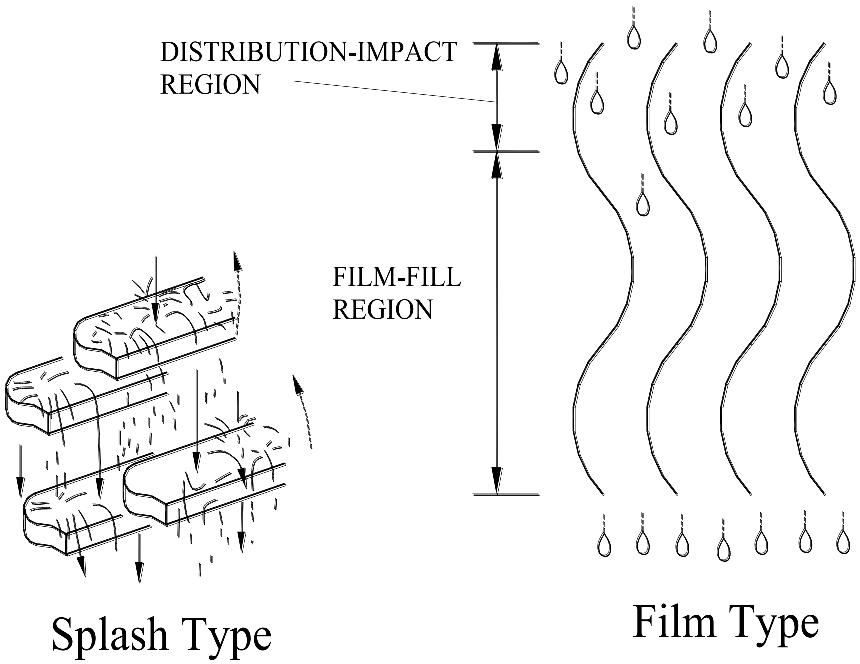
Cooling Tower

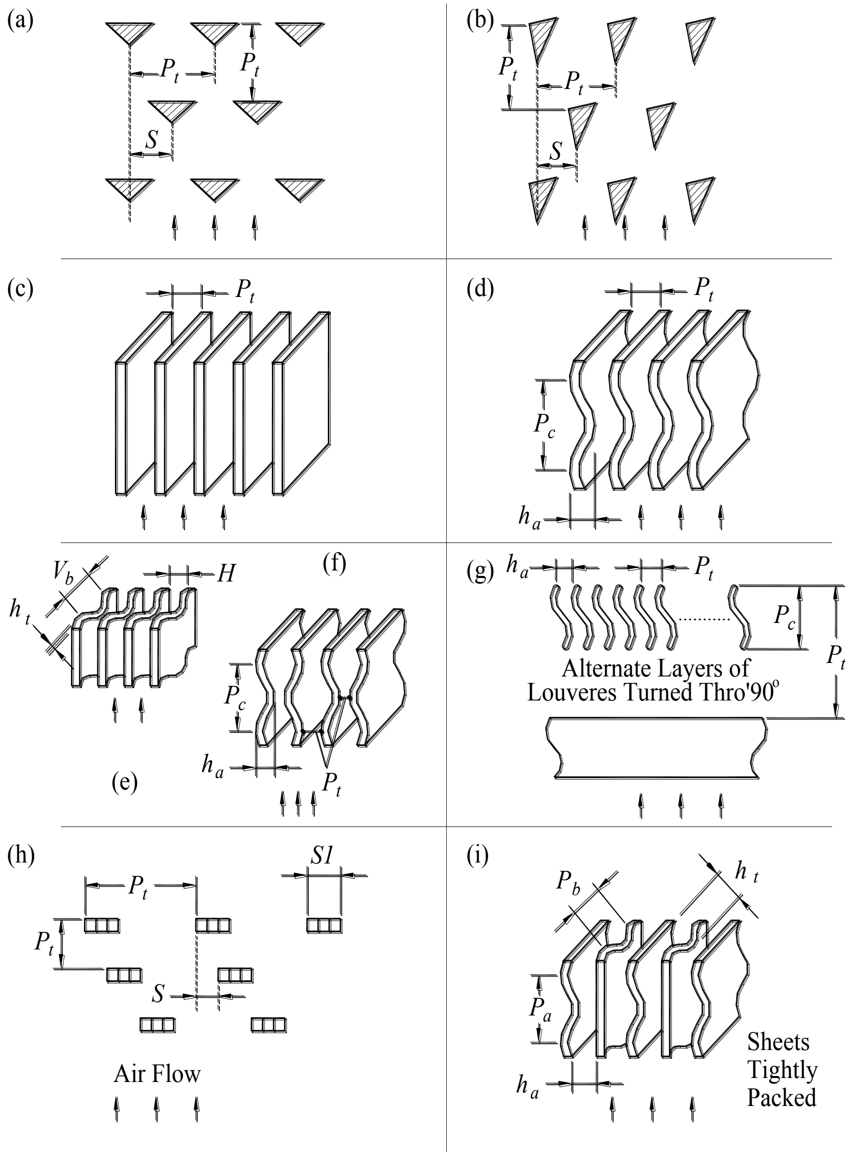




Tray-type direct HX & Spray condenser

典型填料形狀


DISTRIBUTION-IMPACT REGION
FILM-FILL REGION
Splash Type
Film Type


(a)
P_t
P_t
S
(b)
P_t
P_t
S
(c)
P_t
(d)
P_t
P_c
h_a
V_b
H
h_t
(f)
P_c
h_a
P_t
(e)
(g)
h_a
P_t
P_c
P_t
Alternate Layers of Louveres Turned Thro'90°
(h)
P_t
P_t
S
$S1$
Air Flow
(i)
P_b
h_t
P_a
h_a
Sheets Tightly Packed

熱交換器原理介紹(Cont.)

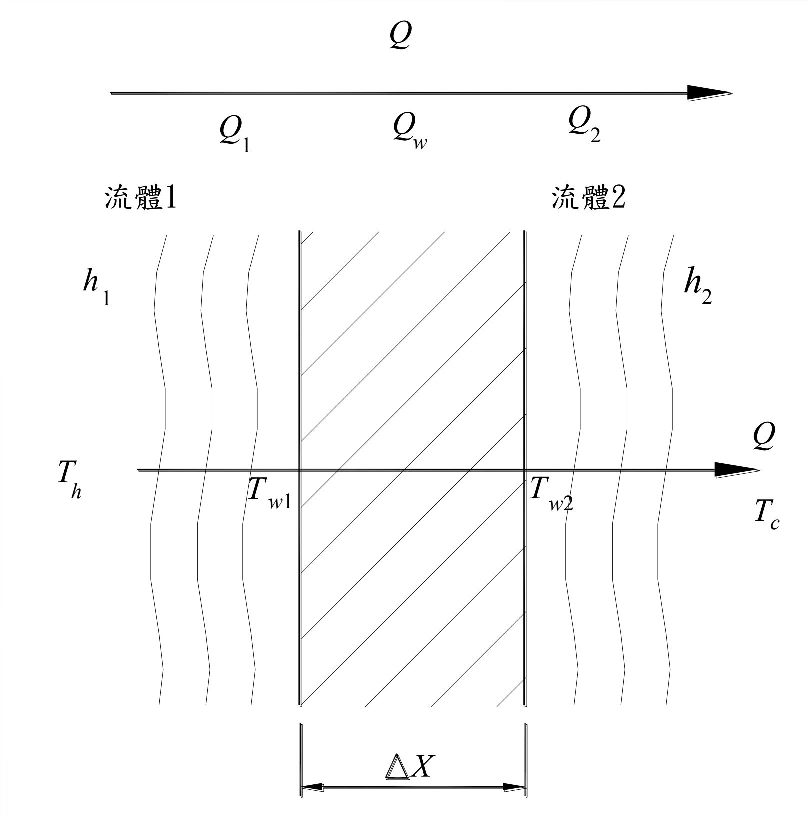
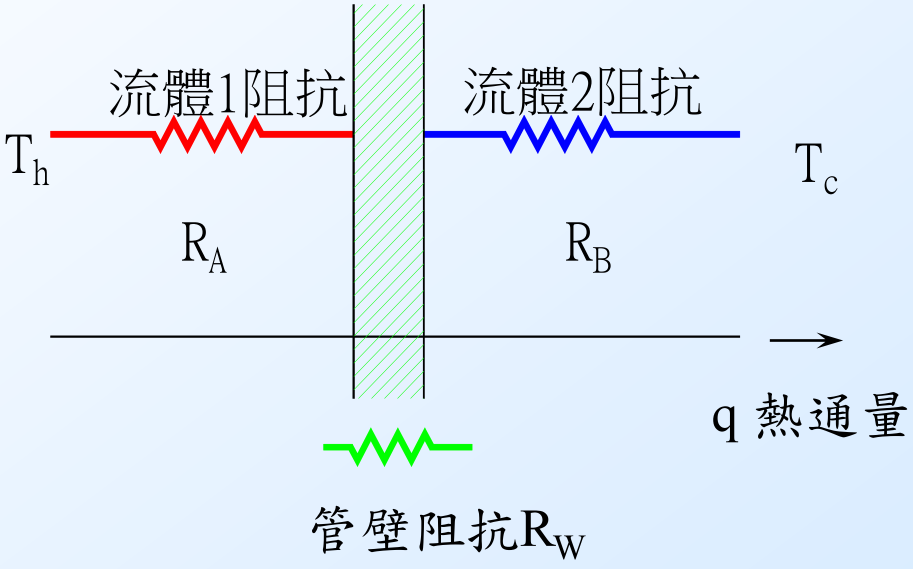


Fig.熱交換器兩側熱對流熱傳示意圖



總阻抗：$R=R_A+R_W+R_B$

熱通量 $q=(T_h-T_c)/R$

$Q=q\cdot A$ ⇒ 總熱傳量

因此若能確切掌握流體阻抗，即能精確設計熱交換器

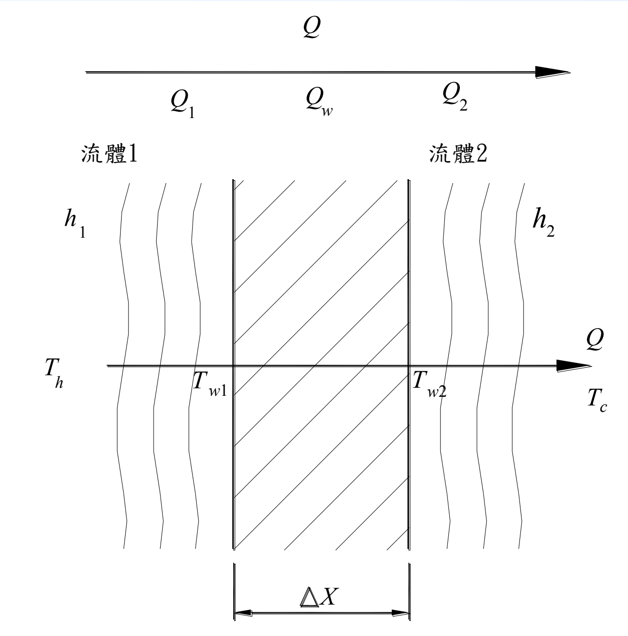
熱交換器原理介紹(Cont.)

在穩定狀態(steady state)下熱傳量是平衡的，故$Q_1 = Q_w = Q_2 = Q$

$$\dot{Q} = h_1 A_{w1}\left(T_h - T_{w,1}\right) \Rightarrow \frac{\dot{Q}}{h_1 A_{w1}} = T_h - T_{w,1}$$

$$\dot{Q} = kA\frac{\left(T_{w,1} - T_{w,2}\right)}{\Delta X} \Rightarrow \frac{\dot{Q}}{\frac{kA}{\Delta X}} = T_{w,1} - T_{w,2}$$

$$\dot{Q} = h_2 A_{w2}\left(T_{w,2} - T_c\right) \Rightarrow \frac{\dot{Q}}{h_2 A_{w2}} = T_{w,2} - T_c$$



三式相加，得

$$\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)\dot{Q} = T_h - T_c$$

$$\Rightarrow \dot{Q} = \frac{1}{\underbrace{\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)}_{UA}}(T_h - T_c)$$

熱交換器原理介紹(Cont.)

熱傳量Q表示成：

$$\therefore \dot{Q} = \frac{1}{\underbrace{\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)}_{UA}}(T_h - T_c)$$

$$\Rightarrow \dot{Q} = UA\Delta T_m$$

$$\boxed{\left(\dot{Q} = \frac{\Delta T_m}{\frac{1}{UA}} = \frac{\Delta T_m}{R_t}\right)}$$

其中 U一般習稱為總熱傳係數(單位： W/m^2.K)，A為參考面積

$$\therefore \underbrace{\frac{1}{UA}}_{\text{總阻抗}} = \underbrace{\frac{1}{h_1 A_{w1}}}_{\text{熱側阻抗}} + \underbrace{\frac{\Delta X}{kA}}_{\text{管壁阻抗}} + \underbrace{\frac{1}{h_2 A_{w2}}}_{\text{冷側阻抗}} \qquad \Rightarrow \mathrm{R_t = R_1 + R_w + R_2}$$

$$\boxed{U_o A_o = U_i A_i = \frac{1}{R_t} = \frac{1}{\underbrace{\frac{1}{h_i A_i}}_{\text{冷媒側阻抗}} + \underbrace{\frac{X_p}{k_p A_p}}_{\text{管壁阻抗}} + \underbrace{\frac{1}{h_o A_o}}_{\text{空氣側阻抗}}}}$$

管壁組抗

$$\begin{cases} R_w = \dfrac{X_p}{k_p A_p} & \text{For plane wall} \\[2ex] R_w = \dfrac{\ln(r_o / r_i)}{2\pi k_p L} & \text{For tube wall} \end{cases}$$

熱交換器原理介紹(Cont.)

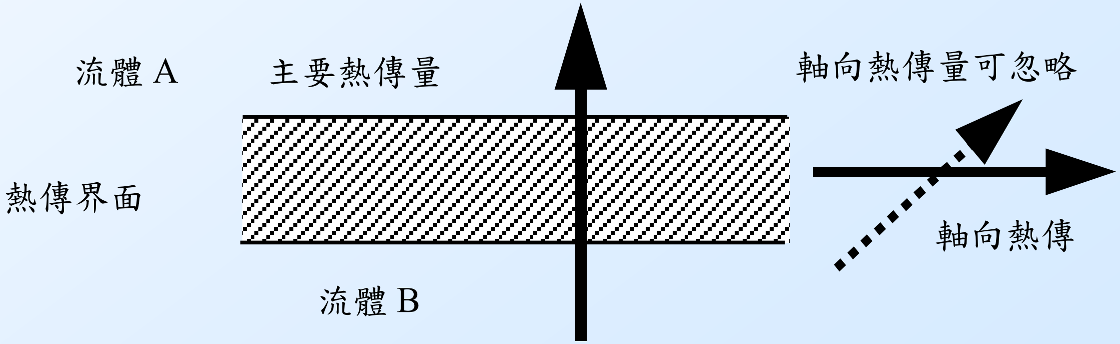
一些常見流體的總熱傳係數(overall heat transfer coefficient)

FLUID COMBINATION	*U(W/m^2.K)*
Water to water	850-1700
Water to oil	110-350
Steam condenser (water in tubes)	1000-6000
Ammonia condenser (water in tubes)	800-1400
Alcohol condenser (water in tubes)	250-700
Finned – tube heat exchanger (water in tubes, air in cross flow)	25-50

熱交換器性能計算方法

□ 介紹兩種熱交換器熱傳設計基本方法，即*UA-LMTD-F* 與*ε-NTU* 方法，這兩種設計方法作如下的一些基本的假設：

1. 熱交換過程是在一穩定狀態下進行（即與時間的改變無關）。
2. 溫度、速度為一維分佈 (one-dimensional)，且流體無分層的現象。
3. 流體的比熱為定值，且在操作時間內，整個熱交換器內部的比熱值(Cp)不變。
4. 總熱傳係數 U 值為固定。
5. 熱交換器的軸向熱傳可忽略。
6. 熱量損失到外界的部分可予以忽略。
7. 熱交換器內並無產生熱量的裝置或反應(例如裝加熱器或化學反應等) 。

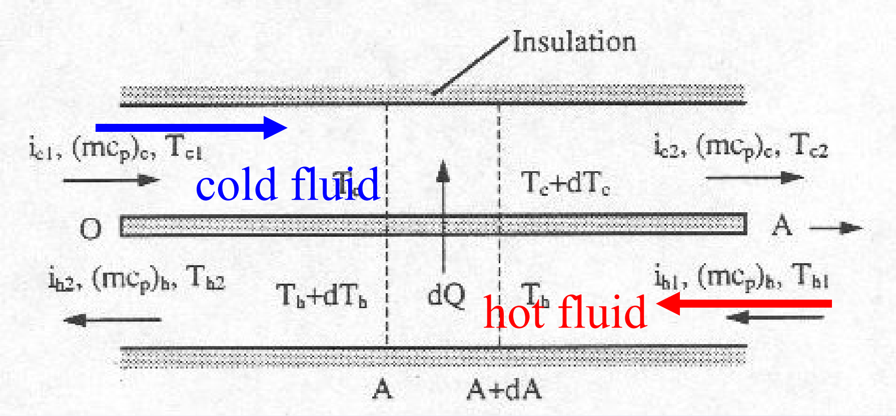


熱傳基本模式介紹(Cont.)

能量平衡關係：

- 冷側或熱側的熱傳量

$$\dot{Q} = \dot{m}(i_2 - i_1)$$



- 冷側與熱側之間的能量平衡

$$\dot{Q} = \dot{m}_h(i_{h1} - i_{h2}) = \dot{m}_c(i_{c2} - i_{c1})$$

$$\dot{Q} = (\dot{m}C_{p,c})_c(T_{c,o} - T_{c,i}) = (\dot{m}C_{p,h})_h(T_{h,i} - T_{h,o})$$

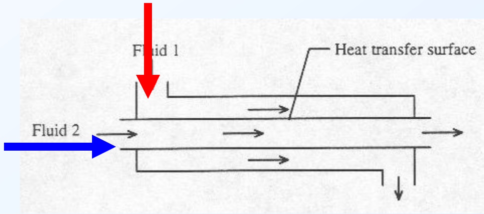
其中 $C_h = (\dot{m}c_{p,h})_h$ and $C_c = (\dot{m}c_{c,h})_c$

常用的熱交換器符號及其單位

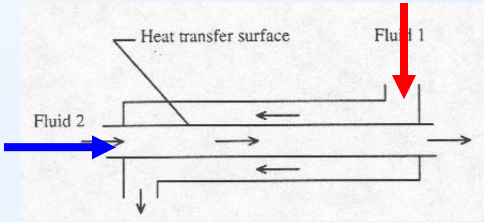
符號	說明	單位
A	熱交換面積	m^2
C_c	冷側流體熱容量流率$(\dot{m}c_p)_c$	W/K
C_h	熱側流體熱容量流率$(\dot{m}c_p)_h$	W/K
Q	熱傳量	W
Q_{max}	最大熱傳量	W
ε	有效度 (Q/Q_{max})	無因次
$\dot{m}$	質量流率	kg/s
c_p	比熱	J/kg·K
q	熱通量 $= Q/A$	$W/m^2 \cdot K$
C_{min}	C_c 與 C_h 中較小者	W/K
U	總熱傳係數	$W/m^2 \cdot K$
NTU	傳遞單位，UA/C_{min}	無因次
C^*	C_{min}/C_{max}	無因次
C_{max}	C_c 與 C_h 中較大者	W/K
$T_{h,i}$	熱側流體進口溫度	°C，K
$T_{h,o}$	熱側流體出口溫度	°C，K
$T_{c,i}$	冷側流體進口溫度	°C，K
$T_{c,o}$	冷側流體出口溫度	°C，K
F	校正係數	無因次
P	溫度有效度，$(T_{c,o}-T_{c,i})/(T_{h,i}-T_{c,i})$	無因次
R	C_c/C_h，$(T_{h,i} - T_{h,,o})/(T_{c,o} - T_{c,i})$	無因次
$LMTD$	對數平均溫度差	°C，K
Δ_o	熱交換器冷熱進口之最大溫差，$(T_{h,i} - T_{c,i})$	°C，K

熱交換器之流體流動型態

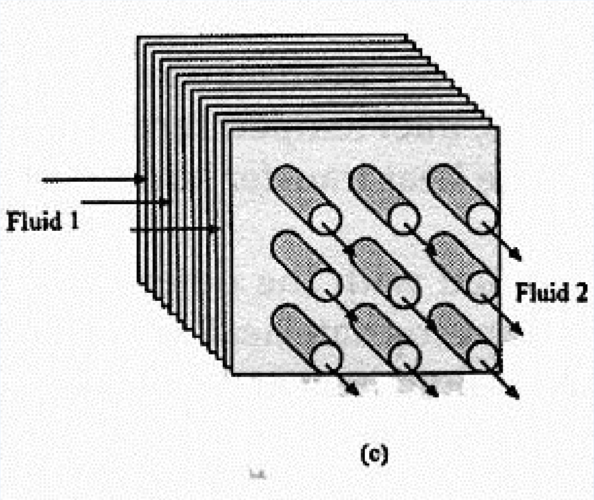
(1)平行流(parallel flow)



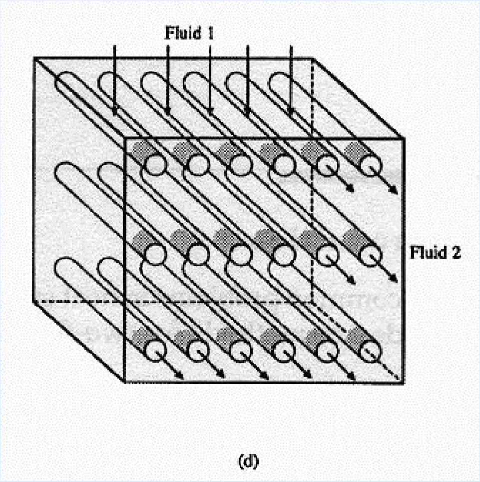
(2)逆向流(counter flow)



(3)交叉流(cross flow)



(a) fin with both fluid unmixed

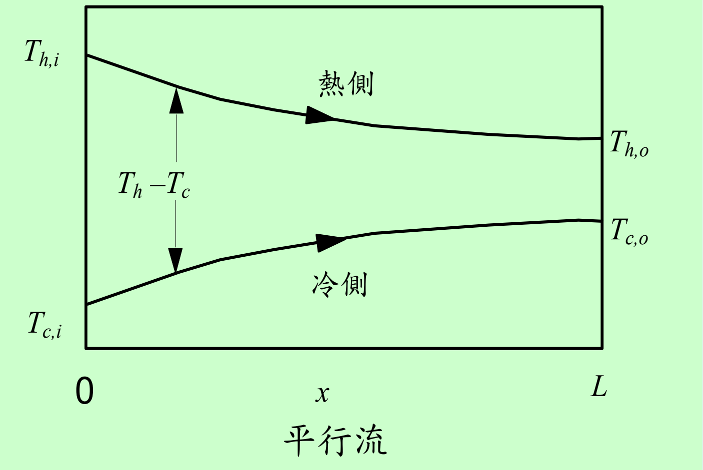


(b) unfin with one fluid mixed and the other unmixed

熱交換器之流體流動型態(Cont.)

- 熱交換器的流體流動型態，在設計安排上大致可分成下列的方式：

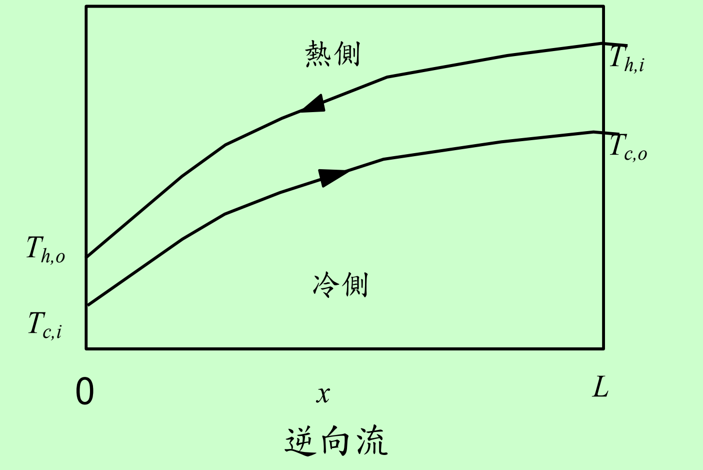
(1)平行流 (parallel flow)，此時的熱側與冷側進出口在同一處，典型的溫度變化如右圖所示：



平行流之流體流動方向示意圖

(2)逆向流 (counter flow)，此時熱側的進口與冷側的出口溫度在同一側，而熱側的出口與冷測的進口在同一側，典型的溫度變化如右圖所示：

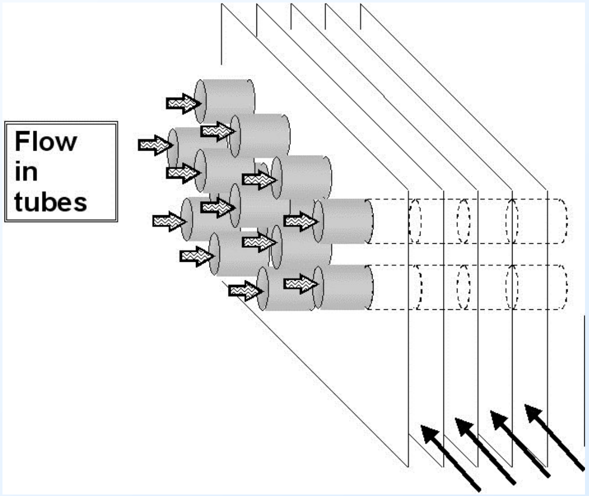
※逆向流冷側的出口溫度有可能高於熱側的出口溫度，但是平行流動下則不可能出現此一現象。



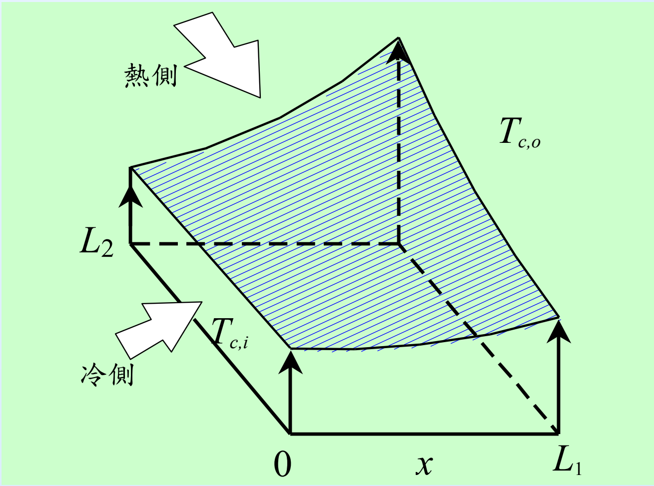
逆向流之流體流動方向示意圖

熱交換器之流體流動型態(Cont.)

(3)交錯流 (crossflow)，係熱側的進口與冷側的進口處剛好垂直，典型的溫度變化如所示，由右圖可知交錯流動下出口溫度呈現一較為複雜的變化。交錯流動時，有時候為了適度提升另一側的溫度差，管路的安排經常會由幾個回數 (pass)所組成，它的另一個優點就是可將熱交換器較為緊密地安排，一些常見的熱交換器回數安排如後圖說明。



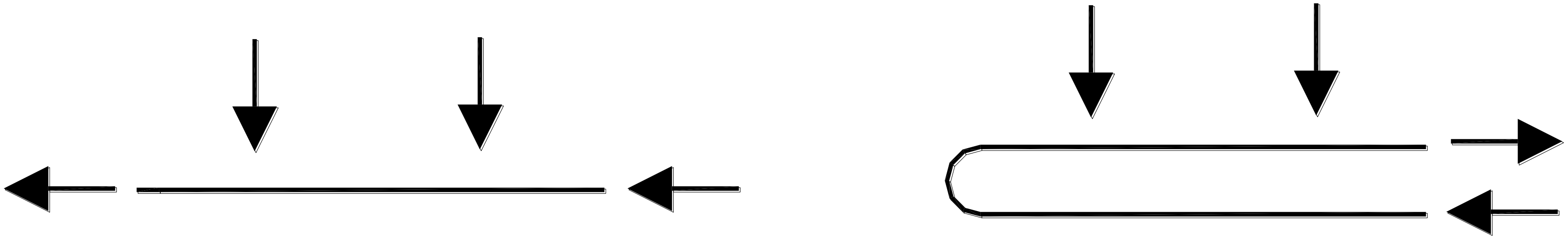
交錯流之流體流動方向示意圖



交錯流之溫度變化示意圖

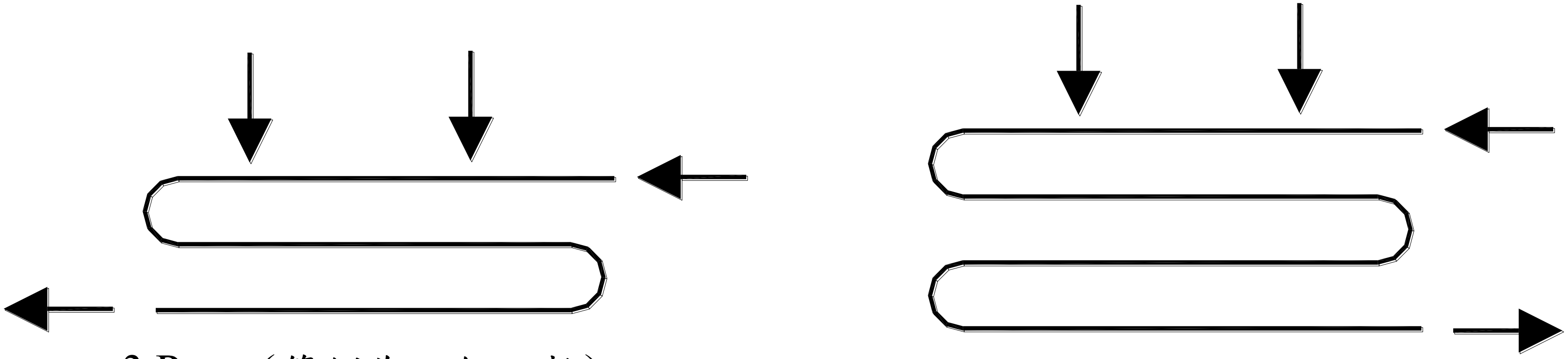
(4)混合流動方式 (mixed flow)，此時的流動型態係上述三者流動方式的綜合。

熱交換器之流體流動型態(Cont.)



1 Pass（管側為一個回數）

2 Pass（管側為兩個回數）



3 Pass（管側為三個回數）

4 Pass（管側為四個回數）

熱交換器的回數安排示意圖

熱交換器之流體流動型態(Cont.)

❑上述流動型式在熱交換器設計上，所代表的意義可由 $\dot{Q}=UA\Delta T_m$ 來說明，即由於增加熱交換性能的方法不外乎：

(1) 增加總熱傳係數 U

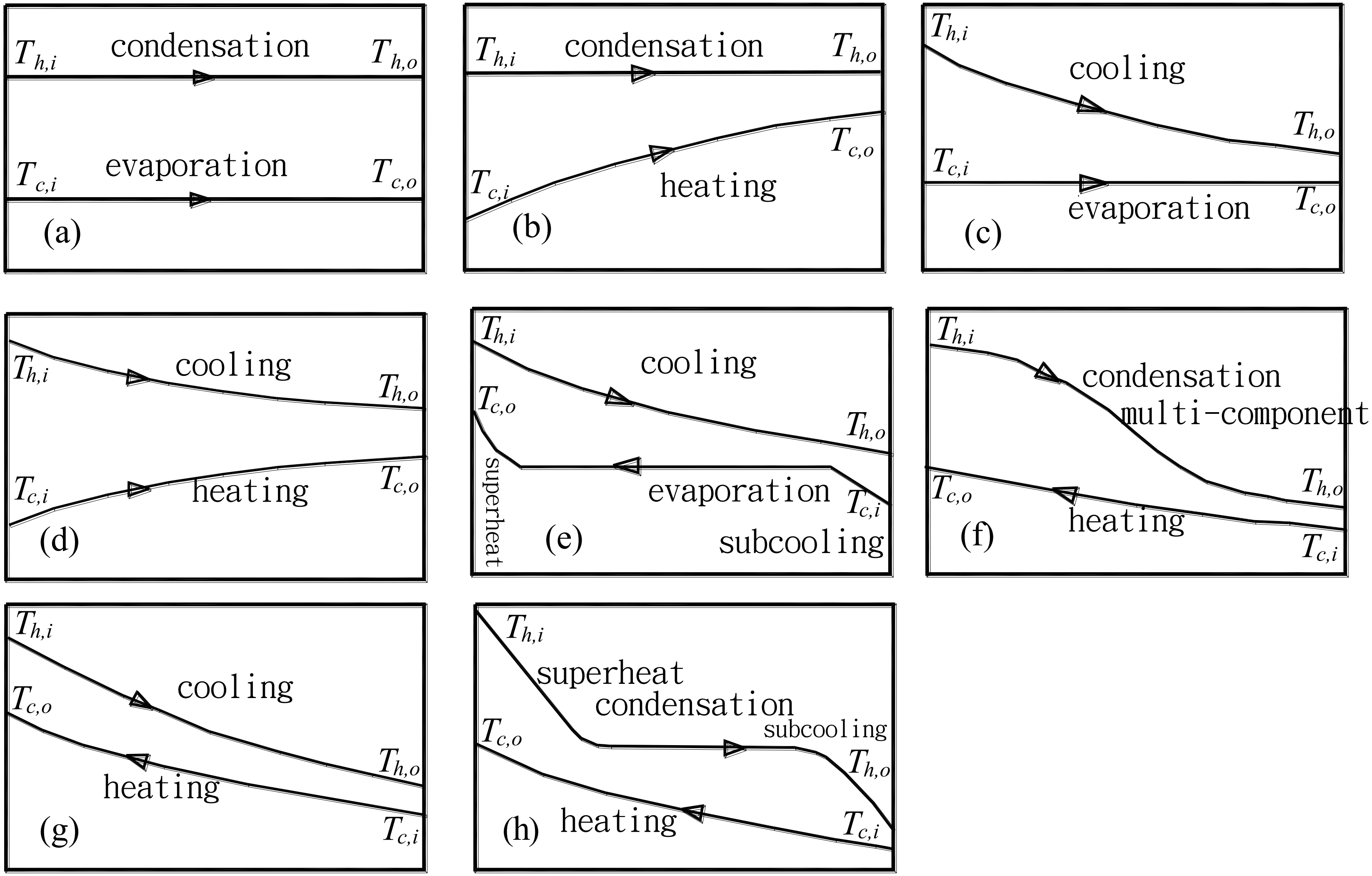
(2) 增加總熱傳面積 A

(3) 增加有效溫度差 ΔT_m

※ 流動型式在熱交換器設計上所扮演的角色即為調整有效溫度差，另外上述的流動型態中，以逆向流的安排具有最大的溫度差。這可從逆向流的出口溫度有可能高於熱側的出口溫度，但是平行流動下則不可能發生此現象看出。

※在逆向流動的熱交換器中，熱側與冷側流體的溫差保持的最為「均勻」，而平行流動時，熱側與冷側流體的溫差變化較大，以平均值而言，熱交換器內各處保有最為「均勻」溫差者，將會擁有最大的有效溫差。

熱交換器之流體流動型態(Cont.)



熱交換器之不同流動型態的溫度變化示意圖

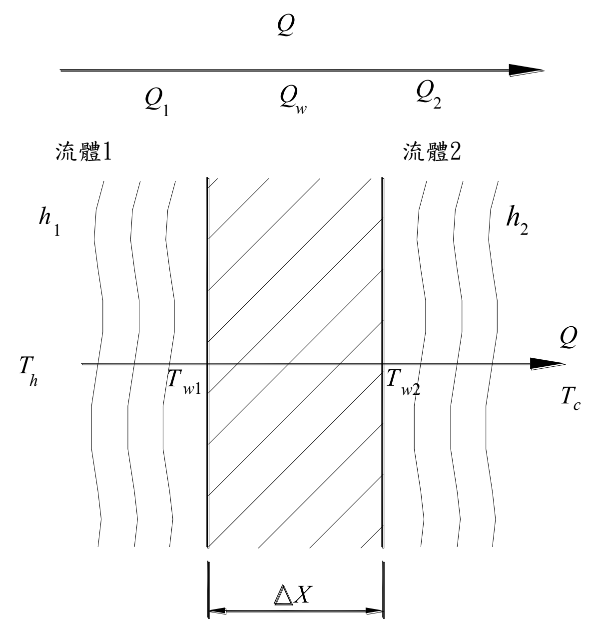
熱交換器原理介紹(Cont.)

在穩定狀態(steady state)下熱傳量是平衡的，故$Q_1 = Q_w = Q_2 = Q$

$$\dot{Q} = h_1 A_{w1}\left(T_h - T_{w,1}\right) \Rightarrow \frac{\dot{Q}}{h_1 A_{w1}} = T_h - T_{w,1}$$

$$\dot{Q} = kA\frac{\left(T_{w,1} - T_{w,2}\right)}{\Delta X} \Rightarrow \frac{\dot{Q}}{\frac{kA}{\Delta X}} = T_{w,1} - T_{w,2}$$

$$\dot{Q} = h_2 A_{w2}\left(T_{w,2} - T_c\right) \Rightarrow \frac{\dot{Q}}{h_2 A_{w2}} = T_{w,2} - T_c$$



三式相加，得

$$\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)\dot{Q} = T_h - T_c$$

$$\Rightarrow \dot{Q} = \frac{1}{\underbrace{\left(\frac{1}{h_1 A_{w1}} + \frac{1}{\frac{kA}{\Delta X}} + \frac{1}{h_2 A_{w2}}\right)}_{UA}}(T_h - T_c)$$

如何進行熱交換器的Rating/Sizing?

- **性能計算(rating problem)**：已知熱交換器相關幾何尺寸和流體進口條件，計算其熱傳性能(熱傳量、壓損及流體出口狀態等)。
- **尺寸設計(sizing problem)**：已知熱交換器性能(熱傳量及流體進出口狀態)，計算其幾何尺寸(熱傳管有效長度或管排數)。
- 能量平衡方程式(Energy balance equation)：

$$\dot{Q} = \dot{m} \times Cp \times \Delta T \quad \Rightarrow \dot{Q} = (\dot{m}C_{p,c})_c(T_{c,o} - T_{c,i}) = (\dot{m}C_{p,h})_h(T_{h,i} - T_{h,o})$$

- 一般熱交換器的熱量傳遞方程式均可寫成如下：

$$\boxed{\dot{Q} = (UA)(\mathit{Driving\,Potential}, \text{驅動勢})}$$

$$\dot{Q} = U \times A \times \Delta T_m$$

，ΔT_m 為有效流體之溫度差

問題：如何求得決定U & ΔT_m?

UA-LMTD-F Method

- 熱傳遞為顯熱變化，驅動力為乾球溫度差，**驅動勢為平均溫差**

－若是逆向流，則

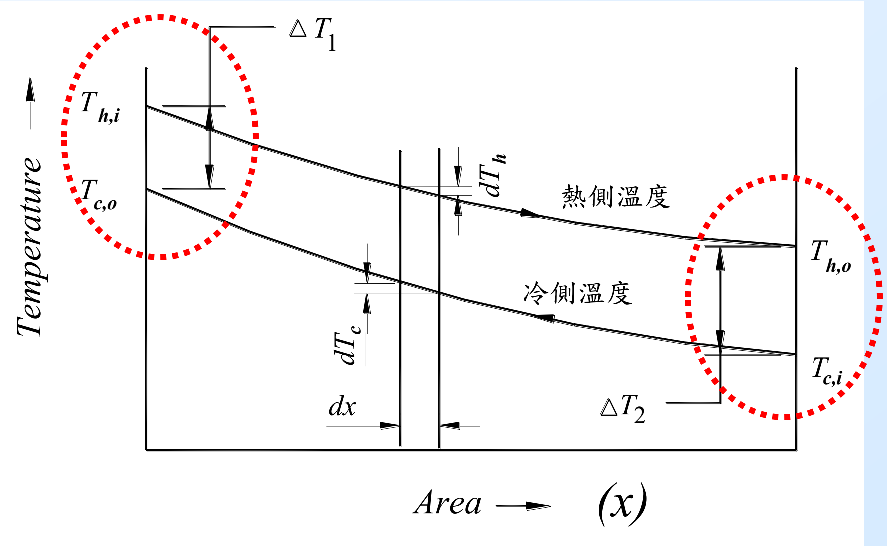
$\Delta T_m = ?$

$$\boxed{\dot{Q} = UA\Delta T_m}$$

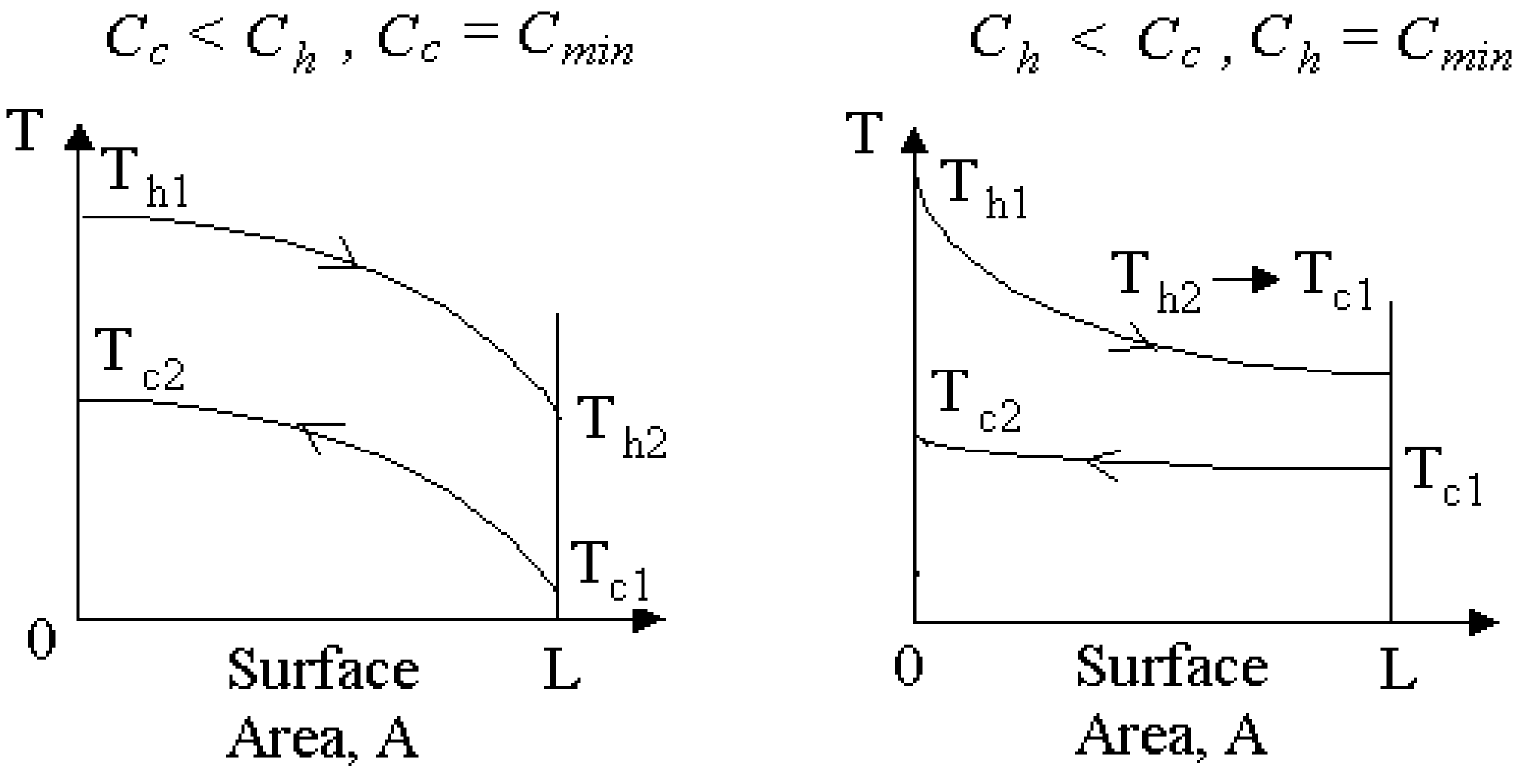
$$LMTD = \frac{(\Delta T_1 - \Delta T_2)}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \Delta T_m = \Delta T_{lm}$$

$$\Delta T_1 = (T_{h,i} - T_{c,o})$$

$$\Delta T_2 = (T_{h,o} - T_{c,i})$$



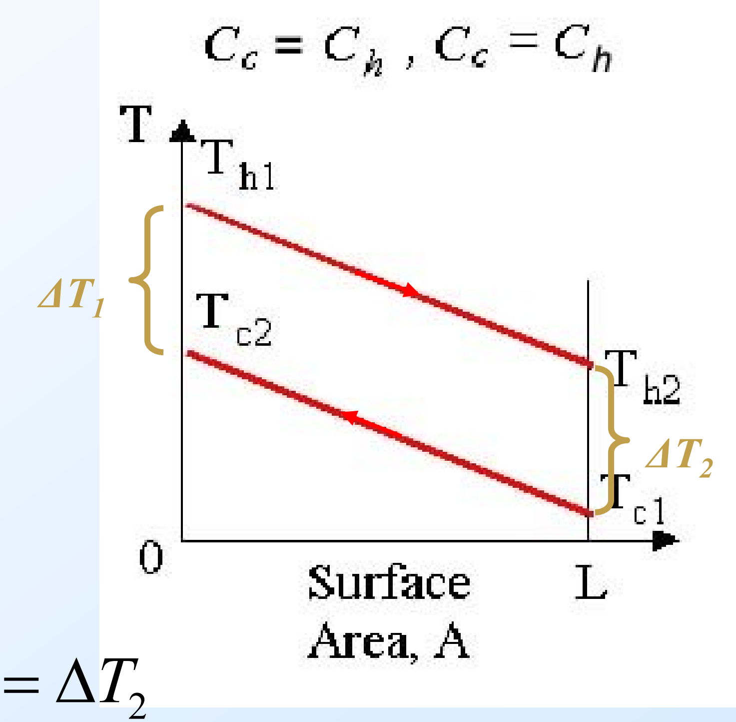
Counter Flow heat exchanger



(a) Fig2.逆向流之流體流動方向示意圖 (b)

$$\dot{Q} = \underbrace{(\dot{m}C_{p,c})_c}_{C_c}(T_{c,o} - T_{c,i}) = \underbrace{(\dot{m}C_{p,h})_h}_{C_h}(T_{h,i} - T_{h,o})$$

UA-LMTD-F Method(Cont.)



- ➢定義 LMTD

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

- ➢ The total heat transfer rate for all single-pass flow arrangements

$$\dot{Q} = UA\Delta T_{lm}$$

◆若是逆向流(CF HX)，且 $C_h=C_c$ ，

$$(T_{c,o} - T_{c,i}) = (T_{h,i} - T_{h,o}) \quad and \quad \Delta T_1 = \Delta T_2$$

因此熱傳量

$$\dot{Q} = UA(T_h - T_c) \quad and \quad (T_h - T_c) = \Delta T_1 = \Delta T_2$$

◆若是平行流(PF HX)

$$\Delta T_1 = (T_{h,i} - T_{c,i}) \quad \Delta T_2 = (T_{h,o} - T_{c,o})$$

UA-LMTD-F Method(Cont.)

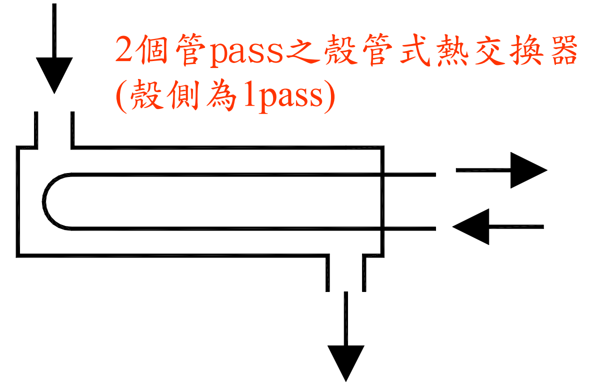
- 如果流動不是逆向流動時(例如Multipass and Crossflow HX)，ΔT_m 可證明等於ΔT_{lm}再乘上一個校正係數 F，F值也可證明與流動型態及 P、R 有關，即 $F = F$(流動型態，P，R)

$$\boxed{\dot{Q} = (UA)F(\Delta T_{lm,cf})}$$

$$\Delta T_{lm,cf} = \frac{(T_{h,i}-T_{c,o})-(T_{h,o}-T_{c,i})}{\ln\left((T_{h,i}-T_{c,o})\big/(T_{h,o}-T_{c,i})\right)}$$

$$P = \frac{T_{c,o}-T_{c,i}}{T_{h,i}-T_{c,i}} = \frac{\Delta T_c}{\Delta T_{\max}}$$

$$R = \frac{C_c}{C_h} = \frac{T_{h,i}-T_{h,o}}{T_{c,o}-T_{c,i}}$$



2個管pass之殼管式熱交換器
(殼側為1pass)

※F值小於1 →for cross flow & multipass arrangement，約為0.9~1.0。

F=1 → For pure counterflow

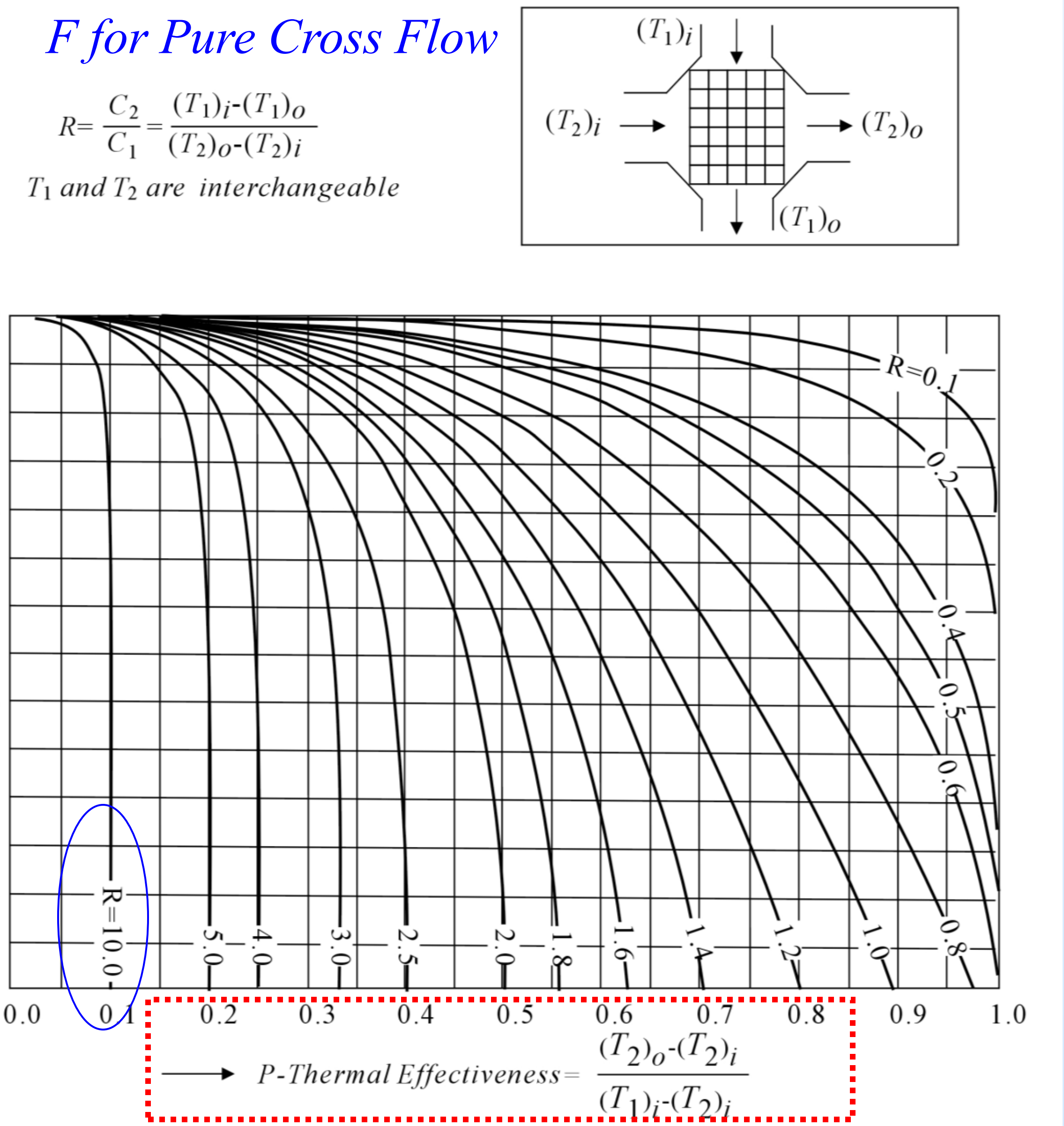
UA-LMTD-F Method(Cont.)

F與P、R 間的關係圖

$$\dot{Q} = (\dot{m}_c C_{pc})(T_{c,o} - T_{c,i})$$
$$= (\dot{m}_h C_{ph})(T_{h,i} - T_{h,o})$$

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{\Delta T_c}{\Delta T_{\max}}$$

$$R = \frac{C_c}{C_h} = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}}$$



F for Pure Cross Flow

$$R = \frac{C_2}{C_1} = \frac{(T_1)_i - (T_1)_o}{(T_2)_o - (T_2)_i}$$

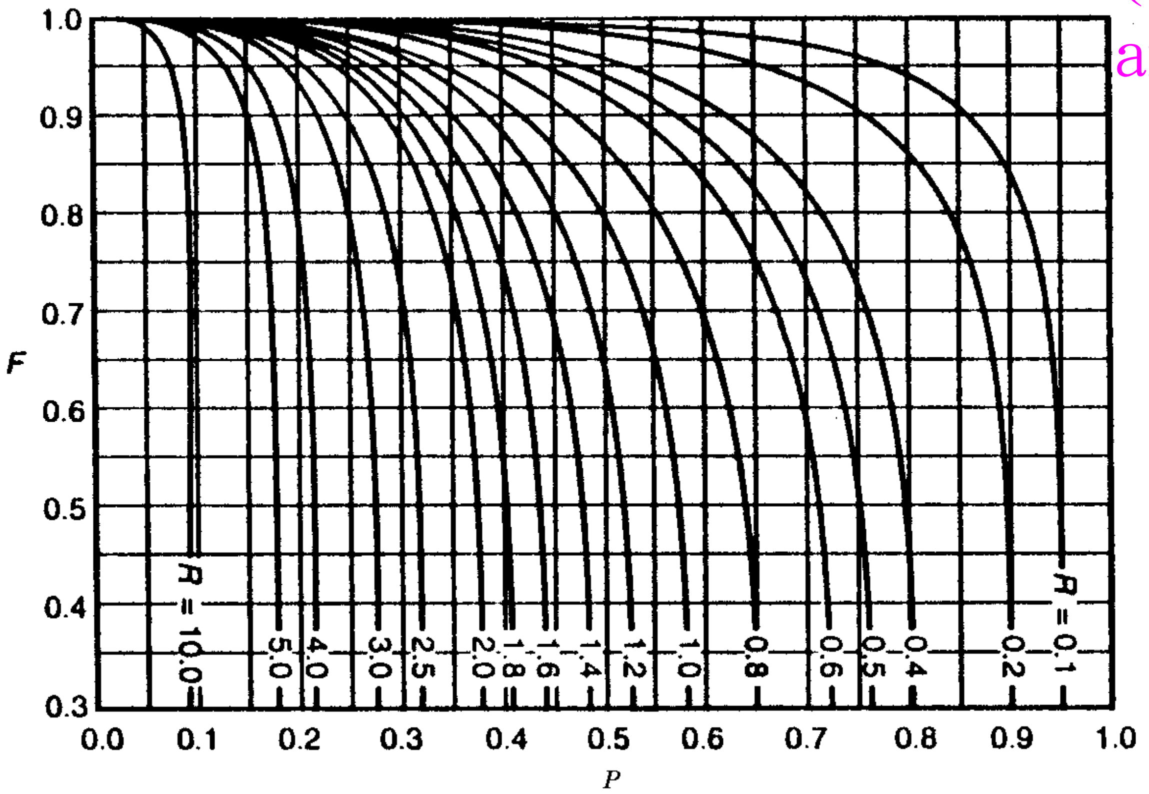
T_1 and T_2 are interchangeable

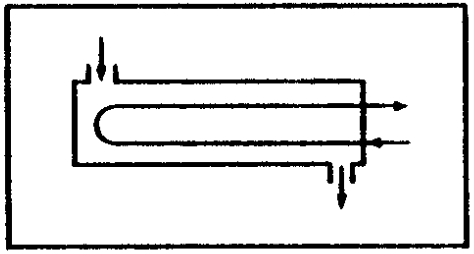
F-Correction Factor $= \frac{\Delta T_m}{\Delta T_{\ell m}}$

$\longrightarrow$ P-Thermal Effectiveness $= \frac{(T_2)_o - (T_2)_i}{(T_1)_i - (T_2)_i}$

UA-LMTD-F Method(Cont.)

F與P、R間的關係圖
(one shell pass and two tube pass)





- LMTD correction factor *F* for a shell-and-tube heat exchanger – one shell pass and two or multiple of two tube passes

ε-NTU Method

- 首先定義熱流 (hot stream)和冷流 (cold stream) 的熱容量 (heat capacity) ：

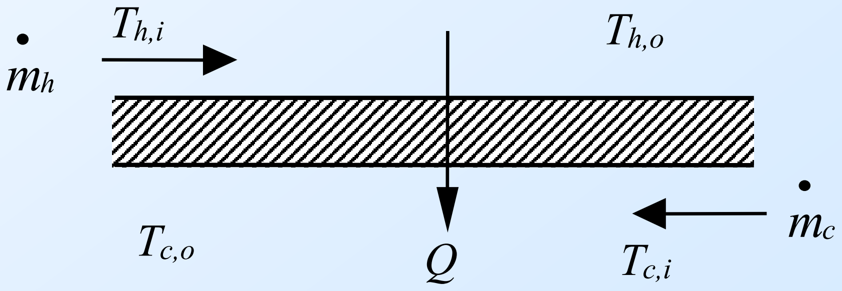
$$C_h = (\dot{m}C_p)_h \qquad C_c = (\dot{m}C_p)_c$$

C_{min} 為兩者之間熱容量較小者

- 定義 ε (effectiveness，有效度) 與 *NTU* (number of transfer unit，傳遞單位)，有效度定義如下：

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}}$$

其中 Q 為熱交換器實際熱交換量，Q_{max} 為熱交換器在該操作條件下之最大可能熱交換量。

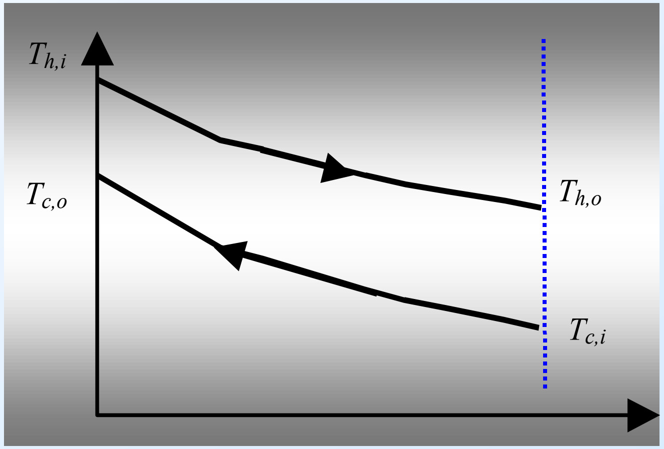


熱傳路徑示意圖

What is Q_{max}?

●假設 $C_h=(\dot{m}C_p)_h$ 較 $\dot{m}_c c_{p,c}$ 為大，此時若我們漸漸增大熱交換器的面積，則冷側的出口溫度會逐漸地趨近熱側進口溫度，當熱交換器的面積趨近無窮大時，此時熱交換器的冷側出口溫度會等於熱側的進口溫度(溫度變化如圖所示)。因此，最大可能熱傳量為

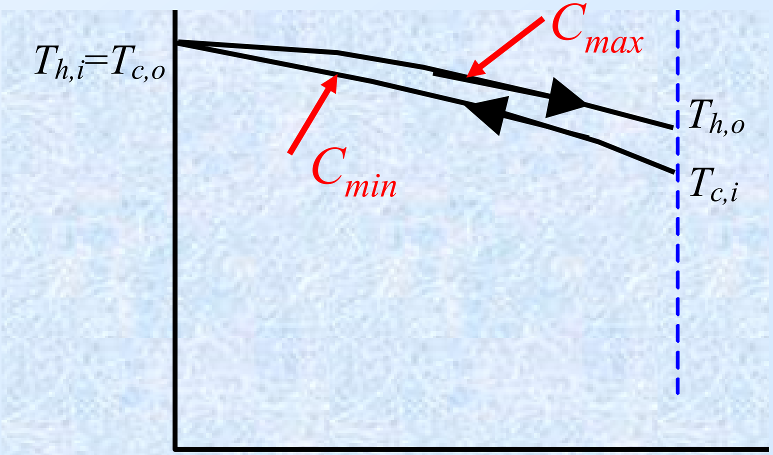
$$\dot{Q}_{\max} = C_{\min}\Delta T_{\max} = C_{\min}(T_{hi}-T_{ci})$$



●所以熱交換器的實際熱傳量為：

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,i}-T_{c,i})$$

其中 C_{min} 為熱側與冷側流體，兩者之間熱容量(heat capacity)較小者。$T_{h,i}$和$T_{c,i}$分別為熱側和冷側流體的進口溫度。 ε 稱為有效性(effectiveness) 。



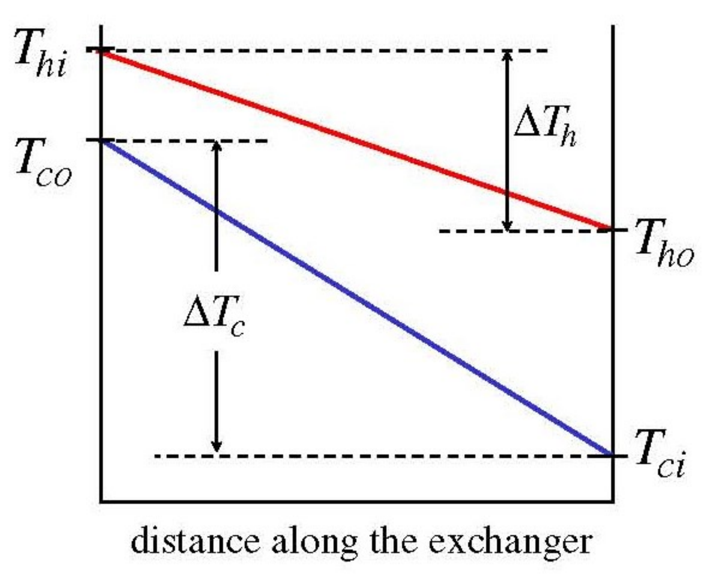
What is Q_{max}(Cont.)?

$$\dot{Q} = (\dot{m}C_{p,c})_c(T_{c,o} - T_{c,i}) = (\dot{m}C_{p,h})_h(T_{h,i} - T_{h,o})$$

其中 $C_c = (\dot{m}C_{p,c})_c$ $C_h = (\dot{m}C_{p,h})_h$

If $C_c < C_h \Rightarrow (T_{c,o}-T_{c,i}) > (T_{h,i}- T_{h,o})$

If $C_c > C_h \Rightarrow (T_{c,o}-T_{c,i}) < (T_{h,i}- T_{h,o})$

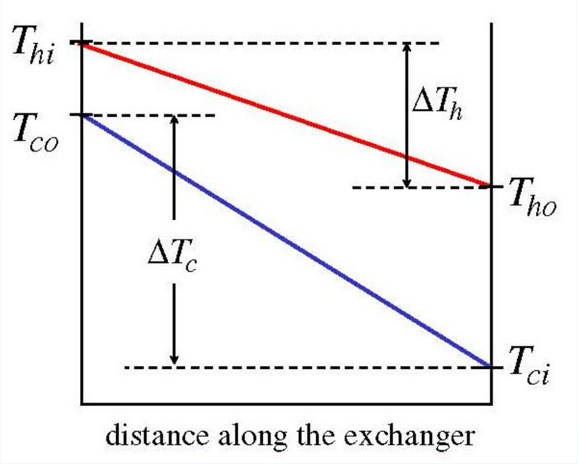


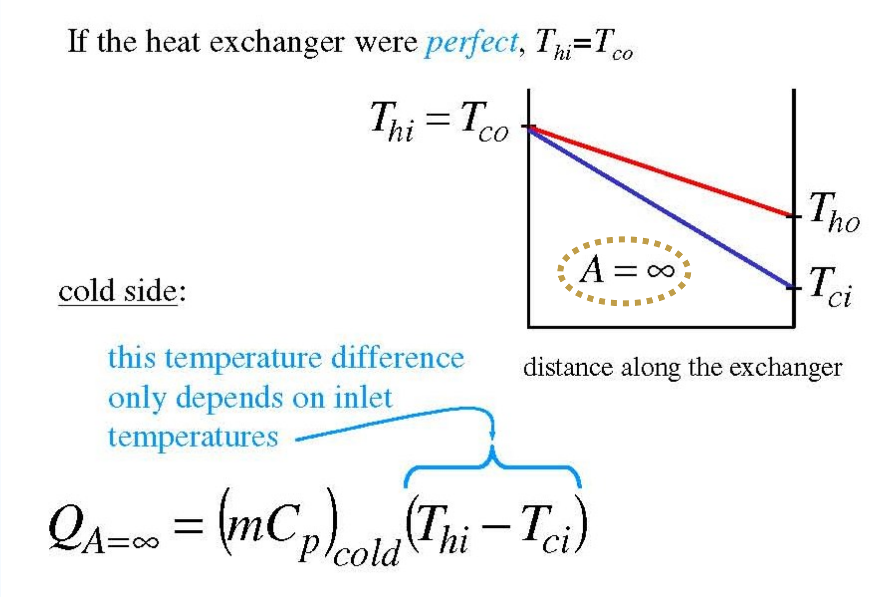
What is Q_{max}(Cont.)?

➢Case1 $\begin{cases}(\dot{m}C_{p,c})_c < (\dot{m}C_{p,h})_h \\ \Delta T_c > \Delta T_h\end{cases}$ *Cold fluid=minimum fluid*

$$\dot{Q}_{\max} = \underbrace{(\dot{m}c_{p,c})_c}_{C_{min}}(T_{h,i}-T_{c,i})$$

if $C_c < C_h$




If the heat exchanger were *perfect*, T_{hi}=T_{co}

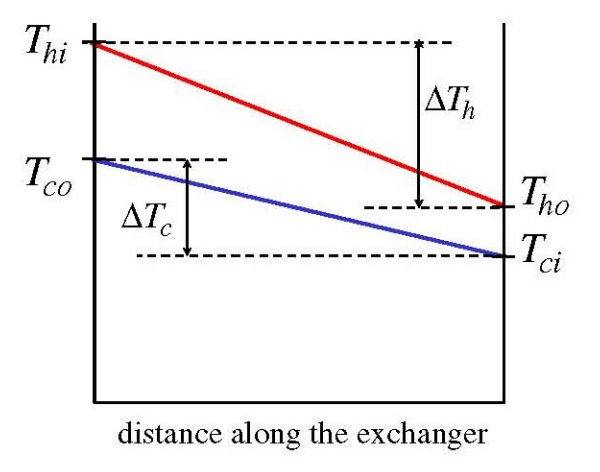
cold side:

this temperature difference only depends on inlet temperatures

$$Q_{A=\infty} = \left(mC_p\right)_{cold}\overbrace{\left(T_{hi}-T_{ci}\right)}$$

What is Q_{max}(Cont.)?

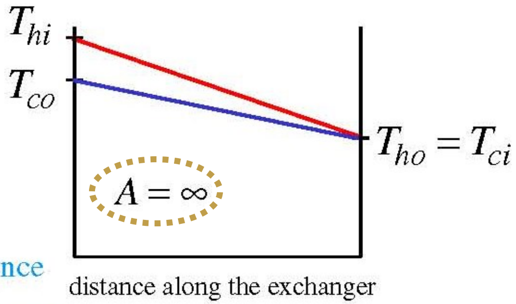
- Case2 $\begin{cases} (\dot{m}C_{p,c})_c > (\dot{m}C_{p,h})_h \\ \Delta T_c < \Delta T_h \end{cases}$ *hot fluid=minimum fluid*



$$\dot{Q}_{\max} = \underbrace{(\dot{m}c_{p,h})_h}_{C_{min}}(T_{h,i} - T_{c,i})$$

if $C_c > C_h$

If the heat exchanger were *perfect*, $T_{hi}=T_{co}$



hot side:

this temperature difference only depends on inlet temperatures

$$Q_{A=\infty} = (mC_p)_{hot}\overbrace{(T_{hi} - T_{ci})}$$

What is Q_{max}(Cont.)?

$$\begin{cases} \dot{Q}_{\max} = (\dot{m}c_{p,c})_c(T_{h,i}-T_{c,i}) \quad \mathit{if} \quad C_c < C_h \\ \dot{Q}_{\max} = (\dot{m}c_{p,h})_h(T_{h,i}-T_{c,i}) \quad \mathit{if} \quad C_h < C_c \end{cases}$$

$$\boxed{\dot{Q}_{\max} \qquad {}_{\min}(T_{h,i}-T_{c,i})}$$

$$\boxed{\dot{Q} = \varepsilon\dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,i}-T_{c,i})}$$

ε (effectiveness，有效度)

$$\boxed{\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,i}-T_{h,o})}{C_{\min}(T_{h,i}-T_{c,i})} = \frac{C_C(T_{c,o}-T_{c,i})}{C_{\min}(T_{h,i}-T_{c,i})}}$$

What is ε?

1. ε稱為有效性(effectiveness)，任何形式之熱交換器，ε可表示為NTU (傳遞單位，number of transfer units)、C^* (C_{min}/C_{max})、和流體流動型態的函數

$$\varepsilon = \varepsilon(NTU，C^*，流體流動型式)$$

2.

$$\dot{Q} = (\dot{m}C_{p,c})_c(T_{c,o} - T_{c,i}) = (\dot{m}C_{p,h})_h(T_{h,i} - T_{h,o}) = UA\Delta T_m$$

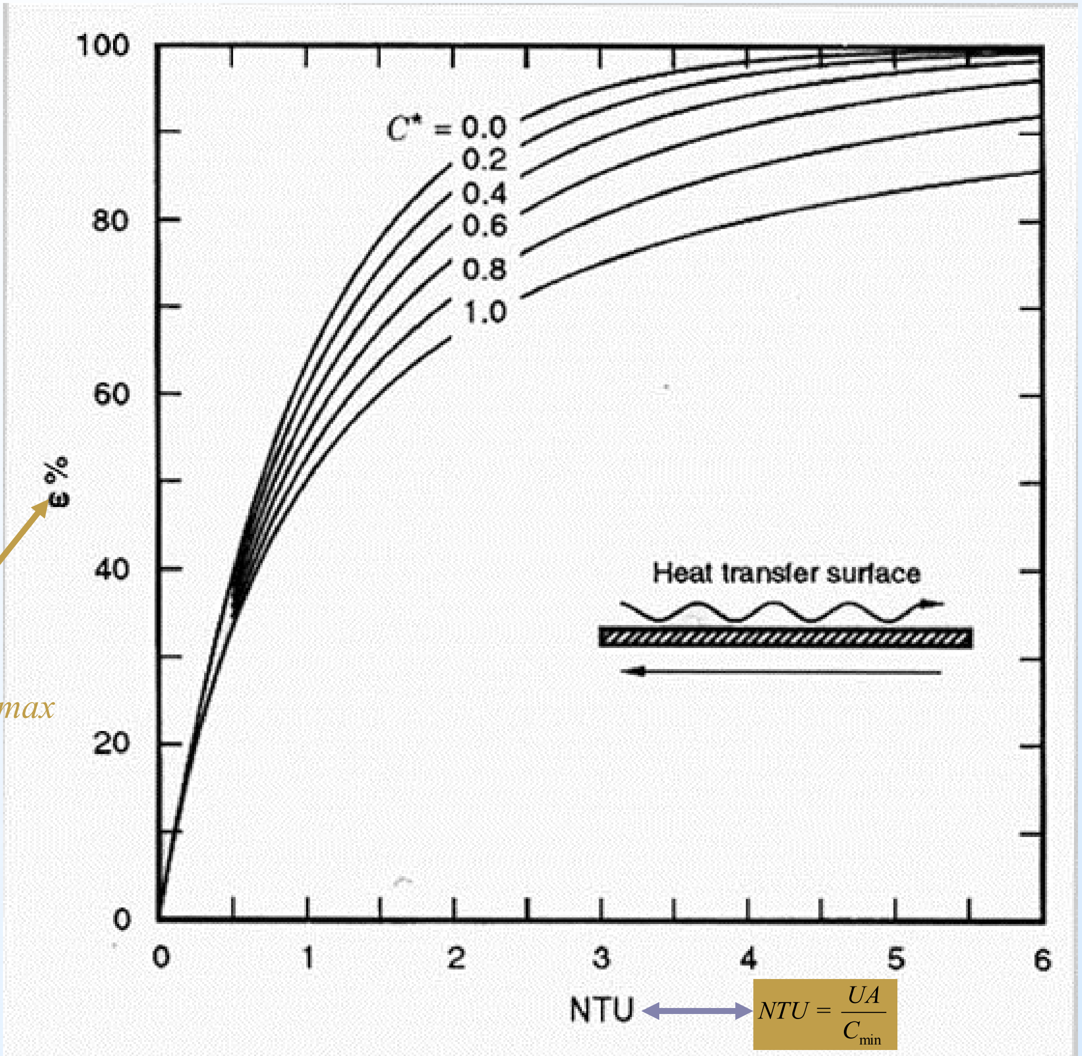
定義傳遞單位NTU為 $NTU \equiv UA/C_{min}$

其意義為熱交換器的熱傳性能的大小 (thermal size)；從定義上即可發現它是熱交換器的面積尺寸A 、總熱傳係數 U 與最小熱容量 $C_{\min}$ 的組合。

- 一般 NTU 一值在應用上多設計在 0.0～4.0間；不過在許多密集式熱交換器上的 NTU 很可能仍會超過 此一範圍。熱交換器的有效度 ε，通常會隨著 NTU 的增加而增加，而且增加到某個程度後，其增加幅度就會呈現飽和狀態，例如下圖A的逆向流動下ε與 NTU、C*間的關係圖。

- 不過也有例外；在某些特殊的流動安排上，ε 會先增加再減少，例如圖B，由於熱量的逆向傳遞，當 NTU 大過一個值後，NTU 增加反而會降低 ε；這個特殊的結果告訴我們，一昧的增加熱交換器的面積不見得一定可以增加熱傳量，熱交換器的流動安排也是非常重要的因素，另外圖2-30(a) 中不會有熱量逆傳的現象。

圖A　逆向流動下，有效度與 NTU 間的關係圖

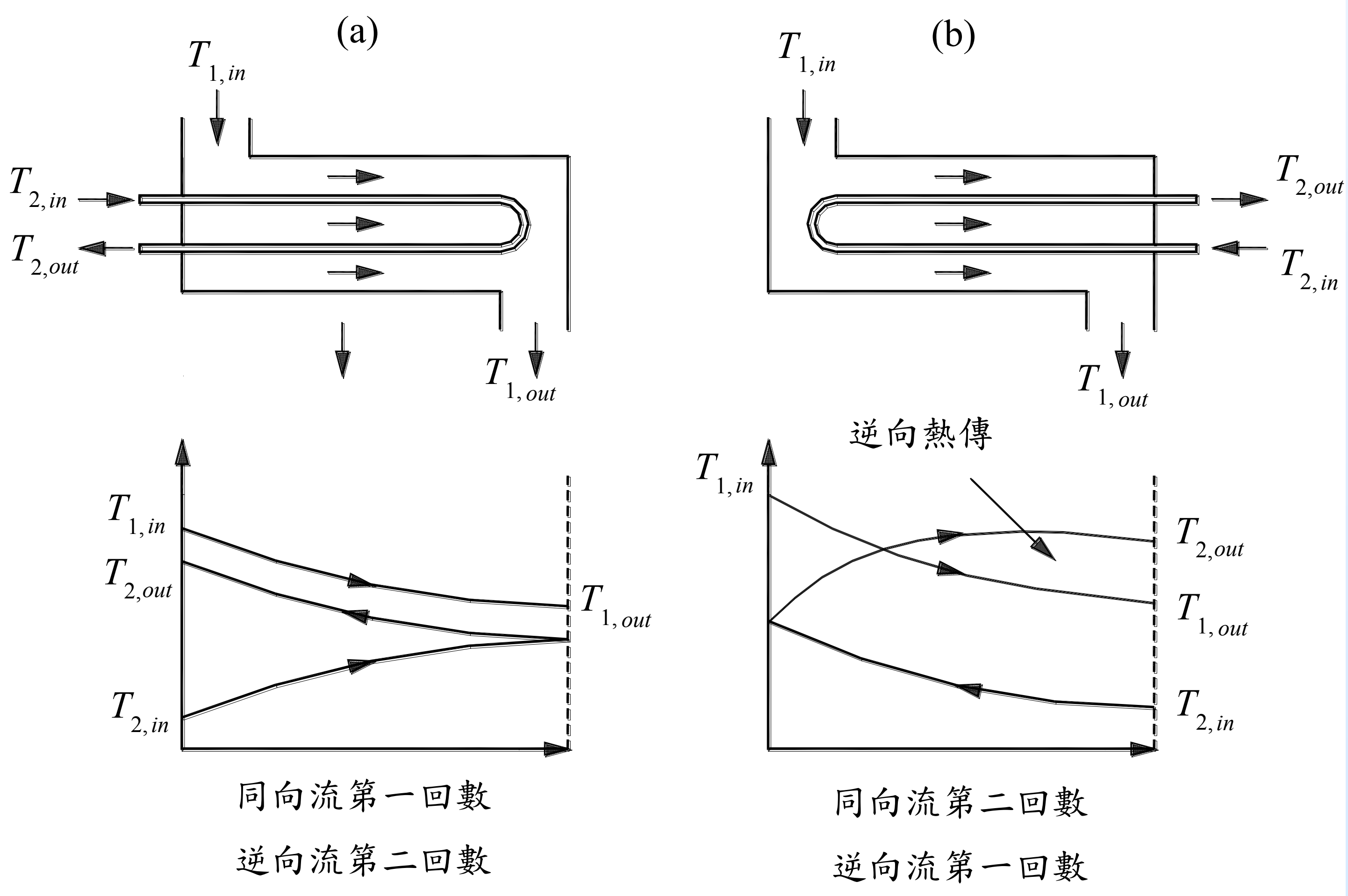


$\varepsilon = Q/Q_{max}$

$NTU = \dfrac{UA}{C_{\min}}$

FIGURE 3.7 Counterflow exchanger ε as a function of NTU and C*.

圖B　2-Pass殼管式熱交換器，溫度變化示意圖



同向流第一回數
逆向流第二回數

同向流第二回數
逆向流第一回數

- 一般而言，最後一個回數(pass)若與另一側流體保持逆向流動時，可避免熱量逆傳的現象

Capacity Heat Ratio C* 的意義？

1. Capacity heat ratio C^*，定義為C_{min}/C_{max}，從定義上可知 $1 \geq C^* \geq 0$，若一個熱交換器中有一側牽涉到兩相蒸發(evaporation)或冷凝(condensation)，則 $C^*=0$，這是因為 $C=\dot{m}Cp$，而 $Cp=\left.\frac{\partial i}{\partial T}\right|_P$，在蒸發或冷凝的過程中溫度與壓力幾乎為一定值，所以 $Cp\equiv\left.\frac{\partial i}{\partial T}\right|_P\rightarrow\infty$，因此 $C^*\approx 0$；(想像一下，兩相蒸發或冷凝過程中，溫度保持不變，是否類似 heat sink 或 heat source? 故C_{max}為此有兩相流體的部分，而C_{min}就是另外的流體。)

- 若熱交換器中牽涉到兩相蒸發或冷凝的過程時，則此一熱交換器的有效度與 NTU 間的關係式將唯一決定(與熱交換器型式及流動型式無關)
- 若 $C^*\neq 0$，每一種型式的熱交換器都會有它的有效度與NTU 間的關係圖，一般而言，合理的熱交換器設計點約在最大有效度的 95%以內。

ε-NTU Method(Cont.)

- Single pass heat exchanger $C_c > C_h$

  ⇨$C_h = C_{min}$ and $C_c = C_{max}$

  可以推得

$$\varepsilon = \frac{1-\exp\left[-\mathrm{NTU}(1\text{-}C_{\min}/C_{\max}\right]}{1-(C_{\min}/C_{\max})\exp\left[-\mathrm{NTU}(1\text{-}C_{\min}/C_{\max})\right]}$$

(1).For $C_{\min}/C_{\max}=1$ and Counter Flow $\varepsilon = \frac{\mathrm{NTU}}{1+\mathrm{NTU}}$

(2) .For $C_{\min}/C_{\max}=1$ and Parallel Flow $\varepsilon = \frac{1}{2}(1-e^{-2\mathrm{NTU}})$

(3) For $C_{\min}/C_{\max}=0$, both CF and PF HXs $\varepsilon = 1-e^{-\mathrm{NTU}}$

有效度ε與 NTU 間的關係(公式計算)

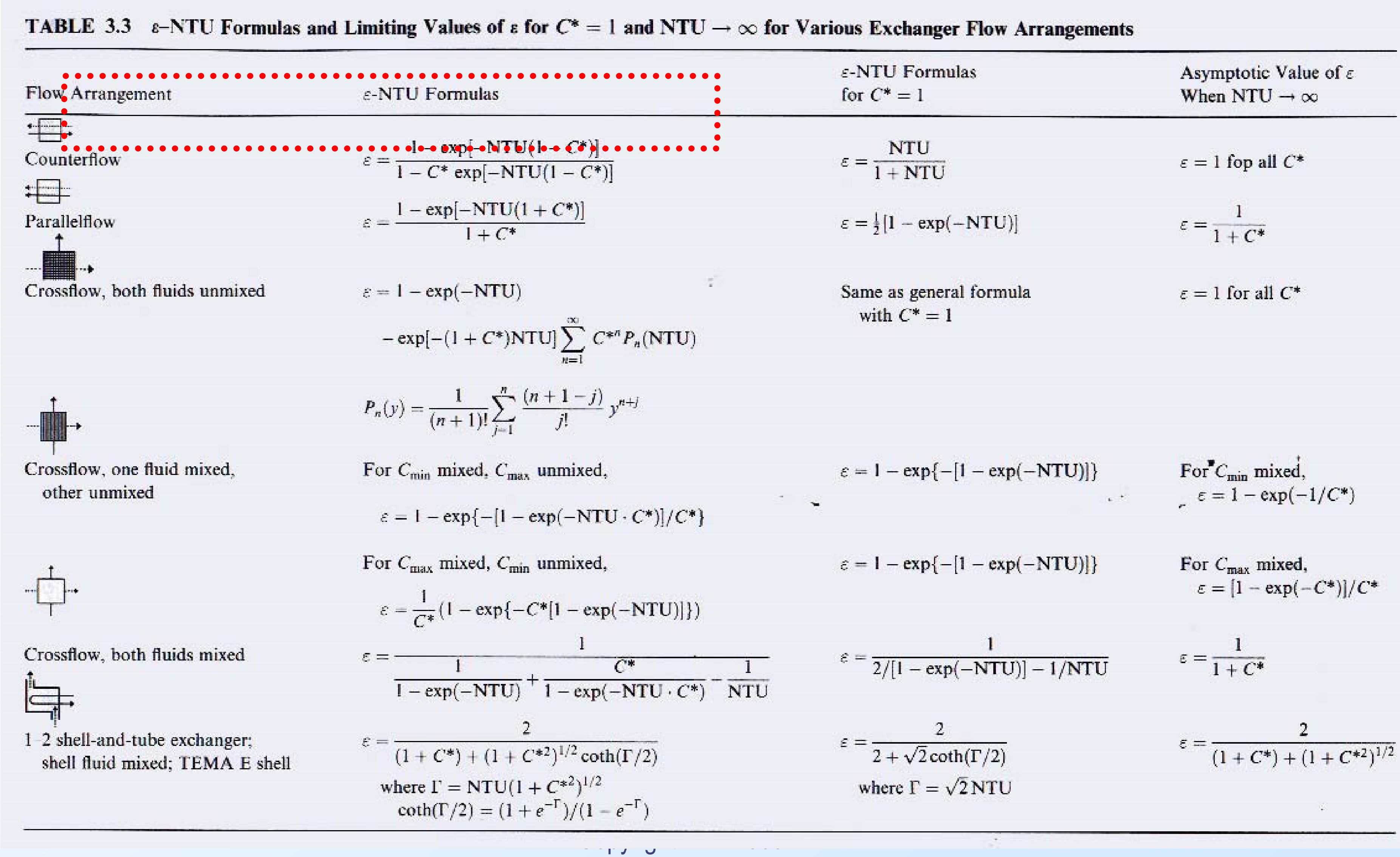


TABLE 3.3 ε–NTU Formulas and Limiting Values of ε for $C^* = 1$ and NTU → ∞ for Various Exchanger Flow Arrangements

Flow Arrangement	ε-NTU Formulas	ε-NTU Formulas for $C^* = 1$	Asymptotic Value of ε When NTU → ∞
Counterflow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 - C^*)]}{1 - C^* \exp[-\mathrm{NTU}(1 - C^*)]}$	$\varepsilon = \dfrac{\mathrm{NTU}}{1 + \mathrm{NTU}}$	$\varepsilon = 1$ fop all C^*
Parallelflow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 + C^*)]}{1 + C^*}$	$\varepsilon = \frac{1}{2}[1 - \exp(-2\mathrm{NTU})]$	$\varepsilon = \dfrac{1}{1 + C^*}$
Crossflow, both fluids unmixed	$\varepsilon = 1 - \exp(-\mathrm{NTU})$ $- \exp[-(1 + C^*)\mathrm{NTU}] \sum_{n=1}^{\infty} C^{*n} P_n(\mathrm{NTU})$ $P_n(y) = \dfrac{1}{(n+1)!} \sum_{j=1}^{n} \dfrac{(n+1-j)}{j!} y^{n+j}$	Same as general formula with $C^* = 1$	$\varepsilon = 1$ for all C^*
Crossflow, one fluid mixed, other unmixed	For $C_{\min}$ mixed, $C_{\max}$ unmixed, $\varepsilon = 1 - \exp\{-[1 - \exp(-\mathrm{NTU} \cdot C^*)]/C^*\}$	$\varepsilon = 1 - \exp\{-[1 - \exp(-\mathrm{NTU})]\}$	For $C_{\min}$ mixed, $\varepsilon = 1 - \exp(-1/C^*)$
	For $C_{\max}$ mixed, $C_{\min}$ unmixed, $\varepsilon = \dfrac{1}{C^*}(1 - \exp\{-C^*[1 - \exp(-\mathrm{NTU})]\})$	$\varepsilon = 1 - \exp\{-[1 - \exp(-\mathrm{NTU})]\}$	For $C_{\max}$ mixed, $\varepsilon = [1 - \exp(-C^*)]/C^*$
Crossflow, both fluids mixed	$\varepsilon = \dfrac{1}{\dfrac{1}{1 - \exp(-\mathrm{NTU})} + \dfrac{C^*}{1 - \exp(-\mathrm{NTU} \cdot C^*)} - \dfrac{1}{\mathrm{NTU}}}$	$\varepsilon = \dfrac{1}{2/[1 - \exp(-\mathrm{NTU})] - 1/\mathrm{NTU}}$	$\varepsilon = \dfrac{1}{1 + C^*}$
1–2 shell-and-tube exchanger; shell fluid mixed; TEMA E shell	$\varepsilon = \dfrac{2}{(1 + C^*) + (1 + C^{*2})^{1/2} \coth(\Gamma/2)}$ where $\Gamma = \mathrm{NTU}(1 + C^{*2})^{1/2}$ $\coth(\Gamma/2) = (1 + e^{-\Gamma})/(1 - e^{-\Gamma})$	$\varepsilon = \dfrac{2}{2 + \sqrt{2}\coth(\Gamma/2)}$ where $\Gamma = \sqrt{2}\,\mathrm{NTU}$	$\varepsilon = \dfrac{2}{(1 + C^*) + (1 + C^{*2})^{1/2}}$

有效度ε與 NTU 間的關係(公式計算整理)

Type	$\varepsilon(\mathrm{NTU},C^*)$	$\mathrm{NTU}(\varepsilon,C^*)$
Counter Flow	$\varepsilon = \dfrac{1-\exp\left[-\left(1-C^*\right)\mathrm{NTU}\right]}{1-C^*\exp\left[-\left(1-C^*\right)\mathrm{NTU}\right]}$	$\mathrm{NTU} = \dfrac{1}{1-C^*}\ln\left(\dfrac{1-\varepsilon C^*}{1-\varepsilon}\right)$
Parallel	$\varepsilon = \dfrac{1}{1+C^*}\left[1-\exp\left[-\left(1+C^*\right)\mathrm{NTU}\right]\right]$	$\mathrm{NTU} = -\dfrac{1}{1+C^*}\ln\left[1+\varepsilon\left(1+C^*\right)\right]$
Cross flow, $C_{\min}$ mixed	$\varepsilon = 1-\exp\left[\dfrac{1-\exp\left(-C^*\mathrm{NTU}\right)}{C^*}\right]$	$\mathrm{NTU} = -\dfrac{1}{C^*}\ln\left[1+C^*\ln\left(1-\varepsilon\right)\right]$
Cross flow, $C_{\max}$ mixed	$\varepsilon = \dfrac{1}{C^*}\left[1-\exp\left\{-C^*\left[1-\exp\left(-\mathrm{NTU}\right)\right]\right\}\right]$	$\mathrm{NTU} = -\ln\left[1+\dfrac{1}{C^*}\ln\left(1-\varepsilon C^*\right)\right]$
1-2 shell-and-tube heat exchanger	$\varepsilon = \dfrac{2}{1+C^*+\left(1+C^{*2}\right)^{1/2}\dfrac{1+\exp\left\{-\mathrm{NTU}\left(1+C^{*2}\right)^{1/2}\right\}}{1-\exp\left\{-\mathrm{NTU}\left(1+C^{*2}\right)^{1/2}\right\}}}$	$\mathrm{NTU} = \dfrac{1}{\left(1+C^{*2}\right)^{1/2}}\ln\dfrac{2-\varepsilon\left\{1+C^*-\left(1+C^{*2}\right)^{1/2}\right\}}{2-\varepsilon\left\{1+C^*+\left(1+C^{*2}\right)^{1/2}\right\}}$

Summary for ε-NTU & UA-LMTD-F

UA-LMTD-F

$$\dot{Q} = (UA)F(\Delta T_{lm})$$

$$LMTD = \Delta T_{lm} = \frac{(\Delta T_1 - \Delta T_2)}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)}$$

$$\Delta T_1 = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,o} - T_{c,i}$$

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{\Delta T_c}{\Delta T_{\max}}$$

$$R = \frac{C_c}{C_h} = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}}$$

F = fn (P，R，流體流動型式)

ε-NTU

$$\dot{Q} = \varepsilon(\dot{m}c_p)_{\min}(T_{h,i} - T_{c,i})$$

$$\varepsilon = \frac{\dot{Q}_{actual}}{\dot{Q}_{\max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} = \frac{C_C\left(T_{c,o} - T_{c,i}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)}$$

$$C^* = \frac{C_{\min}}{C_{\max}} = \frac{(\dot{m}c_p)_{\min}}{(\dot{m}c_p)_{\max}},\ \left(0 \le C^* \le 1\right)$$

$$NTU = \frac{UA}{C_{\min}}$$

ε = fn (NTU， C^*，流體流動型式)

Rating by ε-NTU Method

1.從已知的熱交換器幾何尺寸、操作條件(冷熱流體進口條件)、熱傳係數及流體輸送性質，冷熱流體出口狀態未知，來計算 *NTU* 與 *C**。

⇨ $NTU \equiv UA/C_{min}$，$C^{*}=C_{min}/C_{max}$

2.由已知的 *NTU*、*C**與流動型式；再根據相關的 ε-*NTU* 的圖表或方程式來算出有效度 ε。 ⇨ $\varepsilon = \varepsilon(NTU$， C^{*}，流體流動型式$)$

Note： $0 < \varepsilon < 1$

3.由 $\boxed{\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,i} - T_{c,i})}$ ，來算出熱交換量 $\dot{Q}$

⇨ $\boxed{\dot{Q} = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i})}$ ，然後可一併算出冷熱側之出口溫度如下：

$$\boxed{T_{h,o} = T_{h,i} - \dot{Q}/C_h}$$

$$\boxed{T_{c,o} = T_{c,i} + \dot{Q}/C_c}$$

Rating by UA-LMTD-F

1. 先猜一個熱側出口溫度$T_{h,o}$或冷側出口溫度$T_{c,o}$。

2. 由 $R = C_c/C_h$算出 R 值。再由$P = (T_{c,o} - T_{c,i})/(T_{h,i} - T_{c,i})$，可算出 P 值。

由熱側和冷側的進出口溫度得 ΔT_{lm}。

3.由 P、R值再搭配相關圖表或方程 可得到校正因子 F。

4.由 $\dot{Q} = (UA)F(\Delta T_{lm})$，算出熱傳量 $\dot{Q}$。

5.$\dot{Q}$值可推算出口溫度，$T_{h,o} = T_{h,i} - \dot{Q}/C_h$ 或 $T_{c,o} = T_{c,i} + \dot{Q}/C_c$

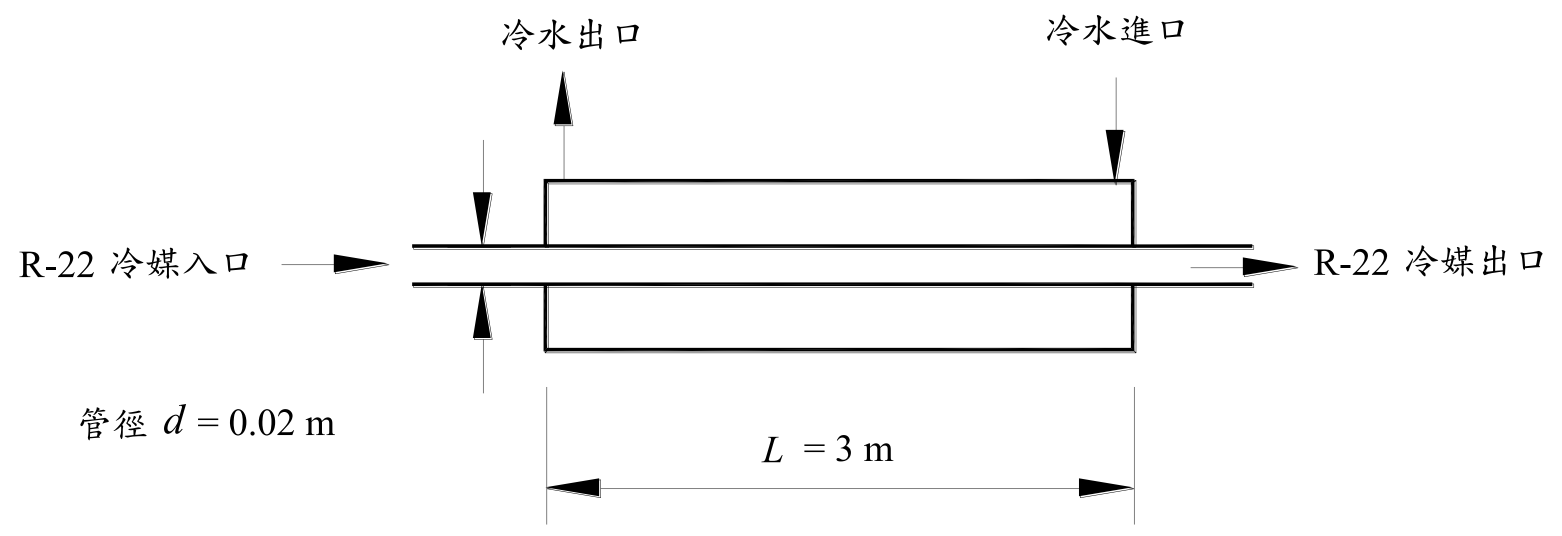
然後再比較此一出口溫度是否與原先假設一致？

6.若與假設值不同，則需重新假設熱側或冷側出口溫度，再重複步驟(2)-(5)，直至出口溫度相符合為止。

- Rating for *UA-LMTD-F* Method need to iterate a outlet temperature.

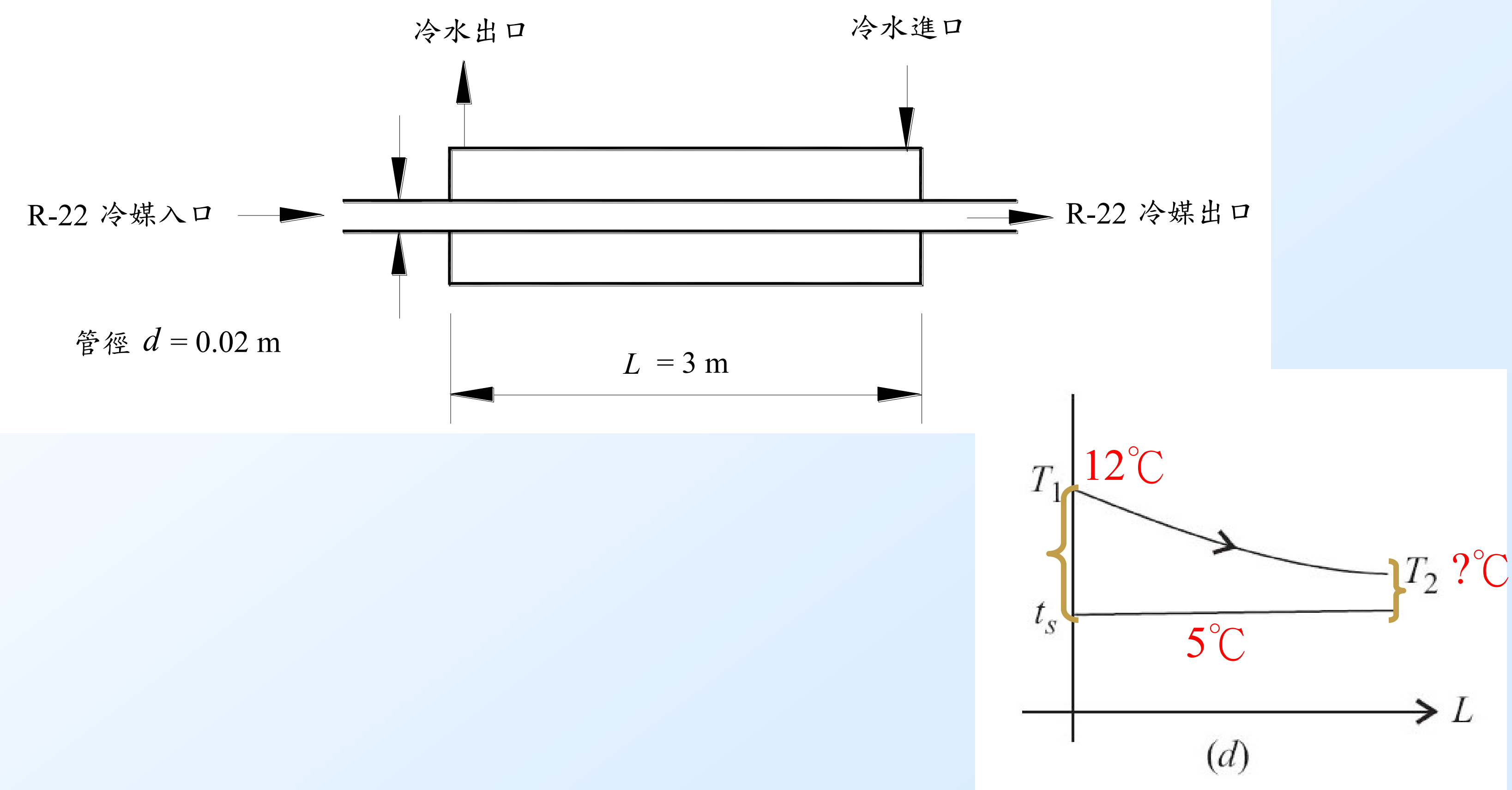
一個Rating計算實例

例題：一雙重管蒸發器，管內為R-22冷媒，其蒸發溫度為 5 °C，環側為冷水，水入口溫度為 12 °C，水流量為 0.1 kg/s，比熱為 4180 J/kg·K，蒸發器之總熱傳係數U為 2000 W/m^2·K。熱交換器總長度 3 m，管徑為2 cm，試問冷水出口溫度為何？



一個Rating計算實例

例題：一雙重管蒸發器，管內為R-22冷媒，其蒸發溫度為 5 °C，環側為冷水，水入口溫度為 12 °C，水流量為 0.1 kg/s，比熱為 4180 J/kg·K，蒸發器之總熱傳係數*U*為 2000 W/m²·K。熱交換器總長度 3 m，管徑為2 cm，試問冷水出口溫度為何？



A Rating Example

(1) Solution By *UA-LMTD-F*

$$\boxed{UA \cdot LMTD \cdot F = \dot{m} c_p \Delta T}$$

Counter flow， $F = 1$

$A = \pi \times 0.02 \times 3 = 0.189\ \mathrm{m}^2$

$\Delta T_1 = 12 - 5 = 7$ °C

$\Delta T_2 = x - 5$ (assume outlet water is x °C)

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{7-(x-5)}{\ell n\left(\dfrac{7}{x-5}\right)}$$

$$2000 \times 0.189 \times \frac{12-x}{\ell n\left(\dfrac{7}{x-5}\right)} = 0.1 \times 4180 \times (12-x)$$

$$\ell n \frac{7}{x-5} = 0.9043 \rightarrow x = 7.83\ °\mathrm{C}$$

$$\boxed{\begin{aligned}\dot{Q} &= 0.1 \times 4180 \times (12-7.83) \\ &= 1741.6\mathrm{W}\end{aligned}}$$

(2) Solution by *ε-NTU*

$$C_{min} = 0.1 \times 4180 = 418\ \mathrm{W/K}$$

$$NTU = \frac{UA}{C_{min}} = \frac{2000 \times 0.189}{418} = 0.9043$$

$$C^* = \frac{C_{min}}{C_{max}} = 0$$

$$\boxed{\varepsilon = 1 - \exp(-NTU) = 0.5952}$$

$$\dot{Q}_{max} = C_{min} \Delta T_{\max} = 418 \times (12-5) = 2926\mathrm{W}$$

$$\dot{Q} = \varepsilon \dot{Q}_{max} = 0.5952 \times 2926 = 1741.6\ \mathrm{W}$$

$$T_{c,out} = 12 - 1741.6/418 = 7.83\,°\mathrm{C}$$

結論：

不管是使用 *UA-LMTD-F* 或 *ε-NTU* 法，其計算結果應相同。

Sizing Problem

Sizing by UA-LMTD-F

1.由於熱傳量Q為已知，故可算出出口溫度、P 與R 值。

2. 由 P、R 算出該設計條件的校正因子 F。

3.由端點溫度算出對數平均溫差 LMTD；ΔT_{lm} 。

4.由$A=Q/(UF\Delta T_{lm})$，算出所需要的熱交換器面積。

Sizing by ε-NTU Method

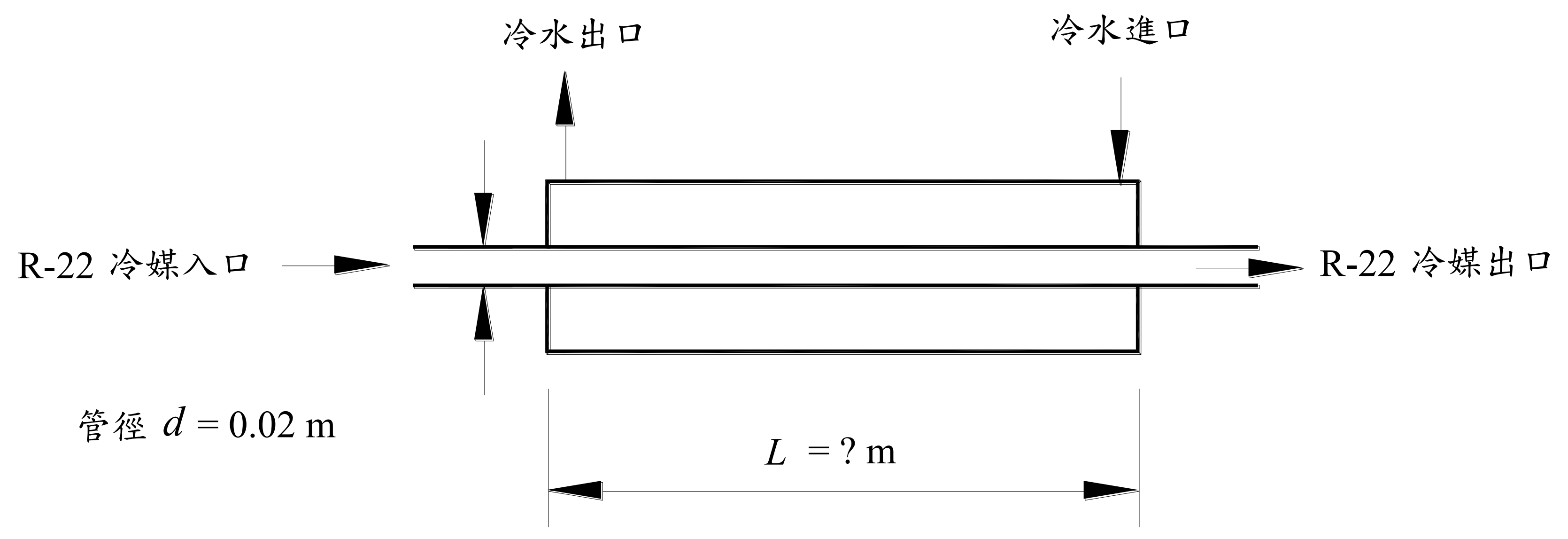
1.由於熱傳量Q為已知，故可算出熱側出口溫度$T_{h,o}$和冷側出口溫度$T_{c,o}$與 $\varepsilon=Q/Q_{max}$ ，同時也算出$C^*=C_{min}/C_{max}$ 。

2.根據已知的 ε-NTU 的關係式，可算出此一設計點的 NTU 值(可能需要疊代)。

3 由$A=NTU\cdot C_{min}/U$，算出所需要的熱交換器面積。

一個Sizing計算實例

例題：雙重管蒸發器，管內為R-22冷媒，其蒸發溫度為 5 °C，環側為冷水，水流量為 0.1 kg/s，水入出口溫度分別為 12 ° 與 7 °C，比熱為 4180 J/kg·K，蒸發器之總熱傳係數*U*為 2000 W/m²·K 。試問雙重管熱交換器的長度要多長才足以滿足此一條件？



A Sizing Example

(1) Solution By *UA-LMTD-F*

$$Q = \dot{m}c_p\Delta T = 0.1\times4180\times(12-7) = 2090\,\text{W}$$

$$UA\cdot LMTD\cdot F = \dot{m}c_p\Delta T$$

$$\Delta T_1 = 12 - 5 = 7\ ℃$$
$$\Delta T_2 = 7 - 5 = 2\ °\text{C}$$

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{7-2}{\ell n\left(\dfrac{7}{2}\right)} = 3.99\,°\text{C}$$

counterflow， $F = 1$

$$\therefore UA\cdot LMTD = \dot{Q}$$

$$A = \dot{Q}/LMTD/U$$

$$= 2090/3.99/2000 = 0.2618\,\text{m}^2$$

$$A = \pi\times0.02\times L \;\Rightarrow L = 4.167\ \text{m}$$

(2) Solution by *ε-NTU*

$$Q = \dot{m}c_p\Delta T = 0.1\times4180\times(12-7) = 2090\ \text{W}$$

$$C_{min} = 0.1\times4180 = 418\,\text{W/K}$$

$$\dot{Q}_{max} = C_{min}\Delta T_{\max} = 418\times(12-5) = 2926\,\text{W}$$

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{2090}{2926} = 0.7143$$ ，Also

$$\varepsilon = 1 - \exp(-NTU) = 0.7143$$

$$\therefore NTU = -\ln(1-\varepsilon) = -\ln(1-0.7143) = 1.253$$

$$NTU = \frac{UA}{C_{min}} \Rightarrow A = \frac{NTU\times C_{min}}{U} = \frac{1.253\times418}{2000} = 0.2618$$

$$\text{A} = \pi\times0.02\times\text{L} \;\Rightarrow\; \text{L} = 4.167\ \text{m}$$

結論：

不管是Rating或Sizing 使用 *UA-LMTD-F* 或 *ε-NTU* 法，其計算結果應相同。

UA-LMTD-F法與 ε-NTU法的比較

1. ε-NTU 法為完全無因次化 。

2. ε 代表能量的效率，ε 較高代表熱交換器的熱傳效率指標較高，F 值則否，較大的 F 值代表流動型式較接近逆向流，並不代表其效率較高。

3.使用 ε-NTU 法時，使用者必須隨時掌握 C_{min} 究竟在熱側或冷側。

4. ε-NTU 圖表較 F 圖表容易查詢。

5.在rating時，使用UA-LMTD-F法需要疊代，ε-NTU 法則不需要。

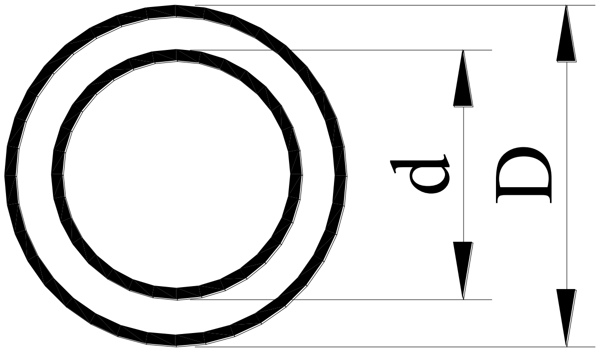
6.在sizing時，UA-LMTD-F較直接，由於熱傳量與進出口溫度均為已知，所以熱交換器面積可以較為迅速的算出，即$A = Q/(UF\Delta T_{lm})$。

換你了…

- Sizing case: (假設管壁厚度為1.5 mm)
  - 同Rating case，但管內外側工作流体為水，入口溫度同樣為12 °C與5°C，管內外水流分別為1 kg/s & 2 kg/s，熱水出口溫度為7°C，假設U 仍為2000 W/m^2·K，則熱傳管長度為何？
  - 如果僅知水量為 0.2 kg/s & 0.4 kg/s，則長度為何？(該如何估算U？)
  - 這個設計好嗎？(從ε-NTU chart)

$D_o = 3$ cm
$d_o = 2$ cm

管壁厚度 = 1.5 mm



END of
the First Talk,
Questions?